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EIGENVALUES AND EIGENFUNCTIONS OF THE LIMIT q-BERNSTEIN OPERATOR

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The Bernstein operator maps $f \in C[0,1]$ to its Bernstein polynomial $B_n f$. The eigenvalues and eigenfunctions of the Bernstein operator on $C[0,1]$ have been described in [1], where the authors also demonstrated various applications of their results. Similar description has been obtained for the q-Bernstein polynomials in [3].

The study of q-Bernstein polynomials in the case $0 < q < 1$ leads to the definition of the *limit q-Bernstein operator*, see [2, 8]. Various properties of this operator have been studied in [5, 7]. A survey of results on the q-Bernstein polynomials and the limit q-Bernstein operator are given in [4, 6].

Let $0 < q < 1$. We denote by $\psi(z)$ the entire function:

$$\psi(z) := \prod_{j=0}^{\infty} (1 - q^j z).$$

Definition. Let $0 < q < 1$. The *limit q-Bernstein operator* on $C[0,1]$ is given by:

$$B_{\infty,q} : f \mapsto B_{\infty,q} f,$$

where

$$(B_{\infty,q} f)(x) = \begin{cases} \psi(x) \sum_{k=0}^{\infty} \frac{f(1-q^k)\, x^k}{(1-q)\dots(1-q^k)}, & x \in [0,1), \\ f(1), & x = 1. \end{cases}$$

Problem. *Find all $f \in C[0,1]$ so that*

$$B_{\infty,q} f = \lambda f, \quad \lambda \in \mathbf{C} \setminus \{0\}.$$

Conjecture. *If* $B_{\infty,q}f = \lambda f$, $\lambda \neq 0$, *then* f *is a polynomial and* $\lambda \in \{q^{m(m-1)/2}\}_{m=0}^{\infty}$.

Remark. The conjecture has been proved under some additional conditions on the smoothness of f at 1 (for example, for $f \in \mathrm{Lip}\,\alpha$) in [5, Corollary 5.6].

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