Variable $\mathbb{C}$ompleja, e$\mathcal{S}p$acios de $\mathcal{F}un$ciones y $\odot$peradores en$\mathbb{T}$re ellos

Problemas de Investigación | $\mathbb{C}^{\mathcal{F}un}_{\mathcal{S}p\odot\mathbb{T}}$ | Research Problems Section

$\mathbb{C}$omplex variables, $\mathcal{F}un$ction $\mathcal{S}p$aces and $\odot$perators between $\mathbb{T}$hem

CONVEX UNBOUNDED AND BOUNDED DOMAINS

FILIPPO BRACCI

ABSTRACT. We present one open problem concerning unbounded convex domains in several complex variables.

STATEMENT OF THE PROBLEM

By the Riemann mapping theorem all simply connected domains of $\mathbb{C}$ are biholomorphic either to $\mathbb{C}$ or to the unit disc $\mathbb{D}$.

In particular if D is a convex subset of $\mathbb{C}$ then either $D = \mathbb{C}$ or D is biholomorphic to a bounded convex domain (the unit disc $\mathbb{D}$). Notice that the convex domain D is biholomorphic to the unit disc if and only if it is *hyperbolic*, namely the Poincaré metric of D is a true metric. Therefore we can state the following:

Proposition 1. *Let $D \subset \mathbb{C}$ be a (possibly unbounded) convex domain. Then D is biholomorphic to a bounded convex domain if and only if D is hyperbolic.*

In higher dimension the Poincaré metric is replaced by the Kobayashi metric [2], and it is well known that a bounded domain in $\mathbb{C}^n$ is Kobayashi hyperbolic. It is also possible to show that a (possibly unbounded) convex domain in $\mathbb{C}^n$ is Kobayashi hyperbolic if and only if it is biholomorphic to a bounded domain [1]. However convexity is not a property invariant by biholomorphism and, in fact, the bounded domain image of the hyperbolic unbounded convex domain by means of the biholomorphism constructed in [1] is not convex in general. This raises up the following question:

Question 1. *Let $D \subset \mathbb{C}^n$ be a Kobayashi-hyperbolic convex domain. Is it true that D is biholomorphic to a bounded convex domain?*

2000 *Mathematics Subject Classification.* Primary 32Q45 Secondary 32A25; 52A20.

Key words and phrases. holomorphic functions, convex domains, Kobayashi hyperbolicity.

References

[1] F. Bracci and A. Saracco, *Hyperbolicity in unbounded convex domains*, Forum Math. (2008), to appear.

[2] Shoshichi Kobayashi, *Hyperbolic complex spaces*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 318, Springer-Verlag, Berlin, 1998. MR1635983 (99m:32026)

F. Bracci: Dipartimento Di Matematica, Università di Roma "Tor Vergata", Via Della Ricerca Scientifica 1, 00133, Roma, Italy

E-mail address: fbracci@mat.uniroma2.it