Málaga Economic Theory Research Center Working Papers

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WP 2009-1
May 2009

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Forecasting the Spanish economy with an Augmented VAR-DSGE model*

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Abstract. During the past ten years Dynamic Stochastic General Equilibrium (DSGE) models have become an important tool in quantitative macroeconomics. However, DSGE models was not considered as a forecasting tool until very recently. The objective of this paper is twofold. First, we compare the forecasting ability of a canonical DSGE model for the Spanish economy with other standard econometric techniques. More precisely, we compare out-of-sample forecasts coming from different estimation methods of the DSGE model to the forecasts produced by a VAR and a Bayesian VAR. Second, we propose a new method for combining DSGE and VAR models (Augmented VAR-DSGE) through the expansion of the variable space where the VAR operates with artificial series obtained from a DSGE model. The results indicate that the out-of-sample forecasting performance of the proposed method outperforms all the considered alternatives.

JEL Classification Numbers: C53; E32; E37.

Key words: DSGE models, forecasting, VAR, BVAR.

(*) We would like to thank J. Pérez for useful comments. The authors acknowledge financial support from SEJ-122 and Junta de Andalucía-Proyecto de Excelencia P07-SEJ-02479.

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1 Introduction

Forecasting macroeconomic variables is a crucial issue for both practitioners and policymakers, since decisions of the former are based on the forecasts of key macroeconomic time series made by the later. As pointed out by Litterman (1986) economic forecasting is a very difficult task for several reasons: there is only a limited amount of data, available data present often important measurement errors and relationships among economic variables are complex. However, for both central banks and governments, forecasting is a key element in the design and implementation of economic policies. Therefore, theoretical models must be designed in order to be a useful tool not only for policy analysis but also for forecasting.

Following Diebold (1998) macroeconomics forecasting follows two distinct approaches: structural and non-structural forecasting methods. Whereas non-structural macroeconomic forecasting methods attempt to exploit the reduced-form correlations among macroeconomic variables, structural macroeconomic forecasting is grounded on economic theory. According to Diebold (1998), the failure of large-scale macroeconomic forecasting models and the Lucas' (1976) critique lead to abandon the structural macroeconometrics approach and to the dramatic growth of non-structural econometric forecasting methods in the 1970s. However, despite of its importance, macroeconomic forecasting performance have shown important failures in both, the structural and non-structural approaches.

After the empire of the Box-Jenkins methodology and following Sims (1980) and Litterman (1986) the use of (Bayesian) vector autoregressive (VAR and BVAR) time series to forecast key macroeconomic variables was standard. The advantages of VARs and BVARs are multiple: they are easy to estimate, generate out-of-sample forecasts, and are very flexible, thus VAR models became very popular in the macroeconomists' toolbox. However, they present no (unrestricted VARs) or little (Structural VARs) economic theory.

The alternative approach to purely statistical methods is structural forecasting, using a theoretical based approach. However, traditional Keynesian structural forecasting approach declined in the 1970s due to the effects of the Lucas critique. The resurgence of structural macroeconomic forecasting arose based on the Dynamic Stochastic General Equilibrium (DSGE) modeling developments. Today DSGE models have become the most popular tool in quantitative macroeconomics. During the 1990's and specially in the last ten years, we have assisted to an impressive development in the specification and empirical application of DSGE models, becoming the laboratory for macroeconomists and the standard tool for policy analysis.[1] The

[1]Most central banks and other public institutions have developed recently DSGE models. Representative examples are the Sveriges Riksbank (RAMSES) model, developed by Adolfson, Laséen, Lindé and Villani (2007), the New Area-Wide model (NAWM) developed at the European Central Bank by Christofell, Coenen and Warne (2008), the model developed at the Federal Reserve Board by Edge, Kiley and Laforte (2008), the SIGMA model by Erceg, Guerrieri and Gust (2006), among

DSGE models have a strong theoretical background as they are firmly grounded on modern micro-foundations. In the last few years, DSGE models have increased considerably in complexity, and size, incorporating several types of rigidities emphasized by the New Keynesian literature.[2]

The increasing number and size of the recently developed models account for the success of this methodology. The REMS model by Boscá, Díaz, Doménech, Ferri, Pérez and Puch (2009); the MEDEA model by Burriel, Fernández-Villaverde and Rubio-Ramírez (2009); the BEMOD model by Andrés, Burriel and Estrada (2006) are some of the DSGE developed recently for the Spanish economy.

However, despite of their success DSGE models were not considered a forecasting tool until very recently. As pointed out by Smets and Wouters (2003, 2004), DSGE models were only rarely applied to forecasting. The seminal works of Smets and Wouters (2003, 2004) lead to an emergent literature studying the forecasting performance of DSGE models compared to alternative non-structural models.

In general, DSGE forecasting implies the estimation of an hybrid model that combines theoretical DSGE models with the flexibility of atheoretical VAR models.[3] Different methods for solving, estimating and forecasting with DSGE models have been proposed in the literature: Sargent (1989) and Altug (1989) proposed augmenting a DSGE model with measurement error terms following a first order autoregressive process, known as the DSGE-AR approach. Ireland (2004) proposed a similar method to that of Sargent (1989) and Altug (1989) but imposing no restriction on the measurement errors, assuming that residuals follow a first-order vector autoregression. We will refer to this method as the DSGE-VAR approach.

An alternative, and somewhat different, approach is the one proposed by DeJong, Ingram and Whiteman (1993) and Ingram and Whiteman (1994) and further developed by Del Negro and Schorfheide (2003, 2004). They proposed the use of general equilibrium models as priors for Bayesian VARs. We will refer to this method as the VAR-DSGE approach.[4] DeJong, Ingram and Whiteman (1993) and Ingram and Whiteman (1994) developed a strategy for improving time series forecast by shrinking vector autoregression coefficient estimates given a prior derived from a DSGE model. They showed that a simple DSGE model can improve the forecasting performance of an unrestricted VAR. However, they also reported that the forecasting performance of the VAR-DSGE is similar to that of a Bayesian VAR with the Minnesota prior.

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others.

[2] Examples of these New Keynesian DSGE models are Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005).

[3] See Canova (2002) for a comparison of the quantitive implications of DSGE models with respect to those of unconstrained VAR models.

[4] Some authors, as Del Negro and Shorfheide (2003), call this method as the DSGE-VAR procedure. However, in order to avoid confusion with the method proposed by Ireland (2004), also called DSGE-VAR method in the literature, we will refer to this method as the VAR-DSGE procedure.

The objective of this paper is twofold. First, this paper puts in a forecasting competition the structural approaches we have already refer to, through a small scale DSGE model (DSGE-AR, DSGE-VAR and VAR-DSGE models) versus the standard VAR and BVAR methods. In addition, we propose a new approach consisting of the expansion of the variables space where the VAR operates with the addition of artificial series obtained from a carefully calibrated dynamic general equilibrium model, as an alternative strategy of combining DSGE and VAR models. This new approach is simple, powerful and of easy empirical implementation. When solving a DSGE model, we obtain time series for variables that are relevant to explaining the dynamics of the economy, with a clear theoretical interpretation, but unobservable. If the specification of the model is a good approximation of the underlying relations between the macroeconomic variables we are interested in, those unobserved variables do contain information about the observable macroeconomic variables. Moreover, the VAR not only can be augmented with unobserved variables but also with observed ones, such as the stock of capital. In general, this approach can be interpreted as a new technique to mix structural forecasting methods through DSGE models with standard non-structural forecasting methods as VAR and BVAR models. We will refer to this new procedure as the Augmented (B)VAR-DSGE approach.

The exercise is conducted for the Spanish economy, focusing on forecasting four key macroeconomic variables: output, consumption, investment and labor, for the period 1980:1-2007:4. The ex post forecast errors are evaluated on the basis of the data from the period 1995:1-2007:4. The results indicate that the out-of-sample forecasting performance of the proposed Augmented (B)VAR model outperforms all the considered alternatives. The AVAR model outperforms a unrestricted VAR model and the ABVAR model also outperforms the BVAR model. The only alternative with a similar forecasting performance is the DSGE-AR model. However, the overall results indicate that the out-of-sample forecasting performance of the proposed method outperforms all the considered alternatives.

The structure of the paper is as follows. In section two, we describe the theoretical DSGE model. The estimation of the model is laid out in Section 3. Section 4 describes the alternative method proposed here to combine DSGE and VAR models. Section 5 describes the data, calibration and estimation of the different models. Section 6 presents some statistics to determine relative performance of the different models. Section 7 concludes with final comments.

2 The model

In this section we describe the prototype DSGE model we will use in the rest of the paper. The scale of DSGE models have grown over time, specially in the last ten years by incorporating a large set of New Keynesian elements. However, as pointed out by Diebold (1998) the scale of DSGE models must be as small as possible for two reasons. First, the demise of large-scale macroeconomics models have shown that bigger models are not necessary better. Second, DSGE models requires that their parameters to be jointly estimated, which implies a limitation to the complexity of these models. On the other hand, the final objective of the paper is to compare the forecasting performance of DSGE based approaches with non-structural models. Therefore, for this comparative exercise we consider more suitable the use of a canonical DSGE model.

2.1 Households

Consider a stand-in consumer whose preferences are represented by the following instantaneous utility function:

$$U(C_t, N_t\overline{H} - L_t) = \gamma \log C_t + (1-\gamma)\log(N_t\overline{H} - L_t), \tag{1}$$

Private consumption is denoted by C_t. Leisure, $N_t\overline{H} - L_t$, is calculated as the number of effective hours in the week times the number of weeks in a year $\overline{H}$, times the population at the age of taking labor-leisure decisions, N_t, minus the aggregated number of hours worked in a year L_t. The parameter γ $(0 < \gamma < 1)$ is the proportion of private consumption to total private income. The budget constraint faced by the stand-in consumer is:

$$(1+\tau_t^c)C_t + K_t - K_{t-1} = (1-\tau_t^l)W_t L_t + (1-\tau_t^k)(R_t - \delta)K_{t-1} + T_t, \tag{2}$$

where T_t is the transfer received by consumers from the government, K_t is private capital stock, W_t is the compensation to employees, R_t is the rental rate, δ is the capital depreciation rate which is modelled as tax deductible, and $\tau_t^c, \tau_t^l, \tau_t^k$, are the private consumption tax, the labor income tax, and the capital income tax, respectively. The budget constraint indicates that consumption and investment cannot exceed income (net of taxes) and lump sum transfers.

The stand-in consumer maximizes the value of his lifetime utility given by:

$$\underset{\{C_t,L_t\}_{t=1}^{\infty}}{Max} \; E\left[\sum_{t=1}^{\infty} \beta^{t-1}\left[\gamma \log C_t + (1-\gamma)\log(N_t\overline{H} - L_t)\right]\right]$$

subject to the budget constraint given $\tau_t^c, \tau_t^l, \tau_t^k$ and K_0 and where $\beta \in (0,1)$, is the consumer's discount factor.

2.2 Firms

The problem of the firm is to find optimal values for the utilization of labor and capital. The production of final output, Y_t, requires labor services, L_t, and capital, K_t. Goods and factors markets are assumed to be perfectly competitive. The firm rents capital and hires labor to maximize period profits, taking factor prices as given. The technology exhibits a constant return to private factors and thus the profits are zero in equilibrium. The technology used by the firm is given by:

$$Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{3}$$

where A_t is a measure of total factor productivity and α is the capital share of output. The technology shock A_t is assumed to follow a first-order autoregressive process:

$$\ln(A_t) = (1-\rho)\ln(\overline{A}) + \rho\ln(A_{t-1}) + \varepsilon_t$$

where $\overline{A} > 0$, $\rho < 1$, and $\varepsilon_t \sim N(0, \sigma^2)$.

2.3 Government

The government uses tax revenues to finance spending trough lump-sum transfers paid out to the consumers. We assume that the government balances its budget period-by-period by returning revenues from distortionary taxation to the agents via lump-sum transfers, T_t. The government budget in each period is given by,

$$\tau_t^c C_t + \tau_t^l W_t L_t + \tau_t^k (R_t - \delta) K_{t-1} = T_t. \tag{4}$$

2.4 Equilibrium

The model has implications for six variables: Y_t, C_t, I_t, K_t, L_t, and A_t. The parameters of the model are: α, β, γ, δ, $\overline{A}$, ρ, σ, the three tax rates, τ_t^c, τ_t^l, τ_t^k, and the Lagrange multiplier λ_t. The first-order conditions for the consumer are:

$$\begin{aligned} \gamma\frac{1}{C_t} - \lambda_t(1+\tau_t^c) &= 0, & (5)\\ -(1-\gamma)\frac{1}{N_t\overline{H} - L_t} + \lambda_t(1-\tau_t^l)W_t &= 0, & (6)\\ \beta\left[\lambda_t\left(1 + (1-\tau_t^k)(R_t - \delta)\right)\right] - \lambda_{t-1} &= 0. & (7) \end{aligned}$$

Combining (5) and (6) we obtain the condition that equates the marginal rate of substitution between consumption and leisure, i.e., the opportunity cost in terms of consumption (the numeraire) of an additional hour of leisure:

$$\frac{1-\gamma}{\gamma}\frac{(1+\tau_t^c)C_t}{N_t\overline{H} - L_t} = (1-\tau_t^l)W_{p,t}. \tag{8}$$

Combining (5) and (7) gives

$$\frac{1}{\beta}\frac{C_{t+1}}{C_t} = \left(1 - \tau_{t+1}^k\right)R_{t+1} + (1-\delta), \tag{9}$$

The first-order conditions from the firm's maximization problem are:

$$R_t = \alpha K_{t-1}^{\alpha-1} L_t^{1-\alpha}, \quad (10)$$
$$W_t = (1-\alpha) K_{t-1}^{\alpha} L_t^{-\alpha}, \quad (11)$$

Thus, the economy satisfies the following feasibility constraint:

$$C_t + I_t = R_t K_{t-1} + W_t L_t = Y_t \quad (12)$$

where investment enters in the permanent inventory equation of capital accumulation as,

$$I_t = K_t + (1-\delta) K_{t-1} \quad (13)$$

Together with the first-order conditions of the firm, the budget constraint of the government (4), and the feasibility constraint of the economy, (12), characterize a competitive equilibrium for the economy.

Definition. A competitive equilibrium for this economy is a sequence of consumption, leisure, and investment $\{C_t, N_t\overline{H} - L_t, I_t\}_{t=1}^{\infty}$ for the consumers, a sequence of capital and labor utilization for the firm $\{K_t, L_t\}_{t=1}^{\infty}$, and a sequence of government transfers $\{T_t\}_{t=1}^{\infty}$, such that, given a sequence of prices, $\{W_t, R_t\}_{t=1}^{\infty}$, a fixed tax code $\{\tau^c, \tau^k, \tau^l\}$, and the state at $t = 0$, (K_0, A_0):

i) The optimization problem of the consumer is satisfied.

ii) Given prices for capital and labor, the first-order conditions of the firm are satisfied with respect to capital and labor.

iii) Given a tax code, the sequence of transfers and current spending are such that the government constraint is satisfied.

iv) The feasibility constraint of the economy is satisfied.

3 Solving the DSGE model

Our model has six variables (Y_t, C_t, I_t, L_t, K_t, and A_t) and the equilibrium behavior of the economy is determined by the following six equations:

$$Y_t = A_t K_{t-1} L_t^{1-\alpha} \quad (14)$$

$$Y_t = C_t + I_t \quad (15)$$

$$K_t = (1-\delta) K_{t-1} + I_t \quad (16)$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{1}{C_{t+1}} \left(1 + (1 - \tau^k) \left(\alpha \frac{Y_{t+1}}{K_{t+1}} - \delta \right) \right) \right] \tag{17}$$

$$\frac{1-\gamma}{\gamma} \frac{(1+\tau^c)C_t}{N_t \overline{H} - L_t} = (1 - \tau^l)(1 - \alpha) \frac{Y_t}{L_t} \tag{18}$$

$$\ln A_t = (1 - \rho) \ln(\overline{A}) + \rho \ln A_{t-1} + \varepsilon_t \tag{19}$$

Solving the model using the standard Blanchard-Kahn (1980) procedure, we can characterize approximate solutions to the model by using the standard log-linearization procedures. Defining the vectors $\widehat{\mathbf{x}}_t$ and $\widehat{\mathbf{s}}_t$ as the log-deviation of each variable:

$$\widehat{\mathbf{x}}_t = \begin{bmatrix} \widehat{y}_t \\ \widehat{i}_t \\ \widehat{l}_t \\ \widehat{c}_t \end{bmatrix}, \qquad \widehat{\mathbf{s}}_t = \begin{bmatrix} \widehat{k}_t \\ \widehat{a}_t \end{bmatrix} \tag{20}$$

The approximate solution of the model take the form:

$$\widehat{\mathbf{s}}_t = \Gamma \widehat{\mathbf{s}}_{t-1} + \Xi \varepsilon_t \tag{21}$$

$$\widehat{\mathbf{x}}_t = \Lambda \widehat{\mathbf{s}}_t \tag{22}$$

where the elements of the matrices Γ, Ξ and Λ are function of the structural parameters of the model (α, β, γ, δ, ρ, σ_A).

Note that the VAR representation of the DSGE model suffers from the stochastic singularity problem, as the dimension of the vector of shocks is smaller than of the vector of variables included in the VAR representation. In fact, the model has six variables and only one shock (the aggregate technology shock). The stochastic singularity problem has been solved in the literature using two alternative strategies: by augmenting the number of shocks introducing additional structural disturbances in the DSGE model, or by considering additional measurement errors in the system of equations. We focus on this last alternative.

3.1 The DSGE-AR method

One approach to solve DSGE models is the one proposed by Sargent (1989) and Altug (1989), by augmenting the model with unobservable errors. Following Sargent (1989) and Altug (1989), we add error terms to the observation equation (22). Therefore, we consider the following system:

$$\widehat{\mathbf{s}}_t = \Gamma \widehat{\mathbf{s}}_{t-1} + \Xi \varepsilon_t \tag{23}$$

$$\widehat{\mathbf{x}}_t = \Lambda \widehat{\mathbf{s}}_t + v_t \tag{24}$$

$$v_t = \Theta v_{t-1} + \epsilon_t \tag{25}$$

The matrix Θ is governing the persistence of the residuals, where its covariance matrix, $E_t \epsilon_t \epsilon_t' = V$, is uncorrelated with the innovation to technology, ε_t.

Sargent (1989) and Altug (1989) assume that the measurement errors are uncorrelated with the data generated by the model, and thus, the matrices Θ and V are diagonal, implying that the residual are uncorrelated across variables,

$$\Theta = \begin{bmatrix} \theta_y & 0 & 0 \\ 0 & \theta_c & 0 \\ 0 & 0 & \theta_l \end{bmatrix} \tag{26}$$

$$V = \begin{bmatrix} v_y^2 & 0 & 0 \\ 0 & v_c^2 & 0 \\ 0 & 0 & v_l^2 \end{bmatrix} \tag{27}$$

Therefore, this method combines the DSGE model with an AR model for the measurement residuals. This approach have been applied by Altug (1989), McGrattan (1994), Ireland (2004), among others.

3.2 The DSGE-VAR method

Another possibility, proposed by Ireland (2004), is to consider a more general process for the measurement errors, allowing the residuals to follow an unconstrained, first-order vector autoregression. As Ireland (2004) pointed out, this alternative has the advantage that imposing no restrictions on the cross-correlation of the measurement errors and thus capturing all movements and co-movements in the data not explained by the DSGE model. In this case, the matrices Θ and V take the following form:

$$\Theta = \begin{bmatrix} \theta_y & \theta_{yc} & \theta_{yl} \\ \theta_{cy} & \theta_c & \theta_{cl} \\ \theta_{ly} & \theta_{lc} & \theta_l \end{bmatrix} \tag{28}$$

$$V = \begin{bmatrix} v_y^2 & v_{yc} & v_{yl} \\ v_{cy} & v_c^2 & v_{cl} \\ v_{ly} & v_{lc} & v_l^2 \end{bmatrix} \tag{29}$$

Ireland (2004) compare the forecasting performance of this method to DSGE-AR alternative approach of Sargent (1989), applied to the Hansen (1985) model for the US economy. Despite of the fact that his approach is more flexible and general in the treatment of the measurement errors, he find that the forecast performance of the more restrictive DSGE-AR model outperforms the DSGE-VAR alternative.

3.3 The VAR-DSGE approach

A different strategy is derived from the method proposed by DeJong, Ingram and Whiteman (1993) and Ingram and Whiteman (1994), who use a Bayesian approach to estimate a DSGE model[5]. Ingram and Whiteman (1994) compare the forecasting performance of a BVAR and a DSGE model over an unrestricted VAR to obtain as a result that the DSGE model is comparable to a Bayesian VAR with the Minnesota prior. This method has been further developed by Schorfheide (2000), Del Negro and Schorfheide (2004), Del Negro (2005), Del Negro, Schorfheide, Smets and Wouters (2007) by incorporating prior from a DSGE model. The idea of the VAR-DSGE approach is to use prior information derived from DSGE model in the estimation of a VAR. DSGE models can be used to provide information about the parameters of a VAR. One possibility is to simulate data from the DSGE and to combine it with observed data and estimate a VAR, given a relative weight placed to the prior information. Therefore, this approach use the DSGE models only to set priors to a VAR.

The intuition behind their approach is that a DSGE model can be used to generate artificial data. A VAR can be estimated using observed data and simulated data of the variables, in a certain proportion. That is, they propose estimating a VAR with an augmented data set of the observations and the artificial data generated by the DSGE model. The key parameter of this procedure is the weight placed on the DSGE models as the prior for the VAR, taking values from zero to infinity. If the weight is large, the resulting model will be close to the DSGE model itself and no weight is placed on the unrestricted VAR. If the weight is small, the resulting model will be close to an unrestricted VAR, with no weight on the DSGE model.

Del Negro and Schorfheide (2004) report that for the US economy, the resulting model out-of-sample forecasting outperforms a VAR. These authors show that even a relative simply DSGE model used as a prior for a VAR is able to improve the forecasting performance relative to an unrestricted VAR.

4 A new method: The Augmented (B)VAR approach

In this section we propose an alternative method to the previous ones for taking DSGE models to the data, consisting on the expansion of the dimension of a VAR using as auxiliary variables sequences of artificial data obtained from de DSGE model. The intuition is that a VAR model only has limited information about the underlying dynamics of the variables, as opposed to the rich dynamics with which the DSGE models are built. The procedure we propose tries to exploit that richness by incorporating some of the unobserved variables delivered by the DSGE

[5]The original idea of this procedure is due to Doan, Litterman and Sims (1984) who propose shrinking vector autoregression coefficient estimates toward a prior view that vector times series are well-described as collections of independent random walks.

model. In practice, the procedure consists of estimating a standard VAR model or a Bayesian VAR model augmented with non-observable variables obtained from a DSGE model. In this case, the combination of the unrestricted VAR with the DSGE model is conducted by increasing the dimension of the VAR.[6]

We first explain trough an example how our method applies to the simple DSGE model we are using for comparisons. We later generalize the method to show how it would apply to different, and more general, DSGE models.

In our simple artificial economy there are two sources of exogenous growth: one is given by population growth, and the other by exogenous technical change. We propose a method to separate the effect of these sources of growth using the DSGE model by creating a new variable Z_t, that will be used to expand the dimension of the VAR. To separate the endowment effect from technical change, we first calibrate the model to the level of observed output at the beginning of the sample as a steady state of the model, and assume next that no technical change took place along the sample time period. If, in addition, population where constant, the economy would experience no variations from the calibration date. The sequence that would have been observed starting from the calibration date onwards is shown in Figure 1 as $Y_t(A_0, N_0)$.

A second counterfactual assumption would be to solve the model assuming that only population varies over time. The sequence that would have been observed starting from the calibration date onwards is shown in Figure 1 as $Y_t(A_0, N_t)$. The change in total output can be therefore be regarded as the variation induced by the change in the endowment of labor. The remaining differences from actual output data is the residual, denoted by Z_t and shown in Figure 2. This residual contains the exogenous technical change, and the endogenous capital accumulation and the change in capital labour ratios induced by technical progress.

The ratio between the actual value of output and the no technical progress output value, computed as $Z_t = Y_t/Y_t(A_0, N_t)$, is the technical progress and the induced growth implied by the model. At this point, there are two strategies that we can follow. One is to add the residual Z_t as a variable in a standard VAR, and a different one is to refine further the residual to disentangle how much of Z_t is due to technical change, and how much is due to the capital accumulation induced by it. In the forecasting exercise, we will incorporate the variable Z_t generated from the model as an additional variable in the VAR with the key macroeconomic variables.

In this particular example, we construct a VAR with the following specification:

$$\begin{bmatrix} \mathbf{x}_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \phi_{11}(L) & \phi_{12}(L) \\ \phi_{21}(L) & \phi_{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^z \end{bmatrix} \tag{30}$$

where $\mathbf{x}_t$ are the macroeconomic data that the DSGE model seeks to explain and

[6] Bernanke, Boivin and Eliasz (2005) proposed a similar method using a dynamic factor model to agument the VAR scale, the FAVAR (Factor-augmented vector autoregressive) model.

Z_t is a vector of a subset of the non-observable state or co-state variables derived from the DSGE model. If the model specification is correct, the relation between $\mathbf{x}_t$ and Z_t should then capture additional economic information relevant to model the dynamics of $\mathbf{x}_t$. Standard unrestricted VAR implies that $\phi_{12}(L) = 0$. Therefore, compared with a DSGE model the VAR omits relevant information. Thus, this method takes into account the information provided by the DSGE model by augmenting the dimension of the VAR.

In more general terms, consider a DSGE model to be characterize by a pair $(\mathbb{R}, \Omega)$ describing a set of functional relations $\mathbb{R}$ between the variables of the model, and a set of parameters Ω. Denoting by $\mathbf{x}_t = \{\mathbf{x}_t^o, \mathbf{x}_t^u\}$, the set of observable (upper script o) and unobservable (upper script u) co-state variables and by $\mathbf{z}_t = \{\mathbf{z}_t^o, \mathbf{z}_t^u\}$, the set of observable (upper script o) and unobservable (upper script u) state variables, a DSGE model can be written as $M[\mathbb{R}(\mathbf{x}_t, \mathbf{z}_t), \Omega(\mathbf{x}_t^o, \mathbf{z}_t^o)] = (\mathbf{x}_{mt}, \mathbf{z}_{mt})$, i.e., a function that transforms actual data $(\mathbf{x}_t^o, \mathbf{z}_t^o)$, into modeled data $(\mathbf{x}_{mt}, \mathbf{z}_{mt})$, with the help of a set of functional relations $\mathbb{R}(\mathbf{x}_t, \mathbf{z}_t)$ and a calibrated set of parameters $\Omega(\mathbf{x}_t^o, \mathbf{z}_t^o)$. Once we have solved the model and obtained an estimation of the unobserved variables $\mathbf{q}_{mt}^u = [\mathbf{x}_{mt}^u, \mathbf{z}_{mt}^u]$, we construct a VAR with the following specification:

$$\begin{bmatrix} \mathbf{x}_t^o \\ \mathbf{q}_{mt}^u \end{bmatrix} = \begin{bmatrix} \phi_{11}(L) & \phi_{12}(L) \\ \phi_{21}(L) & \phi_{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{x}_{t-1} \\ \mathbf{z}_{mt-1}^u \end{bmatrix} + \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^z \end{bmatrix} \tag{31}$$

where $\mathbf{q}_{mt}^u$ is a vector of a subset of the non-observable state or co-state variables derived from a DSGE that has been previously calibrated to match some salient features of the data in $\mathbf{x}_t^o$.

In the example illustrated above, we have assumed a counterfactual value for $\mathbf{z}_t$, denoted by $\overline{\mathbf{z}}_t$. Solving the model, $M[\mathbb{R}(\mathbf{x}_t, \overline{\mathbf{z}}_t), \Omega(\mathbf{x}_t^o, \mathbf{z}_t^o)] = (\mathbf{x}_{mt}(\overline{\mathbf{z}}_t), \mathbf{z}_t(\overline{\mathbf{z}}_t))$. Therefore, the model has implications for the observed variables as a function of the counterfactual. By comparing the actual value of the observed variables to those of the model, we can obtain a measure for $\mathbf{q}_{mt}^u$ as a function of the counterfactual value for $\mathbf{z}_t$. Coming back to our particular example, we have solved the DSGE model assuming that, technical progress (the exogenous non-observable state variable) was constant from the date of calibration onwards. The solution of the model provided the level of output that is consistent with that assumption. So, we can obtain a measure of how technical progress would have affected output by comparing actual level of output to that obtained from the model.

5 Data, calibration and estimation

5.1 The data

The analysis focus on four key variables of the Spanish economy: output, consumption, investment and labor. The models are estimated for the Spanish economy

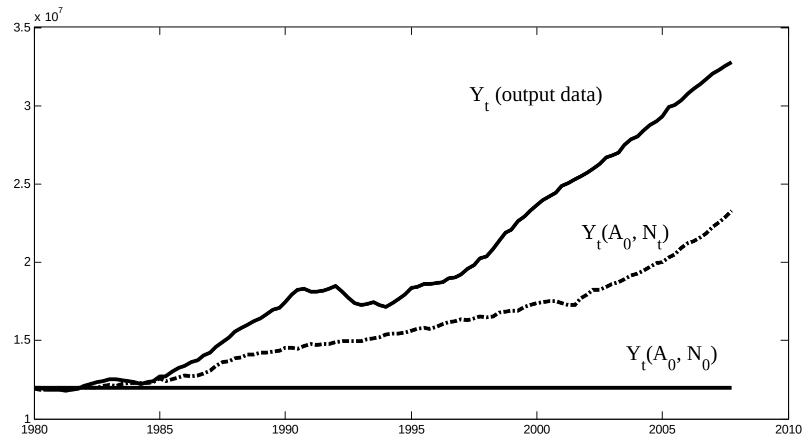

Figure 1: Output with and without technical progress

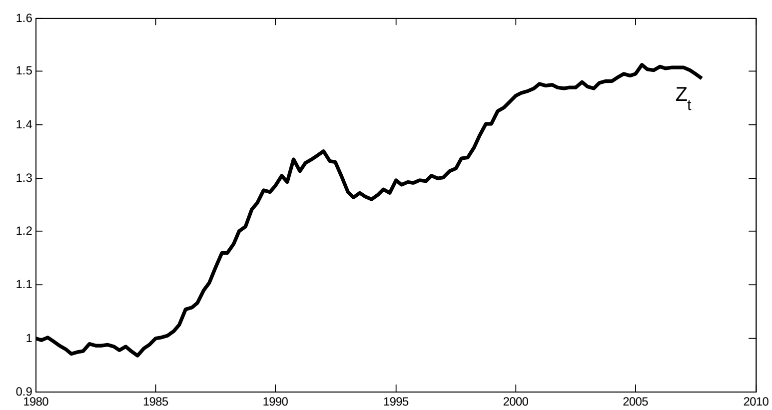

Figure 2: Residual

based on quarterly seasonally-adjusted data on GDP, consumption, investment and hours worked for the period 1980:1-2007:4. Data are taken from the BD-REMS database.[7] Labor is defined as employment full-time equivalents. Data are logged and linearly detrended for all models but the proposed AVAR approach.

5.2 Calibration

Before estimation, we calibrate some parameters of the model. First, both the discount factor and the depreciation rate are fixed as they are difficult to estimate from the model is log-deviations from its steady-state. This is the standard strategy used in the literature, as estimated values lead to unreasonably low estimate of the discount factor and high estimate of the depreciation rate. The inter-temporal discount rate β is set to 0.99, which implies a steady state real interest rate of 4% and the depreciation rate δ is set to 0.025.

Additionally, we keep fix the tax rates. We use effective average tax rates, borrowed from Boscá *et al.* (2005). Table 1 summarizes the values for the calibrated parameters.

Table 1: Calibrated parameters

β	Discount factor	0.99
δ	Depreciation rate	0.025
τ_c	Consumption tax	0.09
τ_l	Labor income tax	0.33
τ_k	Capital income tax	0.22

5.3 Estimation

The rest of the parameters are estimated using Bayesian methods.[8] Fernández-Villaverde and Rubio-Ramírez (2004) argue that Bayesian estimates outperform maximum likelihood results. Therefore, instead of estimate the DSGE model via maximum likelihood, as in Altug (1989), McGrattan (1994) and Ireland (2004), the model is estimated using the Bayesian approach.

Prior distributions for structural parameters have been set by imposing plausible values, whereas for the measurement errors we assume flat priors. Specifically, for non-negative parameters we will assume Inverse Gamma prior distributions. For the parameters α, γ and ρ, we will assume Beta prior distributions in order to keep them bounded between 0 and 1. Finally, for the correlation of measurement residuals added to the model, we will assume Uniform prior distributions with a range of (-2,2).

[7] See Boscá et al. (2007) for a description of the BD-REMS database.

[8] Estimations have been conducted using Dynare 4 running under Matlab R14.

Table 2 summarizes our assumptions regarding prior distributions for the estimated parameters and the posterior distribution corresponding to the DSGE-AR and DSGE-VAR models for the complete sample period. The estimates appears quite reasonable. However, the point estimates for the parameter α are relatively low (0.237 for the DSGE-AR model and 0.207 for the DSGE-VAR model).

Table 2: Prior and Posterior distributions

	Prior distribution			**Posterior distribution**			
				DSGE-AR		DSGE-VAR	
	Distribution	Mean	Std/Range	Mean	Std	Mean	Std
α	Beta	0.35	0.1	0.2378	0.0068	0.2077	0.0059
γ	Beta	0.45	0.1	0.4903	0.0058	0.5066	0.0061
ρ	Beta	0.97	0.01	0.9996	0.0007	0.9998	0.0005
σ_A	Inv. Gamma	0.01	Inf	0.0101	0.0003	0.0079	0.0004
θ_y	Uniform	0	[-2,2]	0.7850	0.086	0.7918	0.2263
θ_c	Uniform	0	[-2,2]	0.5018	0.061	0.9871	0.1557
θ_l	Uniform	0	[-2,2]	0.9993	0.023	0.9540	0.0636
θ_{yc}	Uniform	0	[-2,2]	-	-	-0.0174	0.0027
θ_{yl}	Uniform	0	[-2,2]	-	-	0.0007	0.0001
θ_{cy}	Uniform	0	[-2,2]	-	-	-0.1397	0.0019
θ_{cl}	Uniform	0	[-2,2]	-	-	0.0004	0.0000
θ_{ly}	Uniform	0	[-2,2]	-	-	-0.6540	0.0034
θ_{lc}	Uniform	0	[-2,2]	-	-	-0.6769	0.0046
v_y	Inv. Gamma	0.01	Inf	0.0074	0.0001	0.0018	0.0000
v_c	Inv. Gamma	0.01	Inf	0.0022	0.0000	0.0020	0.0001
v_l	Inv. Gamma	0.01	Inf	0.0068	0.0001	0.0147	0.0023
v_{yc}	Inv. Gamma	0.01	Inf	-	-	0.0076	0.0001
v_{yl}	Inv. Gamma	0.01	Inf	-	-	0.0042	0.0000
v_{cl}	Inv. Gamma	0.01	Inf	-	-	0.0053	0.0001

5.4 VAR models

VARs models are widely used in macroeconomic forecasting. On the other hand, DSGE models are easily represented through a VAR model. For these reasons, VARs models have been extensively used as a benchmark for evaluating forecasting performance of alternative models, in particular, with respect to DSGE models.

One of the main advantage of VAR models is that they can be applied directly to the data, implying the existence of a relationship between each variable and past lagged values of all variables considered in the model. One of the drawbacks of VARs is the problem of overfitting, that results in inefficient estimates and large out-of-sample forecasting errors. Unrestricted VARs models may have too may parameters, and thus, the estimates may be very imprecise, specially in small samples. This is particularly important as we consider longer forecasting horizons, as VAR forecasting performance will deteriorate rapidly. The problem of overfitting with standard VAR can be overcome using Bayesian methods. The Bayesian VAR analysis, developed by Litterman (1981), imposes restrictions on some coefficients by introducing prior information, generating more precise parameter estimates.

In our analysis we estimate VARs and Bayesian VARs for the four macroeconomics variables (output, consumption, investment and labor) of the Spanish economy contained in the DSGE model as a benchmark. Our basic VAR model include a constant and a trend term.

6 Forecast evaluation

This section analyses the out-of-sample performance of the competing models over the four key macroeconomic variables for the Spanish economy. There a relative large number of recent papers that compare the forecasting performance of DSGE and VAR models as Smets and Wouters (2004), Ireland (2004), Del Negro et al. (2005), Adolfson et al (2007), Christoffel et al. (2007), Rubaszek and Skrzypczynski (2008), Ghent (2009), among others. In general, they obtain that the use of DSGE models improve forecasting performance compared with VAR methods.

We report the out-of-sample forecast performance of seven different alternative models: VAR, BVAR, DSGE-AR, DSGE-VAR, VAR-DSGE, AVAR and ABVAR models. The out-of-sample forecast analysis is performed for horizons ranging from one up to eight quarters ahead. The forecast accuracy evaluation period is 1995:1-2007:4. Therefore, all the models are estimated initially over the first 60 periods (1980:1 through 1994:4). These estimations are used to generate forecast for the period 1995:1-1996:4. The model is then re-estimated over 61 periods, incorporating one additional observations, 1980:1-1995:1, and the forecasts are recalculated for the period 1995:2-1997:1, and so on until the end of the sample period. This procedure is repeated quarter to quarter. This procedure implies that the one period ahead forecast is calculated from 1995:1 to 2007:4. The two period ahead forecast is calculated for the period 1995:2-2007:4, and so on. Therefore, each model is re-estimated 52 times.

The forecasting performance of the competing models is evaluated along two dimensions: the bias in errors and the absolute size of errors. The bias in errors is measured by the mean absolute error (MAE), while the absolute size of errors is measured by the root-mean squared error (RMSE). Table 3 summarizes the results for the MAE statistics for the bias errors. The best results are generated by the DSGE-AR and the ABVAR models. For all the four variables, the ABVAR model outperforms all other alternatives for one period ahead forecast. The ABVAR model is also superior forecasting investment in all periods. However, for the other three variables, output, consumption and labor, the forecasting performance of the DSGE-

AR model outperforms those of the other alternatives with the exception of one period ahead where ABVAR is superior. In general, we obtain that the results from the DSGE-AR model and the ABVAR model are comparable, being the alternatives producing more accurate forecasts.

Table 3: Forecasting MAE

	Periods ahead							
	1	2	3	4	5	6	7	8
Output								
VAR	0.607	1.255	1.950	2.675	3.408	4.132	4.830	5.486
BVAR	0.155	0.305	0.452	0.594	0.734	0.871	0.983	1.085
DSGE-AR	0.154	**0.256**	**0.347**	**0.437**	**0.527**	**0.612**	**0.681**	**0.748**
DSGE-VAR	0.320	1.829	2.841	3.604	4.105	4.497	4.476	4.467
VAR-DSGE	1.530	1.614	2.270	3.309	4.454	5.617	5.653	5.690
AVAR	0.553	0.517	1.216	1.513	2.167	2.914	3.759	4.676
ABVAR	**0.120**	0.268	0.409	0.547	0.685	0.817	0.930	1.040
Consumption								
VAR	0.499	1.024	1.581	2.163	2.763	3.365	3.954	4.523
BVAR	0.136	0.267	0.398	0.531	0.670	0.805	0.902	0.993
DSGE-AR	0.229	0.287	0.431	**0.482**	**0.605**	**0.660**	**0.754**	**0.835**
DSGE-VAR	0.179	1.831	2.867	3.561	3.980	4.283	4.277	4.284
VAR-DSGE	1.381	1.649	1.976	2.389	2.845	3.325	3.323	3.325
AVAR	0.496	0.358	0.718	0.850	1.281	1.761	2.295	2.887
ABVAR	**0.109**	**0.243**	**0.375**	0.509	0.644	0.778	0.877	0.971
Investment								
VAR	0.415	0.808	1.181	1.539	1.898	2.244	2.586	3.022
BVAR	0.130	0.252	0.358	0.458	0.564	0.668	0.769	0.864
DSGE-AR	1.091	1.070	1.083	1.085	1.081	1.116	1.138	1.172
DSGE-VAR	1.282	1.339	1.435	1.508	1.612	1.688	1.721	1.751
VAR-DSGE	1.328	1.067	1.367	1.929	2.593	3.233	3.236	3.231
AVAR	0.440	1.579	2.923	4.013	5.177	6.326	7.481	8.658
ABVAR	**0.092**	**0.210**	**0.317**	**0.420**	**0.527**	**0.632**	**0.731**	**0.826**
Labor								
VAR	0.735	1.513	2.324	3.154	3.990	4.831	5.667	6.481
BVAR	0.215	0.432	0.654	0.879	1.107	1.334	1.502	1.666
DSGE-AR	0.196	**0.375**	**0.540**	**0.693**	**0.833**	**0.966**	**1.098**	**1.231**
DSGE-VAR	8.284	9.755	10.440	10.692	10.928	11.112	10.951	10.795
VAR-DSGE	18.059	18.433	18.775	19.089	19.376	19.640	19.756	19.870
AVAR	0.712	0.666	0.845	0.878	1.025	1.224	1.483	1.785
ABVAR	**0.179**	0.397	0.618	0.841	1.066	1.294	1.462	1.626

* Numbers in boldface indicate the lower MAE.

Table 4 summarizes the results in terms of the root-mean squared error. The results are similar to those obtained by the MAE statistic. First, we obtain that the forecasting performance of an AVAR is superior to that of an unrestricted VAR for all the variables and for all forecast horizons, with the only exception of investment, where this relation is inverted. More remarkable is the forecasting performance absolute superiority of the ABVAR model compared to the BVAR model. Note that the Augmented (B)VAR models nest standard (B)VAR models, so they are directly comparable. This comparison reveals that augmenting the number of variables in the VAR with non-observable variables obtained from a DSGE model incorporates useful information for forecasting. For output, the best alternative is the DSGE-AR for longer horizons while for one period ahead the ABVAR alternative produces better results. For consumption the results are mixed, as the DSGE-AR model and the ABVAR model produces very similar RMSE estimates, being the lowest those produce by ABVAR from one to three periods ahead forecast. For investment, again the ABVAR model is the best alternative for all horizons. For employment the forecasting performance of the AVAR model outperforms any other DSGE-based alternative and it is only challenged by the BVAR model forecasts for one and two periods ahead. DSGE-VAR and VAR-DSGE models display relatively large RMSE values, specially for output and labor. It is noticeable than the forecasting performance of the DSGE-VAR is worse than that of the DSGE-AR. Ireland (2004) also obtains a similar result when comparing the forecasting performance of both alternatives estimated by maximum likelihood.

From the results summarizes in table 3 and 4 we can highlight the following preliminary conclusions. First, we obtain that BVAR methods are superior to an unrestricted VAR for all horizons, confirming previous analysis. Second, RMSE increases at a higher speed in the case of a VAR compared to the BVAR, This implies that Bayesian methods are clearly superior for forecasting at longer horizons. Third, the DSGE-AR model outperforms the DSGE-VAR model, confirming the results obtained by Ireland (2004). Finally, the results show that the method proposed in this paper, the Augmented VAR model, can be very useful in macroeconomic forecasting, being superior to the alternatives. The AVAR model outperforms the VAR and the ABVAR model outperforms the BVAR for all cases. The only alternative with a similar forecasting performance accuracy is the DSGE-AR model. The analysis conducted in this paper confirms previous results, highlighting that DSGE models with a deep theoretical background, combined with the flexibility of the standard VAR approach, can be a very useful tool in macroeconomic forecasting.

Table 4: Forecasting RMSE

	Periods ahead							
	1	2	3	4	5	6	7	8
				Output				
VAR	0.674	1.386	2.127	2.883	3.722	4.488	5.229	5.931
BVAR	0.252	0.532	0.816	1.098	1.379	1.654	1.864	2.056
DSGE-AR	0.435	0.628	**0.773**	**0.898**	**1.022**	**1.145**	**1.249**	**1.356**
DSGE-VAR	1.662	4.501	6.722	8.349	9.394	10.230	10.122	10.032
VAR-DSGE	2.897	2.947	4.399	6.228	8.092	9.915	9.930	9.952
AVAR	0.627	0.681	0.946	1.904	2.702	3.599	4.586	5.658
ABVAR	**0.249**	**0.515**	0.780	1.044	1.302	1.550	1.755	1.944
				Consumption				
VAR	0.551	1.121	1.708	2.311	3.004	3.636	4.259	4.861
BVAR	**0.229**	0.482	0.735	0.987	1.238	1.488	1.666	1.834
DSGE-AR	0.733	0.585	0.964	**0.932**	**1.213**	**1.225**	**1.369**	**1.503**
DSGE-VAR	0.536	4.016	6.132	7.507	8.228	8.732	8.634	8.554
VAR-DSGE	2.847	3.310	3.823	4.409	5.054	5.746	5.765	5.789
AVAR	0.555	**0.440**	**0.475**	1.149	1.650	2.214	2.836	3.519
ABVAR	0.233	0.483	0.731	0.978	1.222	1.461	1.637	1.804
				Investment				
VAR	0.600	1.128	1.586	1.997	2.377	2.811	3.256	3.723
BVAR	0.198	0.399	0.592	0.768	0.938	1.104	1.279	1.451
DSGE-AR	1.907	1.920	1.958	1.974	1.988	2.030	2.047	2.062
DSGE-VAR	2.437	2.504	2.945	3.359	3.773	4.114	4.091	4.077
VAR-DSGE	2.751	2.138	2.657	2.664	4.722	5.710	5.668	5.637
AVAR	0.564	1.699	2.817	4.122	5.300	6.478	7.678	8.918
ABVAR	**0.192**	**0.388**	**0.581**	**0.757**	**0.925**	**1.094**	**1.268**	**1.441**
				Labor				
VAR	0.775	1.585	2.416	3.257	4.206	5.064	5.913	6.742
BVAR	**0.320**	**0.683**	1.050	1.420	1.794	2.170	2.441	2.705
DSGE-AR	0.583	0.982	1.297	1.569	1.812	2.037	2.195	2.368
DSGE-VAR	15.682	17.997	18.919	19.149	19.390	19.557	19.269	18.997
VAR-DSGE	28.119	28.510	28.855	29.157	29.422	29.652	29.668	29.684
AVAR	0.771	0.762	**0.757**	**1.004**	**1.234**	**1.520**	**1.850**	**2.224**
ABVAR	0.329	0.685	1.046	1.409	1.773	2.136	2.406	2.669

* Numbers in boldface indicate the lower RMSE.

7 Conclusions

In this paper we propose a new approach to combining DSGE and VAR models. The proposed method is different from existing methods and consists in augmenting the space of the VAR with non-observables variables artificially generated by a DSGE model. The intuition behind our proposal is that DSGE models contain additional information about the underlying dynamics of actual data and that this information can be incorporated in an otherwise standard VAR model.

The results obtained from the forecasting exercise conducted for the Spanish economy show that the proposed Augmented (B)VAR model is superior to the alternative approaches in forecasting key macroeconomic variables of the economy being only challenged by the DSGE-AR model. We want to stress the fact that our method establishes a metric with a DSGE model can be directly compared to non-structural methods. Given that A(B)VAR models encompass the (B)VAR counterparts, we can attribute all the gains in forecasting accuracy to the artificial variables obtained from the DSGE model. What we have shown in the paper is just a simple example of what could be done with a larger scale DSGE model. There it would be possible to asset the relative improvement in forecasting accuracy gained from augmenting the VAR with the additional variables derived from the larger scale DSGE model.

All the analysis conducted in the paper have been done using a very simple DSGE model. The election of the small scale of the model is not casual and serves to check if even a simple DSGE model specification can be useful as a forecasting tool. A natural extension to our work would be to consider richer DSGE models, in order to asses how the forecasting performance of DSGE models is related to the their scale. Finally, our analysis supports the view, consistently with previous literature, that DSGE models are useful to policymakers for forecasting macroeconomic variables.

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