Topics in Complex Analysis and Operator Theory
Proceedings of the Winter School held in Antequera, Málaga, Spain (February 5–9, 2006)
Pages 153–154

INTERPOLATING SEQUENCES FOR WEIGHTED SPACES OF ENTIRE FUNCTIONS

JOSÉ BONET

Let Φ be a subharmonic function on $\mathbb{C}$. Define the weight $v = v_\Phi$ by $v_\Phi(z) := \exp(-\Phi(z))$, $z \in \mathbb{C}$, and consider the following weighted Banach space of entire functions:

$$Hv(\mathbb{C}) := \{f \in H(\mathbb{C});\ ||f||_v = \sup_{z\in G} v(z)|f(z)| < +\infty\}.$$

We say that a sequence $(z_j)_j$ is *interpolating for* $Hv(\mathbb{C})$ if, for every sequence $(\alpha_j)_j$ of complex numbers such that $sup_j v(z_j)|\alpha_j| < \infty$, there is $g \in Hv(\mathbb{C})$ such that $g(z_j) = \alpha_j$ for each $j \in \mathbb{N}$.

Problem. *Characterize the subharmonic functions Φ on $\mathbb{C}$ such that every discrete sequence in $\mathbb{C}$ contains an interpolating subsequence for $Hv(\mathbb{C}) = Hv_\Phi(\mathbb{C})$.*

Using the work of Marco, Massaneda and Ortega-Cerdà [3], it is proved in [1, Proposition 9] that, if the Laplacian $\mu = \Delta\Phi$ is a doubling measure, then every discrete sequence in $\mathbb{C}$ has a subsequence which is interpolating for $Hv_\Phi(\mathbb{C})$. For example, $\Phi(z) = |z|^\beta(\log(1 + |z|^2))^\alpha$, $\alpha \geq 0$ and $\beta > 0$, yield functions Φ which satisfy this assumption, while $\Phi(z) = \exp|z|$ does not.

Using the natural extension of the definition of the Banach space $Hv(G)$ for a strictly positive continuous weight on an open connected domain G in $\mathbb{C}$, one gets the following positive answer to the problem for other sets G different from the complex plane, and in particular for the open unit disc $\mathbb{D}$.

This result is used in [1] to study compactness of the inclusion between two weighted Banach spaces of the type defined above.

Theorem ([1, Proposition 7]). *Let G be an open connected subset of $\mathbb{C}$ such that, for the Riemann sphere $\mathbb{C}^*$, $\mathbb{C}^* \setminus G$ does not have a connected component consisting of only one point. Let v be a strictly positive, continuous weight on G such that $Hv(G) \neq \{0\}$ and such that there is $C > 0$ such that for every $z \in G$ there is $f \in H(G)$ such that $|f| \leq C/v$ on G and $f(z) = 1/v(z)$. Then every discrete sequence in G contains a subsequence which is interpolating for $Hv(G)$.*

Borichev, Dhuez and Kellay [2] have studied very recently interpolation in radial weighted spaces of entire functions for more general weights than those considered by Marco, Masaneda and Ortega-Cerdà.

References

[1] K.D. Bierstedt and J. Bonet, *Weighted (LB)-spaces of holomorphic functions: $\mathcal{V}H(G) = \mathcal{V}_0H(G)$ and completeness of $\mathcal{V}_0H(G)$*, J. Math. Anal. Appl. **323** (2006), 747–767.

[2] A. Borichev, R. Dhuez, and K. Kellay, *Sampling and interpolation in radial weighted spaces of analytic functions* (2006), Preprint.

[3] N. Marco, X. Massaneda, and J. Ortega-Cerdà, *Interpolating and sampling sequences for entire functions*, Geom. Funct. Anal. **13** (2003), 862–914.

J. Bonet: Universidad Politécnica de Valencia, SPAIN
E-mail address: jbonet@mat.upv.es