Variable $\mathbb{C}$ompleja, e$\mathcal{S}p$acios de $\mathcal{F}un$ciones y $\odot$peradores en$\mathbb{T}$re ellos

Problemas de Investigación | $\mathbb{C}^{\mathcal{F}un}_{\mathcal{S}p\odot\mathbb{T}}$ | Research Problems Section

$\mathbb{C}$omplex variables, $\mathcal{F}un$ction $\mathcal{S}p$aces and $\odot$perators between $\mathbb{T}$hem

A PROBLEM ON PARABOLIC FIXED POINTS

WALTER BERGWEILER

Problem. *Let f be rational, with $f(z) = z + a_2z^2 + a_3z^3 + \dots$ near 0, where $a_2 \neq 0$. The (unique) invariant component U of the Fatou set of f where the iterates of f tend to 0 contains at least one critical point of f. If U contains exactly one critical point, then $\mathrm{Re}(a_3/a_2^2) \leq \frac{3}{4}$, as shown by Shishikura* [3], *Buff and Epstein* [2], *and Bergweiler* [1]. *While $a_3/a_2^2 = \frac{3}{4}$ for the Koebe function, Buff and Epstein have given improved estimates for polynomials. What is the sharp upper bound for $\mathrm{Re}(a_3/a_2^2)$, if f is a polynomial? For degree 3 the bound seems to be $\frac{1}{4}$.*

Similar questions may be asked if $a_2 = 0$; see [1, 2].

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