Rough convergence and Chebyshev centers in Banach spaces

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Antequera
VI CIDAMA

Introduction and background

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Namely, consider a "roughness index" $r \geq 0$ and a bounded sequence $(x_n)_n$ in a normed space X. Then $(x_n)_n$ is said to r-converge to $y \in X$ if

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Accordingly, the following notation is adopted:

$$\lim{}^r x_n = \{y \in X : \limsup_n \|x_n - y\| \leq r\}$$

and of particular interest is the infimum of the roughness indices such that the rough set is not empty, in other words,

$$\bar{r}(x_n) = \bar{r} = \inf\{r > 0 : \lim{}^r x_n \neq \varnothing\}.$$

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Phu proved that the set $\lim^r x_n$ is bounded, closed and convex for every r. Moreover, he also studied the set $\lim^{\bar{r}} x_n$ and proved that it has empty interior in finite-dimensional spaces and is nonempty in reflexive spaces.

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- Show that the behaviour of this set is closely related to Chebyshev centers.

Definitions and our big mistake

Two new properties (at least, we thought so)

Definitions

Let X be a Banach space. We will say that X has

- *the* **empty rough interior** *(ERI) property if* $\operatorname{int}(\operatorname{lim}^{\bar{r}} x_n) = \varnothing$ *for every bounded sequence* $(x_n)_n \subseteq X$.
- *the* **nonempty rough limit** *(NRL) property if* $\operatorname{lim}^{\bar{r}} x_n \neq \varnothing$ *for every bounded sequence* $(x_n)_n \subseteq X$.

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The concepts of $\lim^{\bar{r}} x_n$, $\bar{r}$ and the *NRL* property were well-known and they are due to M. Edelstein, as follows:

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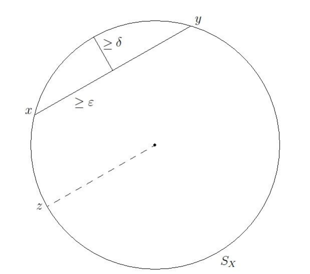

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- *Let $1 \leq p \leq \infty$, X is said to have* **property** *m_p if*

$$\limsup_n \|x + x_n\| = \|(\|x\|, \limsup_n \|x_n\|)\|_p$$

whenever $x_n \xrightarrow{w} 0$ and $x \in X$.

Proposition

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- $\mathcal{C}(K)$ is UR_{ξ_1} for every compact space K, where $\xi_1 : K \to \mathbb{R}$ is the function constantly equal to 1.

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PROPOSITION

Every finite-dimensional space, as well as $\mathcal{C}(K)$ and ℓ_p are sequentially asymptoticaly boundary-center whenever K is a compact space and $1 \leq p \leq \infty$.

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Let us recall that if L is a locally compact, noncompact space then ∞ denotes the only point added in its Alexandroff compactification $\hat{L}$, which is assumed to contain L, and $\mathcal{C}_0(L)$ is the space of continuous functions in L which vanish at infinity.

Proposition

If L is a locally compact, noncompact space such that ∞ has a countable neighbourhood basis in $\hat{L}$ then there exists $(f_n)_n \subseteq S_{\mathcal{C}_0(L)}$ such that $ac(f_n) = B_{\mathcal{C}_0(L)}$. In particular, $\mathcal{C}_0(L)$ is not sabc.

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If X is a Banach space that has the m_∞ property and lacks the Schur property then X is not sabc.

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- *X is said to be* **Chebyshev center complete** *(cc) if every bounded nonempty set has Chebyshev center.*

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Proposition

Let X be a separable Banach space which is also sacc, then X is cc.

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Let X be a separable Banach space. If X is sacc then X is acc.

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$$\textit{acc} \overset{+\text{ separable}}{\Longleftrightarrow} \textit{sacc}$$

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Referencias

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