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Abstract

We analyze the problem of choosing the w contestants who will win a competition within a group of $n > w$ competitors when all jurors commonly observe who are the w best contestants but may be biased. We study conditions on the configuration of the jury so that it is possible to induce the jurors to always choose the best contestants, whoever they are. If the equilibrium concept used by the jurors is dominant strategies, the necessary and sufficient conditions incorporate very strong informational requirements on the mechanism designer. If we relax the equilibrium concept to Nash or subgame perfect equilibria the necessary and sufficient conditions are less demanding. Moreover, these conditions are also necessary for any other equilibrium concept. Finally, we study one specific application: we propose a simple and natural mechanism for the case where each juror is biased in favor of one and only one (different) contestant.

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1 Introduction

A group of $n \geq 3$ contestants are involved in a competition. A jury must choose a subset of $w < n$ contestants who will win the competition. All jurors know who are the w best contestants. We call this group "true subset of winners". Each juror, however, may be biased in favor of/against some contestants (*i.e.*, the juror always prefers some contestants to be/not to be among the chosen winners, whatever the true subset of winners is).

Examples of this situation are very common. Think, for instance, of the Olympic Games host selection. The candidate cities are the contestants and the members of the International Olympic Committee (IOC) are the jurors. The IOC has to choose the city where the next Olympic Games will be held (*i.e.*, $w = 1$). Suppose that all the members of the IOC know that, if the decision was taken purely on the quality and merit of the candidatures, then "city a" should be the chosen one. Some members of the IOC, however, might be biased in favor of/against certain candidatures. There are different reasons why this could happen, ranging from nationality to politics. The biased jurors will try to favor one candidature over another, regardless of which is the best one. A different example is the selection process for hiring civil servants (like judges, professors, etc.) in some countries. A commission has to choose the w candidates who will be hired between a number of applicants. Rather than administering a merit-based selection process for hiring the employees, some of the members of the commission might try to reward political allies with the posts.

In many cases, the problem is that only biased jurors have the relevant information about who are the best contestants. Consider an open tender to build a public infrastructure. Only expert engineers, who work in the subject area and probably have connections with some of the firms that tender for the contract, know which is the best design of the public construction. This is the reason why fair jurors are sometimes useless: they are ignorant about the truth. Fortunately, the fact that the jurors are biased and look for their own interests does not necessarily imply that the decision of the jury will be unfair. Sometimes, when individuals pursue their self-interests, they promote the good of the society. This has been a main topic in economics since the days of Adam Smith and is the real point of the theory of implementation. Of course, in order to be possible to induce the jurors to choose the best contestants, there must be some limits on their self-interests (for instance, if all jurors are biased against the same contestant, he will never be chosen as one of the winners, no matter how good he is). This is precisely one of the main objectives of this paper: to provide restrictions on the configuration of the jury so that it is possible to induce the jurors to always choose the "true subset of winners", whoever they are. For that we use the theory of implementation.

The socially optimal rule is E-implementable if, under the assumption that the jurors take their decisions according to the E equilibrium concept, there exists a mechanism that induces them to always choose the true subset of winners, whatever it is. We study restrictions on the configuration of the jury so that the socially optimal rule is E-implementable. For that, we introduce the notion of being fair with respect to a group of contestants. If a juror is fair with respect to contestants a and b then, when comparing any two subsets of winners which only differ in a and b, if a is in the true subset of winners but b is not, the juror prefers the subset of winners that includes a rather than the subset of winners that includes b.

We first provide necessary conditions for the E-implementability of the social choice rule, whatever equilibrium concept E the jurors use. Propositions 1 and 2 state that, if the socially optimal rule is E-implementable in some equilibrium concept E then, for every two contestants, the social planner must know at least one juror who is fair with respect to them. Whether these conditions are fulfilled or not depends on the specific application being considered. It is important to note, however, the implications of their not being met. If the conditions fail the social planner will not be able to induce the jurors to always chose the best contestants, no matter how the jurors behave.

We focus next on Nash and subgame perfect implementation. Proposition 3 shows that the previous conditions not only are necessary for the E-implementability of the socially optimal rule in any equilibrium concept E, but they are also sufficient when the equilibrium concept used by the jurors is Nash equilibria and there are at least three jurors. The proof of this result is constructive. We propose a mechanism "a la Maskin" that does the job (in contrast to the "canonical mechanism for Nash implementation" proposed by Maskin, 1999, in our mechanism the jurors do not have to announce the state of the world). Moreover, since this mechanism also implements the socially optimal rule in subgame perfect equilibria, an immediate corollary is that the necessary conditions stated in Propositions 1 and 2 are sufficient for subgame perfect implementation of the socially optimal rule as well.

We also study implementation in dominant strategies. Not surprisingly, the conditions for the implementability of the socially optimal rule are much stronger under this equilibrium concept. Proposition 4 shows that if the socially optimal rule is implementable in dominant strategies then there must be some juror who is fair with respect to all contestants. In addition, Proposition 5 states that the social planner must know who this juror is. Obviously, this conditions are also sufficient: the trivial mechanism where the juror who is fair with respect to all contestants chooses his favorite subset of winners implements the socially optimal rule in dominant strategies. The latter necessary and sufficient conditions involve the strongest informational requirements that one can imagine and actually prevent implementation in dominant strategies.

The conditions for the implementability of the social choice rule in Nash and subgame perfect equilibria tell us the minimum degree of impartiality that we must require of the jury, since these conditions are also necessary for any other equilibrium concept. The mechanism proposed in the proof of Proposition 3, however, is quite abstract. The purpose of this mechanism is the characterization of what can be implemented and therefore it has to handle a large number of different situations. More realistic mechanisms can be constructed when one deals with a specific application where the social planner has more information about certain aspects of jurors' preferences. We study one of these situations: the case where each juror is biased in favor of one (and only one) different contestant and is fair with respect to the rest (this may be the case, for example, when the contestants themselves are the jurors). We propose a simple and natural dynamic mechanism that implements the socially optimal rule in subgame perfect equilibria in this situation.

Our results bear a resemblance to those in Amorós et al. (2002) and Amorós (2009), who also consider the problem of eliciting the "truth" from a group of partial jurors. In the model analyzed in these works, however, alternatives are rankings of all contestants instead of subsets of winners. This makes the problem different from that studied in the present paper. Our paper is also related to the literature on information transmission between multiple informed experts and an uninformed decision maker. Austen-Smith (1993) assumes that the decision maker gets advice from two biased and imperfectly informed experts, and compares simultaneous and sequential reporting. Krishna and Morgan (2001) analyze a situation in which two experts observe the same information, but they differ in their preferences, and show that if both experts are biased in the same direction there is no equilibrium in which full revelation occurs. Wolinsky (2002) analyzes a model where the experts share the same preferences, which differ from those of the decision maker, and possess different pieces of information. Gerardi et al. (2009) investigate how the decision maker can extract information from the experts by distorting the decisions that will be taken and show that when the number of informed agents become large one can extract the information at small cost (their focus, however, is not full implementation in the sense that they do not require that all equilibria implement the social choice rule). Finally, our paper is connected with the literature on strategic voting (e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996; 1997; Duggan and Martinelli, 2001; Martinelli, 2002). The problem studied in these papers, however, is different from the problem studied here: the jurors only have to choose to convict or acquit a defendant.

The paper is organized as follows: Section 2 provides definitions, Section 3 states general necessary conditions for implementation, Section 4 analyzes Nash and subgame perfect implementation, Section 5 studies implementation in dominant strategies, Section 6 analyzes the case in which each juror is biased in favor of one contestant, and Section 7 provides the conclusions. The Appendix shows that the set of admissible profiles of preference relations may not have a Cartesian product structure and that the socially optimal rule cannot be impelemented via some "natural" mechanisms.

2 The model

Consider a set N of $n \geq 3$ contestants who are involved in a competition. A group of jurors J must choose a subset of w contestants who will win the competition, where $0 < w < n$. Let 2^N_w denote the set of all subsets of N of size w. All jurors know who are the w best contestants. We call this group **true subset of winners**, $W_t \in 2^N_w$. The **socially optimal rule** is that the winners finally selected are the true winners.

Jurors have **preferences** defined over 2^N_w. The preferences of a juror may depend on the true subset of winners. For example, if $N = \{a, b, c, d\}$ and $w = 2$, a juror $i \in J$ may prefer $W = \{a, b\}$ to $\hat{W} = \{a, c\}$ if the true subset of winners is $W_t = \{b, d\}$, but prefer $\hat{W} = \{a, c\}$ to $W = \{a, b\}$ if the true subset of winners is $W_t = \{c, d\}$. However, the preferences of a juror may also depend on "external factors". For instance, contestant a might be a "friend" of juror i, so that this juror always prefers a being one of the winners. In this case, for example, juror i might prefer $W = \{a, b\}$ to $\tilde{W} = \{b, c\}$ whatever the true subset of winners is.[1]

Let $\Re$ be the class of preference orderings defined over 2^N_w. Each juror $i \in J$ has a **preference function** $R_i : 2^N_w \longrightarrow \Re$ which associates with each feasible true subset of winners, $W_t \in 2^N_w$, a preference relation $R_i(W_t) \in \Re$. Let $P_i(W_t)$ denote the strict part of $R_i(W_t)$. Let $\mathcal{R}$ denote the class of all possible preference functions.

We say that juror i is **fair** with respect to the contestants in a set $F_i \subset N$ (with $|F_i| \geq 2$) if, for every $a, b \in F_i$, when comparing any two subsets of winners $W, \hat{W} \in 2^N_w$ which only differ in that a is in W but not in $\hat{W}$ and b is in $\hat{W}$ but not in W, if a is in the true subset of winners but b is not, then juror i strictly prefers W to $\hat{W}$.

Definition 1 *Given the set of contestants with respect to whom juror i is fair, F_i, the* ***preference function*** *$R_i \in \mathcal{R}$ is* ***admissible*** *for i if for all $a, b \in F_i$, all $W_t \in 2^N_w$ and all $W, \hat{W} \in 2^N_w$ with:*
(i) $a \in W$,
(ii) $b \in \hat{W}$,
(iii) $W \backslash \{a\} = \hat{W} \backslash \{b\}$
(iv) $a \in W_t$, and
(v) $b \notin W_t$,
we have $W P_i(W_t) \hat{W}$.

Let $\mathcal{R}(F_i) \subset \mathcal{R}$ denote the class of admissible preference functions for juror i when the set of contestants with respect to whom i is fair is F_i. Next, we provide an example that illustrates these notions.

Example 1 *Let $N = \{a, b, c, d\}$, $w = 3$ and $i \in J$. Then $2^N_w = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Suppose that $F_i = \{a, b\}$. Then, any admissible preference function for i, $R_i \in \mathcal{R}(F_i)$, is such that if the true subset of winners is $W_t = \{a, c, d\}$, then $\{a, c, d\} P_i(W_t) \{b, c, d\}$. To see this note that the only two contestants who change their winner status between the two alternatives are a and b, both contestants are in F_i and, while a is in the true subset of winners, b is not. The concept of being fair with respect to the contestants in F_i, however, is weakly enough not to impose any other restriction on the preference relation $R_i(W_t)$. For example, when*

[1] Similarly, a could be an "enemy" of juror i, so that he always prefers $\tilde{W} = \{b, c\}$ to $W = \{a, b\}$, whatever the true subset of winners is.

comparing $\{a,b,c\}$ with $\{a,c,d\}$, the only two contestants who change their winner status are b and d. Since $d \notin F_i$, for all $\hat{W}_t \in 2_w^N$ both possibilities, $\{a,b,c\}R_i(\hat{W}_t)\{a,c,d\}$ and $\{a,c,d\}R_i(\hat{W}_t)\{a,b,c\}$, are admissible (something similar happens when comparing any two possible subsets of winners different from $\{a,c,d\}$ and $\{b,c,d\}$). Similarly, it is easy to see that, if the true subset of winners is $\tilde{W}_t = \{b,c,d\}$, then $\{b,c,d\}P_i(\tilde{W}_t)\{a,c,d\}$. Finally, if $\bar{W}_t \in \{\{a,b,c\},\{a,b,d\}\}$, the preference relation $R_i(\bar{W}_t)$ does not need to fulfill any special requirement since in this case every contestant in F_i is in the true subset of winners as well. Table I summarizes these restrictions.

Table I. The case in which $n = 4$, $w = 3$ and $F_i = \{a,b\}$.

$R_i : 2_w^N \longrightarrow \Re$

$W_t = \{a,b,c\}$	$W_t = \{a,b,d\}$	$W_t = \{a,c,d\}$	$W_t = \{b,c,d\}$
No restriction on $R_i(W_t)$	No restriction on $R_i(W_t)$	The only restriction on $R_i(W_t)$ is $\{a,c,d\}$ $P_i(W_t)\{b,c,d\}$	The only restriction on $R_i(W_t)$ is $\{b,c,d\}$ $P_i(W_t)\{a,c,d\}$

Let $F = (F_i)_{i\in J}$ denote a profile of sets of contestants with respect to whom the jurors are fair (one set for each juror). Given F, a **profile of preference relations is admissible** if there exists a profile of admissible preference functions $R = (R_i)_{i\in J} \in \times_{i\in J}\mathcal{R}(F_i)$ and a possible subset of winners $W_t \in 2_w^N$ such that $(R_i(W_t))_{i\in J}$ coincides with these preference relations. Let $\Re(F) \subset \Re^{|J|}$ denote the set of admissible profiles of preference relations when the profile of sets of contestants with respect to whom the jurors are fair is F. In the Appendix we provide an example that shows that the set $\Re(F)$ may not have a Cartesian product structure.

Given $F = (F_i)_{i\in J}$, a **state of the world** is a pair (R, W_t), where $R = (R_i)_{i\in J} \in \times_{i\in J}\mathcal{R}(F_i)$ is a profile of admissible preference functions and W_t is the true subset of winners observed by all jurors. Let $S(F) = \times_{i\in J}\mathcal{R}(F_i) \times 2_w^N$ be the set of admissible states of the world when the profile of sets of contestants with respect to whom the jurors are fair is F.

A **mechanism** is denoted $\Gamma = (M, g)$, where $M = \times_{i \in J} M_i$, M_i is a message space for juror i, and $g : M \to 2^N_w$ is an outcome function.[2] Given a mechanism and a state of the world, the jurors must decide the messages that they announce. Let E equilibrium be a game theoretic solution concept. For each mechanism Γ and each state of the world $(R, W_t) \in S(F)$, let $E(\Gamma, R, W_t) \subset M$ denote the set of profiles of messages that are an E equilibrium of Γ when the state of the world is (R, W_t). For example, $m \in M$ is a **dominant strategy equilibrium** of $\Gamma = (M, g)$ at $(R, W_t) \in S(F)$ if $g(m_i, \hat{m}_{-i}) R_i(W_t)\ g(\hat{m}_i, \hat{m}_{-i})$ for all $i \in J$, all $\hat{m}_i \in M_i$, and all $\hat{m}_{-i} \in M_{-i}$. Similarly, $m \in M$ is a **Nash equilibrium** of $\Gamma = (M, g)$ at $(R, W_t) \in S(F)$ if $g(m) R_i(W_t)\ g(\hat{m}_i, m_{-i})$ for all $i \in J$ and all $\hat{m}_i \in M_i$. Let $D(\Gamma, R, W_t)$ and $N(\Gamma, R, W_t)$ denote the sets of dominant strategy and Nash equilibria of Γ at (R, W_t), respectively.

Given $F = (F_i)_{i \in J}$, a mechanism implements the socially optimal rule if, in equilibrium, the true subset of winners is selected in each state of the world. We call this notion implementation of the socially optimal rule.

Definition 2 *Given the profile of sets of contestants with respect to whom the jurors are fair $F = (F_i)_{i \in J}$, the mechanism $\Gamma = (M, g)$* **E** ***implements*** *the socially optimal rule when, for all $(R, W_t) \in S(F)$:*

(1) There exists $m \in E(\Gamma, R, W_t)$ such that $g(m) = W_t$.

(2) If $m \in M$ is such that $g(m) \neq W_t$, then $m \notin E(\Gamma, R, W_t)$.

If such a mechanism exists then the socially optimal rule is E-implementable.

3 Necessary conditions for implementation

Our aim in this section is to study restrictions on the configuration of the jury so that the social planner is able to induce its members to select the true subset of winners, whatever it is. More specifically, we study which conditions must satisfy the profile of sets of contestants with respect to whom the jurors are fair $F = (F_i)_{i \in J}$, in order to allow the implementability of the socially optimal rule.

There are some profiles F for which implementation of the socially optimal rule is not possible, whatever equilibrium concept the jurors use. Suppose for example that it is admissible a situation in which all jurors want to favor

[2] This kind of mechanism is sometimes called "normal form mechanism" to distinguish it from "extensive form mechanisms" in which jurors make choices sequentially.

the same contestant a over the rest and suppose that $w = 1$.[3] It is clear that then, no matter what mechanism is used, the jurors would always agree to choose a as the winner, whatever the true winner is. If the planner wants to induce the jurors to always choose the true subset of winners, he must be sure that this kind of situations are not possible. Technically that is equivalent to imposing some restrictions on the profile F.

Our first result states that, given any game theoretic solution concept E, if the socially optimal rule is E-implementable then for each two contestants there must be at least one juror who is fair with respect to them. The intuition of this result is the following: if there are two contestants who are not simultaneously in the set F_i of any juror i, then we can find two states of the world $(R, W_t), (\hat{R}, \hat{W}_t) \in S(F)$ such that $W_t \neq \hat{W}_t$ but $R_i(W_t) = \hat{R}_i(\hat{W}_t)$ for all juror $i \in J$ (*i.e.*, the true subsets of winners are different but the preferences of the jurors are the same in both states of the world); therefore, for any mechanism Γ, the set of E-equilibria of Γ at (R, W_t) and the set of E-equilibria of Γ at $(\hat{R}, \hat{W}_t)$ coincide, and E-implementation is not possible.

Proposition 1 *Let E equilibrium be any game theoretic solution concept. If the socially optimal rule is E-implementable then for each two contestants there must be at least one juror who is fair with respect to them (i.e., for all $a, b \in N$ there must be at least one $i \in J$ such that $a, b \in F_i$).*

Proof. Let $F = (F_i)_{i \in J}$. Suppose on the contrary that there are $a, b \in N$ such that, for all $i \in J$, either $a \notin F_i$ or $b \notin F_i$. Let $W_t, \hat{W}_t \in 2_w^N$ be such that (1) $a \in W_t$, (2) $b \in \hat{W}_t$, and (3) $W_t \backslash \{a\} = \hat{W}_t \backslash \{b\}$. Then we can find a profile of admissible preference functions $R = (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ such that $R_i(W_t) = R_i(\hat{W}_t)$ for all $i \in J$; *i.e.*, the preference relation of each juror i at state $(R, W_t) \in S(F)$ is the same than at state $(R, \hat{W}_t) \in S(F)$. Hence, given any game theoretic solution concept E and any mechanism $\Gamma = (M, g)$, we have $E(\Gamma, R, W_t) = E(\Gamma, R, \hat{W}_t)$. If Γ E-implements the socially optimal rule, there must exist some $m \in E(\Gamma, R, W_t)$ such that $g(m) = W_t$. But then $m \in E(\Gamma, R, \hat{W}_t)$ and $g(m) \neq \hat{W}_t$, which contradicts that Γ E-implements the socially optimal rule. ■

Table II shows an example of the profile $R = (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ defined in the proof of Proposition 1 for the case in which $N = \{a, b, c\}$, $J = \{1, 2, 3\}$,

[3] In terms of the concepts introduced in Section 4, a is the only friend of all jurors.

$w = 2$, and F is such that $F_1 = \{a, c\}$ and $F_2 = F_3 = \{b, c\}$ (higher alternatives in the table are strictly preferred to lower alternatives). Notice that, given F, the preference functions represented in Table II are admissible and $R_i(\{a, c\}) = R_i(\{b, c\})$ for all $i \in J$).[4]

Table II. An example in the proof of Proposition 1.

	$R_1 : 2^N_w \longrightarrow \Re$			$R_2 : 2^N_w \longrightarrow \Re$			$R_3 : 2^N_w \longrightarrow \Re$		
$W_t =$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$
	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$	$\{a, b\}$	$\{a, c\}$	$\{a, c\}$	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$
Pref.	$\{a, b\}$	$\{b, c\}$	$\{b, c\}$	$\{a, c\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, c\}$	$\{a, c\}$
	$\{b, c\}$	$\{a, b\}$	$\{a, b\}$	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$	$\{a, c\}$	$\{a, b\}$	$\{a, b\}$

Suppose then that for each two contestants there is at least one juror who is fair with respect to them. Our next result shows that this is not sufficient to guarantee that the socially optimal rule is implementable. If the socially optimal rule is E-implementable in some equilibrium concept E then the social planner must know at least one of the jurors who is fair with respect to each two contestants. In other words, it is not possible to design a mechanism that gives the jurors incentives to reveal who is fair with respect to whom. The social planner must have this information and any mechanism implementing the socially optimal rule must depend on it. Suppose that the social planner knows that for each $a, b \in N$ there is some juror $i \in J$ such that $a, b \in F_i$, but he does not know who these jurors are. In that case we should extend the definitions of the previous section so that different profiles of sets, $F = (F_i)_{i \in J}$, $\hat{F} = (\hat{F}_i)_{i \in J}$, etc., are admissible. It implies an enlargement of the set of admissible states of the world that prevents E-implementability of the socially optimal rule, whatever the equilibrium concept E the jurors use.

Proposition 2 *Let E equilibrium be any game theoretic solution concept. If the socially optimal rule is E-implementable then for each two contestants the*

[4] Given F, the only conditions that must satisfy a preference function of juror 1, $R_1 \in \mathcal{R}$, in order to be admissible are $\{a, b\} P_1(\{a, b\}) \{b, c\}$ and $\{b, c\} P_1(\{b, c\}) \{a, b\}$. Similarly, given F, the only conditions that must satisfy a preference function of juror $i \in \{2, 3\}$, $R_i \in \mathcal{R}$, in order to be admissible are $\{a, b\} P_i(\{a, b\}) \{a, c\}$ and $\{a, c\} P_i(\{a, c\}) \{a, b\}$.

social planner must know at least one of the jurors who is fair with respect to them (i.e., for all $a, b \in N$ the social planner must know some $i \in J$ for whom $a, b \in F_i$).

Proof. Suppose that the socially optimal rule is E-implementable in some equilibrium concept E. Suppose that, for each two contestants different from a and b, the social planner knows at least one juror who is fair with respect to them. Suppose by contradiction that, although the social planner knows that there is at least one juror $i \in J$ for whom $a, b \in F_i$, he does not know whether this juror is juror 1 or juror 2. Slightly abusing notation, let $R^* = (R_i^*)_{i \in J} \in \Re^{|J|}$ be a profile of preference relations such that, when comparing any two subsets of winners, $W, \hat{W} \in 2_w^N$, which only differ in two contestants, then:

(1) if the only two contestants that interchange their winner status between W and $\hat{W}$ are a and b, then juror 1 strictly prefers the subset of winners where a is included, while juror 2 strictly prefers the subset of winners where b is included, and

(2) if the only two contestants that interchange their winner status between W and $\hat{W}$ are not a and b, then those jurors that the social planner knows that are fair with respect to these two contestants strictly prefer the subset of winners that includes the contestant who comes before in alphabetical order.

Let $\tilde{W}, \bar{W} \in 2_w^N$ be two subsets of winners such that:

(i) $a \in \tilde{W}$ and $b \notin \tilde{W}$,

(ii) $a \notin \bar{W}$ and $b \in \bar{W}$, and

(iii) $\tilde{W} \backslash \{a\} = \bar{W} \backslash \{b\}$; in particular, if $w > 1$, then any contestant different from a, b, and c is in $\tilde{W}$ ($\bar{W}$) if and only if his alphabetical predecessor is also in $\tilde{W}$ ($\bar{W}$).[5]

Taking as given the jurors that the social planner knows that are fair with respect to each two contestants different from a and b, consider two profiles of sets of contestants with respect to whom the jurors are fair, $\tilde{F} = (\tilde{F}_i)_{i \in J}$ and $\bar{F} = (\bar{F}_i)_{i \in J}$, such that:

(1) the only juror $i \in J$ with $a, b \in \tilde{F}_i$ is juror 1,

(2) the only juror $i \in J$ with $a, b \in \bar{F}_i$ is juror 2, and

(3) for each two contestants different from a and b, the only jurors who have these contestants in their sets $\tilde{F}_i$ ($\bar{F}_i$) are those jurors that the social planner knows that are fair with respect to them.

[5] *E.g.*, $d \in \tilde{W}$ if and only if $c \in \tilde{W}$.(similarly, $d \in \bar{W}$ if and only if $c \in \bar{W}$).

Notice that we can always find a profile of preference functions that are admissible at $\tilde{F} = (\tilde{F}_i)_{i\in J}$, $\tilde{R} = (\tilde{R}_i)_{i\in J} \in \times_{i\in J}\mathcal{R}(\tilde{F}_i)$, such that, for all $i \in J$, $\tilde{R}_i(\tilde{W}) = R_i^*$. Similarly, we can always find a profile of preference functions that are admissible at $\bar{F} = (\bar{F}_i)_{i\in J}$, $\bar{R} = (\bar{R}_i)_{i\in J} \in \times_{i\in J}\mathcal{R}(\bar{F}_i)$, such that, for all $i \in J$, $\bar{R}_i(\bar{W}) = R_i^*$. Since the social planner does not know whether the profile of sets of contestants with respect to whom the jurors are fair is $\tilde{F}$ or $\bar{F}$, he cannot distinguish between $(\tilde{R}, \tilde{W}) \in S(\tilde{F})$ and $(\bar{R}, \bar{W}) \in S(\bar{F})$. Therefore, we must consider a more general setting in which both states of the world are admissible and the same mechanism Γ should work for both situations. Since $\tilde{R}_i(\tilde{W}) = \bar{R}_i(\bar{W})$ for all $i \in J$, given any game theoretic solution concept E we have $E(\Gamma, \tilde{R}, \tilde{W}) = E(\Gamma, \bar{R}, \bar{W})$. If Γ E-implements the socially optimal rule, there must exist some $m \in E(\Gamma, \tilde{R}, \tilde{W})$ such that $g(m) = \tilde{W}$. But then $m \in E(\Gamma, \bar{R}, \bar{W})$, which contradicts that Γ E-implements the socially optimal rule. ■

Table III provides an example of the profile of preference relations $R^* = (R_i^*)_{i\in J} \in \Re^{|J|}$ proposed in the proof of Proposition 2 for the case in which $N = \{a, b, c\}$, $J = \{1, 2, 3\}$, $w = 1$, and the social planner knows that $a, c \in F_1$, $a, c \in F_2$, and $b, c \in F_3$ (but he does not know whether $b \in F_1$ or $b \in F_2$). The subsets of winners, $\tilde{W}$ and $\bar{W}$, and the profiles of sets of contestants with respect to whom the jurors are fair, $\tilde{F} = (\tilde{F}_i)_{i\in J}$ and $\bar{F} = (\bar{F}_i)_{i\in J}$, defined in the proof of Proposition 2 are, in this case, $\tilde{W} = \{a\}$, $\bar{W} = \{b\}$, $\tilde{F}_1 = \{a, b, c\}$, $\tilde{F}_2 = \{a, c\}$, $\tilde{F}_3 = \{b, c\}$, $\bar{F}_1 = \{a, c\}$, $\bar{F}_2 = \{a, b, c\}$, and $\bar{F}_3 = \{b, c\}$. Tables IV and V provide examples of the profiles of preference functions $\tilde{R} = \left(\tilde{R}_i\right)_{i\in J} \in \times_{i\in J}\mathcal{R}(\tilde{F}_i)$ and $\bar{R} = \left(\bar{R}_i\right)_{i\in J} \in \times_{i\in J}\mathcal{R}(\bar{F}_i)$ defined in the previous proof. Note that $\tilde{R}_i(\{a\}) = \bar{R}_i(\{b\}) = R_i^*$ for all $i \in J$. Therefore, for all equilibrium concept E and all mechanism Γ, any profile of messages that is an E-equilibrium of Γ at $(\tilde{R}, \{a\})$ is also an E-equilibrium at $(\bar{R}, \{b\})$.

Table III. Example of $R^* \in \Re^{|J|}$ in proof of Proposition 2.

R^*		
R_1^*	R_2^*	R_3^*
$\{a\}$	$\{b\}$	$\{a\}$
$\{b\}$	$\{a\}$	$\{b\}$
$\{c\}$	$\{c\}$	$\{c\}$

Table IV. Example of $\tilde{R} = (\tilde{R}_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(\tilde{F}_i)$ in proof of Proposition 2.

	$\tilde{R}_1 : 2^N_w \longrightarrow \Re$			$\tilde{R}_2 : 2^N_w \longrightarrow \Re$			$\tilde{R}_3 : 2^N_w \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{a\}$	$\{b\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{a\}$
Preferences	$\{b\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{c\}$
	$\{c\}$	$\{c\}$	$\{b\}$	$\{c\}$	$\{c\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$

Table V. Example of $\bar{R} = (\bar{R}_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(\bar{F}_i)$ in proof of Proposition 2.

	$\bar{R}_1 : 2^N_w \longrightarrow \Re$			$\bar{R}_2 : 2^N_w \longrightarrow \Re$			$\bar{R}_3 : 2^N_w \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{a\}$	$\{a\}$	$\{b\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{a\}$	$\{a\}$
Preferences	$\{b\}$	$\{b\}$	$\{c\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{b\}$	$\{b\}$	$\{c\}$
	$\{c\}$	$\{c\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{c\}$	$\{c\}$	$\{b\}$

Since $n \geq 3$, except for the trivial case in which the social planner knows that there is a juror who is fair with respect to all contestants, the necessary conditions formulated in Propositions 1 and 2 cannot be fulfilled if there are only two jurors. In other words, we need at least three jurors to be able to implement the socially optimal rule.

4 Implementation in Nash (and subgame perfect) equilibria

The two conditions stated in Propositions 1 and 2 not only are necessary for the E-implementability of the socially optimal rule in any equilibrium concept E, but they are also sufficient when the equilibrium concept used by the jurors is Nash equilibrium and there are at least three jurors (see Proposition 3).

The "canonical mechanism for Nash implementation" proposed by Maskin (1999) implements any social choice rule that satisfies monotonicity and no veto power if there are three ore more agents (see also Repullo, 1987; Saijo, 1988). In our setting, monotonicity requires that if W is optimal in some state of the world, and when the state changes W does not fall in any juror preference ordering relative to any other $\hat{W}$, then W remains optimal in the new state of the world. No-veto says that a subset of winners must be socially optimal if all jurors but perhaps one rank it at the top of their orderings. It can be proved that if the conditions stated in Propositions 1 and 2 are fulfilled then the socially optimal rule is monotonic. Under the same conditions, however, the socially optimal rule does not satisfy no veto power. To see this, suppose that $N = \{a, b, c\}$, $J = \{1, 2, 3\}$, $w = 1$. Suppose that the social planner knows that $F_1 = \{a, b\}$, $F_2 = \{a, c\}$, and $F_3 = \{b, c\}$. Consider the profile of admissible preference functions $R = (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ depicted in Table IV. Notice that $\{b\}$ is the most preferred subset of winners for two of the three jurors when the state of the world is $(R, \{c\}) \in S(F)$, but $\{b\}$ is not socially optimal at that state.

Since the socially optimal rule fails to satisfy the no veto power condition, the "canonical mechanism for Nash implementation" does not work in our setting. To prove Proposition 3, we propose a variation of that mechanism where each juror has to announce a subset of winners and an integer between 1 and $|J|$. If all jurors send the same message $(W, z) \in 2^N_w \times \{1, 2, ..., |J|\}$, then W is chosen. If there is only one dissident $j \in J$ announcing $(W_j, z_j) \neq (W, z)$, then W_j is chosen if W_j and W only differ in contestants with respect to whom j is fair. If more than two jurors disagree on their messages, then W_j is chosen, where $j \in J$ is such that $j = (\sum_{i \in J} z_i)(\text{mod } |J|)$.[6]

Proposition 3 *Suppose that there are at least three jurors. Suppose that for each two contestants there is at least one juror who is fair with respect to them and the social planner knows at least one of the jurors who is fair with respect to each two contestants. Then the socially optimal rule is Nash implementable.*

Proof. Let $\Gamma^N = (M, g)$ be such that, for all $i \in J$, $M_i = 2^N_w \times \{1, 2, ..., |J|\}$, and for all $m = ((W_i, z_i))_{i \in J} \in M$, $g(m)$ is defined by the following three rules:

Rule 1. If $(W_i, z_i) = (W, z)$ for all $i \in J$, then $g(m) = W$.

Rule 2. If there $j \in J$ such that $(W_i, z_i) = (W, z)$ for all $i \neq j$ but $(W_j, z_j) \neq (W, z)$, then

$$g(m) = \begin{cases} W_j; \text{ if } \{W_j \cup W\} \backslash \{W_j \cap W\} \subseteq F_j \\ W; \text{ otherwise} \end{cases} \tag{1}$$

[6] $\alpha = \beta \,\text{mod}(|J|)$ denotes that integers α and β are congruent modulo $|J|$.

Rule 3. In all other cases $g(m) = W_j$ for $j \in J$ such that $j = (\sum_{i \in J} z_i)(\text{mod}\,|J|)$.

Claim 1. For all $(R, W_t) \in S(F)$ there exists $m \in N(\Gamma^N, R, W_t)$ such that $g(m) = W_t$.

Let $(R, W_t) \in S(F)$. Let $m = ((W_i, z_i))_{i \in J} \in M$ be such that $(W_i, z_i) = (W_t, 0)$ for all $i \in J$. Then Rule 1 applies and $g(m) = W_t$. Furthermore, $m \in N(\Gamma^N, R, W_t)$. To see this consider any $j \in J$ and any $\hat{m}_j = (\hat{W}_j, \hat{z}_j)$ such that $g(\hat{m}_j, m_{-j}) = \hat{W}_j \neq W_t$. Then, by Rule 2, $\{\hat{W}_j \cup W_t\} \backslash \{\hat{W}_j \cap W_t\} \subseteq F_j$. Notice that then there is a sequence W^1, ..., $W^s \in 2^N_w$ such that:

(1) $W^1 = W_t$,

(2) $W^s = \hat{W}_j$, and

(3) for each $q \in \{2, .., s\}$ there is $a^q, b^q \in F_j$ with (3.1) $a^q \in W^{q-1}$, (3.2) $b^q \in W^q$, (3.3) $W^{q-1} \backslash \{a\} = W^q \backslash \{b\}$, and (3.4) $a^q \neq b^r$ for all $r \neq q$.

The only difference between any two consecutive subsets of winners in the sequence, W^{q-1} and W^q, is that a^q is replaced by b^q; *i.e.*, $a^q \in W^{q-1}$, $b^q \in W^q$ and $W^{q-1} \backslash \{a\} = W^q \backslash \{b\}$. Moreover, since $W^1 = W_t$ and $a^q \neq b^r$ for all $r \neq q$, then $a^q \in W_t$ and $b^q \notin W_t$. Therefore, since $a^q, b^q \in F_j$, $W_t = W^1 P_j(W_t) W^2 P_j(W_t) ... W^{s-1} P_j(W_t) W^s = \hat{W}_j$. Hence, juror j cannot improve by deviating from m.

Claim 2. For all $(R, W_t) \in S(F)$ and all $m \in M$ such that $g(m) \neq W_t$, $m \notin N(\Gamma^N, R, W_t)$.

Let $(R, W_t) \in S(F)$ and $m = ((W_i, z_i))_{i \in J} \in M$ be such that $g(m) = W \neq W_t$. Then, there exist $a, b \in N$ such that $a \in W_t$, $a \notin W$, $b \in W$ and $b \notin W_t$. Let $\hat{W} \in 2^N_w$ be such that $\hat{W} \backslash \{a\} = W \backslash \{b\}$ and $a \in \hat{W}$. Let $j \in J$ be the juror for whom the social planner knows that $a, b \in F_j$. Then $\hat{W} P_j(W_t) W$.

Case 1. Suppose that Rule 1 applies to m. Then $W_i = W$ for all $i \in J$. Consider a unilateral deviation by juror j to $\hat{m}_j = (\hat{W}, 0)$. Then Rule 2 applies and, since $\{\hat{W} \cup W\} \backslash \{\hat{W} \cap W\} = \{a, b\} \subseteq F_j$, $g(\hat{m}_j, m_{-j}) = \hat{W}$. Since $\hat{W} P_j(W_t) W$, $m \notin N(\Gamma^N, R, W_t)$.

Case 2. Suppose that Rule 2 applies to m.

Subcase 2.1. Suppose that the deviator in m is juror j. If juror j is not announcing W in m then the rest of jurors are announcing W in m, in which case juror j can improve by unilaterally deviating to $\hat{m}_j = (\hat{W}, 0)$ (as in Case 1). If juror j is announcing W in m then the rest of jurors must be announcing some $\tilde{W} \neq W$ such that $\{W \cup \tilde{W}\} \backslash \{W \cap \tilde{W}\} \subseteq F_j$. Notice that then $\{\hat{W} \cup \tilde{W}\} \backslash \{\hat{W} \cap \tilde{W}\} \subseteq F_j$. Therefore juror j can improve unilaterally deviating to $\hat{m}_j = (\hat{W}, 0)$, since in this case Rule 2 applies and

$g(\hat{m}_j, m_{-j}) = \hat{W}$. Hence, $m \notin N(\Gamma^N, R, W_t)$.

Subcase 2.2. Suppose that the deviator in m is not juror j. Consider a unilateral deviation by juror j to $\hat{m}_j = (\hat{W}, \hat{z}_j)$ where $\hat{z}_j \neq z_i$ for all $i \neq j$ and $j = (\hat{z}_j + \sum_{i \neq j} z_i)(\text{mod}\,|J|)$. Then Rule 3 applies and $g(\hat{m}_j, m_{-j}) = \hat{W}$. Therefore $m \notin N(\Gamma^N, R, W_t)$.

Case 3. Suppose that Rule 3 applies to m. Then juror j can improve by deviating as in Subcase 2.2. Therefore $m \notin N(\Gamma^N, R, W_t)$. ■

It is also interesting to analyze implementation of the socially optimal rule via extensive form mechanisms. It is not possible to explain fully this approach here because it would take us too far. Roughly speaking, an extensive form mechanism is a dynamic mechanism in which agents make choices sequentially. Given the profile of sets of contestants with respect to whom the jurors are fair $F = (F_i)_{i \in J}$, the socially optimal rule is **implementable in subgame perfect equilibria** if and only if there exists an extensive form mechanism such that for all state of the world $(R, W_t) \in S(F)$, the only subgame perfect equilibrium outcome is W_t.[7] The two necessary conditions stated in Propositions 1 and 2 are also sufficient for the implementation of the socially optimal rule in subgame perfect equilibria if there are at least three jurors. To see this note that, since the mechanism proposed in the proof of Proposition 3 is a one-shot-mechanism, then, for all state of the world, a profile of messages m is a Nash equilibrium if and only if it is a subgame perfect equilibria. Therefore, this mechanism also implements the socially optimal rule in subgame perfect equilibria.

5 Implementation in dominant strategies

Unlike what happens with Nash and subgame perfect implementation, the necessary conditions stated in Propositions 1 and 2 are not sufficient for the implementability of the socially optimal rule in dominant strategies.

Proposition 4 *If the socially optimal rule is implementable in dominant strategies then there must be at least one juror who is fair with respect to all contestants (i.e., there must be some $i \in J$ such that $F_i = N$).*

[7]For each extensive form mechanism and each state of the world, a subgame perfect equilibrium induces a Nash equilibrium in every subgame. For the very positive results achieved on implementation with extensive form mechanisms under complete information see Moore and Repullo (1988) and Abreu and Sen (1990).

Proof. Suppose by contradiction that the socially optimal rule is implementable in dominant strategies but, for all $i \in N$, $F_i \neq N$. From Propositions 1 and 2 we know that for each two contestants there is at least one juror who is fair with respect to them and the social planner knows at least one of the jurors who is fair with respect to each two contestants. Suppose w.l.o.g. that $N = \{a, b, c\}$, $J = \{1, 2, 3\}$, $w = 1$, and the social planner knows that $F_1 = \{a, b\}$, $F_2 = \{a, c\}$, and $F_3 = \{b, c\}$. Consider the profile of admissible preference functions $R = (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ depicted in Table VI.[8] Abusing notation, let us denote $R_1^*, R_1^{**} \in \Re$ the two following preference relations for juror 1: $R_1^* = R_1(\{a\}) = R_1(\{c\})$ and $R_1^{**} = R_1(\{b\})$ (*i.e.*, $R_1^*, R_1^{**} \in \Re$ are such that $\{c\}P_1^*\{a\}P_1^*\{b\}$ and $\{c\}P_1^{**}\{b\}P_1^{**}\{a\}$). Similarly, let $R_2^* = R_2(\{a\})$, $R_2^{**} = R_2(\{b\}) = R_2(\{c\})$, $R_3^* = R_3(\{a\}) = R_3(\{b\})$ and $R_3^{**} = R_3(\{c\})$ (see Table VII). Suppose that there exists a mechanism $\Gamma^D = (M, g)$ that implements the socially optimal rule in dominant strategies. Notice that then for each juror $i \in J$ there must exist some $m_i^* \in M_i$ which is a dominant strategy for i when his preference relation is $R_i^* \in \Re$ (*i.e.*, $g(m_i^*, m_{-i})R_i^*g(m_i, m_{-i})$ for all $m_i \in M_i$, and all $m_{-i} \in M_{-i}$). Similarly, for each $i \in J$ there must exist $m_i^{**} \in M_i$ which is a dominant strategy for i when his preference relation is $R_i^{**} \in \Re$ (*i.e.*, $g(m_i^{**}, m_{-i})R_i^{**}g(m_i, m_{-i})$ for all $m_i \in M_i$, and all $m_{-i} \in M_{-i}$). Notice also that $g(m_1^*, m_2^*, m_3^*) = \{a\}$ (since $(R, \{a\}) \in S(F)$, $(m_1^*, m_2^*, m_3^*) \in D(\Gamma^D, R, \{a\})$, and Γ^D implements the socially optimal rule in dominant strategies). In a similar way, $g(m_1^{**}, m_2^{**}, m_3^*) = \{b\}$ and $g(m_1^*, m_2^{**}, m_3^{**}) = \{c\}$. Consider now the profile of messages $(m_1^*, m_2^{**}, m_3^*) \in M$.[9] Notice that (1) $g(m_1^*, m_2^{**}, m_3^*) \neq \{a\}$ (otherwise $\{a\} = g(m_1^*, m_2^{**}, m_3^*)P_3^{**}g(m_1^*, m_2^{**}, m_3^{**}) = \{c\}$, which contradicts that m_3^{**} is a dominant strategy for juror 3 when his preference relation is R_3^{**}), (2) $g(m_1^*, m_2^{**}, m_3^*) \neq \{b\}$ (otherwise $\{b\} = g(m_1^*, m_2^{**}, m_3^*)P_2^*g(m_1^*, m_2^*, m_3^*) = \{a\}$, which contradicts that m_2^* is a dominant strategy for juror 2 when his preference relation is R_2^*), and (3) $g(m_1^*, m_2^{**}, m_3^*) \neq \{c\}$ (otherwise $\{c\} = g(m_1^*, m_2^{**}, m_3^*)P_1^{**}g(m_1^{**}, m_2^{**}, m_3^*) = \{b\}$, which contradicts that m_1^{**} is a dominant strategy for juror 1 when

[8] Notice that, in terms of the concepts that we introduce in Section 4, c is the only friend of juror 1, b is the only friend of juror 2, and a is the only friend of juror 3.

[9] Using the same argument than in Example 3 in the Appendix, it can be proved that the profile of preference relations $(R_1^*, R_2^{**}, R_3^*) \in \Re^{|J|}$ is not admissible; *i.e.*, this profile of preference relations do not correspond with any possible state of the world. The outcome function of mechanism Γ^D, however, must select some subset of winners for the profile of messages (m_1^*, m_2^{**}, m_3^*).

his preference relation is R_1^{**}). Therefore, there is no $W \in 2_w^N$ such that $g(m_1^*, m_2^{**}, m_3^*) = W$, which contradicts the definition of an outcome function.[10] ■

Table VI. Profile $R = (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ in proof of Proposition 4.

	$R_1 : 2_w^N \longrightarrow \Re$			$R_2 : 2_w^N \longrightarrow \Re$			$R_3 : 2_w^N \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{c\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{a\}$
Preferences	$\{a\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{c\}$
	$\{b\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$

Table VII. Preference relations $R_i^*, R_i^{**} \in \Re$ in proof of Proposition 4.

R_1^*	R_1^{**}	R_2^*	R_2^{**}	R_3^*	R_3^{**}
$\{c\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{a\}$	$\{a\}$
$\{a\}$	$\{b\}$	$\{a\}$	$\{c\}$	$\{b\}$	$\{c\}$
$\{b\}$	$\{a\}$	$\{c\}$	$\{a\}$	$\{c\}$	$\{b\}$

By an argument similar to that in Proposition 2 we can prove that having a juror who is fair with respect to all contestants is not enough to guarantee that the socially optimal rule is implementable in dominant strategies: the social planner must know who this juror is.

Proposition 5 *If the socially optimal rule is implementable in dominant strategies then the social planner must know at least one of the jurors who is fair with respect to all contestants.*

Proof. From Proposition 4, if the socially optimal rule is implementable in dominant strategies then there must be at least one juror who is fair with respect to all contestants. Suppose that the social planner knows that there is at least one juror i such that $F_i = N$ but he does not know who this juror

[10] Notice that, since the set of admissible profiles of preference relations $\Re(F)$ have not a Cartesian product structure, the revelation principle for dominant strategies (Gibbard, 1973) cannot be used to prove Proposition 4.

is. Suppose w.l.o.g. that $N = \{a, b, c\}$, $J = \{1, 2, 3\}$, and $w = 1$. Consider the following two situations:

(1) $F = (F_i)_{i \in J}$ such that $F_1 = \{a, b, c\}$, $F_2 = \{b, c\}$, and $F_3 = \{b, c\}$.

(2) $\hat{F} = (\hat{F}_i)_{i \in J}$ such that $\hat{F}_1 = \{b, c\}$, $\hat{F}_2 = \{a, b, c\}$, and $\hat{F}_3 = \{b, c\}$.

Since the social planner does not know which juror is fair with respect to all contestants, he does not know whether the profile of sets of contestants with respect to whom the jurors are fair is F or $\hat{F}$. Let $R \in \times_{i \in J} \mathcal{R}(F_i)$ be the profile of preference functions defined in Table VII (these preference functions are admissible when the profile of sets of contestants with respect to whom the jurors are fair is F). Similarly, let $\hat{R} \in \times_{i \in J} \mathcal{R}(\hat{F}_i)$ be the profile of preference functions defined in Table IX (these preference functions are admissible when the profile of sets of contestants with respect to whom the jurors are fair is $\hat{F}$). Notice that $R_i(\{a\}) = \hat{R}_i(\{b\})$ for all $i \in J$. Since the social planner does not know whether the profile of sets of contestants with respect to whom the jurors are fair is F or $\hat{F}$, he cannot distinguish between $(R, \{a\}) \in S(F)$ and $(\hat{R}, \{b\}) \in S(\hat{F})$. Using the same argument than in the proof of Proposition 2 we can conclude that there is no mechanism such that, in equilibrium, the true subset of winners is selected in both states of the world $(R, \{a\}) \in S(F)$ and $(\hat{R}, \{b\}) \in S(\hat{F})$. ■

Table VIII. Example of $R = (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ in proof of Proposition 5.

	$R_1 : 2^N_w \longrightarrow \Re$			$R_2 : 2^N_w \longrightarrow \Re$			$R_3 : 2^N_w \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{a\}$	$\{b\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{c\}$
Preferences	$\{b\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{c\}$	$\{c\}$	$\{b\}$
	$\{c\}$	$\{c\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$

Table IX. Example of $\hat{R} = (\hat{R}_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(\hat{F}_i)$ in proof of Proposition 5.

	$\hat{R}_1 : 2^N_w \longrightarrow \Re$			$\hat{R}_2 : 2^N_w \longrightarrow \Re$			$\hat{R}_3 : 2^N_w \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{c\}$
Preferences	$\{b\}$	$\{b\}$	$\{c\}$	$\{b\}$	$\{c\}$	$\{b\}$	$\{c\}$	$\{c\}$	$\{b\}$
	$\{c\}$	$\{c\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$

If there is one juror i who is fair with respect to all contestants and the social planner knows who this juror is, the socially optimal rule is implementable in dominant strategies via the trivial mechanism where juror i simply chooses a subset of winners. Therefore, the following is a corollary to Propositions 4 and 5.

Corollary 1 *The socially optimal rule is implementable in dominant strategies if and only if there is at least one juror who is fair with respect to all contestants and the social planner knows who this juror is.*

6 The case where each juror is biased in favor of one different contestant

In the previous sections we have identified necessary and sufficient conditions on the jury under which the socially optimal rule is implementable. The necessary and sufficient conditions stated in Corollary 1 incorporate informational requirements that are so strong that they actually prevent the socially optimal rule from being implementable in dominant strategies. The necessary and sufficient conditions for Nash and subgame perfect implementation are less demanding. Moreover, these conditions cannot be relaxed since they are necessary for the implementability of the socially optimal rule, whatever equilibrium concept the jurors use. The mechanism 'a la Maskin (1999) proposed in the proof of Proposition 3, however, is quite abstract. This type of mechanisms have received much criticism for being unnatural (see Jackson, 1992). Nevertheless, as argued by Serrano (2004), the main purpose of these mechanisms is the characterization of what can be implemented. As such, these mechanisms are designed to handle a large number of social choice problems. More realistic mechanisms can be constructed when one deals with a specific application.

This is precisely what happens with the mechanism proposed in the proof of Proposition 3. This mechanism works whenever there are three or more jurors, there is at least one juror who is fair with respect to each two contestants, and the social planner knows who these jurors are. There are many different situations covered by these conditions. Suppose for example that $N = \{a, b, c, d\}$, $J = \{1, 2, 3, 4\}$ and the social planner knows that $F_1 = \{b, c, d\}$, $F_2 = \{a, c, d\}$, $F_3 = \{a, b, d\}$, and $F_4 = \{a, b, c\}$. Then the necessary and sufficient conditions stated in Proposition 3 are fulfilled. Many different situations are still possible however: (1) contestants a, b, c and d could be friends of jurors 1, 2, 3 and 4, respectively; (2) contestants a, b, c and d could be enemies of jurors 1, 2, 3 and 4, respectively; (3) contestants a, b could be friends of jurors 1 and 2, respectively, while contestants c and d could be enemies of jurors 3 and 4, respectively; etc. The mechanism proposed in the proof of Proposition 3 works in all these situations. This is the reason why our mechanism is so abstract and other mechanisms that could seem more "natural" do not work in this general framework (see the Appendix).

In some situations the social planner has more information about certain aspects of jurors' preferences. It allows him to disregard some of the previous cases and therefore to design more realistic mechanisms. Next, we study one of these situations: the case where each juror has one and only one different friend. Let us formally define the concept of a juror's friend.

Definition 3 *We say that $a \in N$ is the only* ***friend*** *of juror i if for all $W_t \in 2^N_w$ and all $W, \hat{W} \in 2^N_w$ with $a \in W$ and $a \notin \hat{W}$, we have $WP_i(W_t)\hat{W}$.*

Suppose that the social planner knows that each juror has only one friend among the contestants and is fair with respect to the rest. For example, this could be the case when the jury is made up of all contestants so that, even though each contestant is fair with respect to the rest, he always wants to be among the winners of the competition. In this case we have:

(i) $N = J = \{1, 2, ..., n\}$,

(ii) for all $i \in J$, all $W_t \in 2^N_w$ and all $W, \hat{W} \in 2^N_w$ with $i \in W$ and $i \notin \hat{W}$, $WP_i(W_t)\hat{W}$ (*i.e.*, each contestant always prefers to be among the winners, whatever the true subset of winners is).

(iii) $F_i = N \backslash \{i\}$ for all $i \in J$ (*i.e.*, each contestant i is fair with respect to all contestants but himself).

Next, we propose a natural mechanism that implements the socially optimal rule in subgame perfect equilibria in this situation when $w = 1$.

Extensive form mechanism 1:

Stage 1: Juror 1 announces who the winner is, $m_1 \in N$. There are two possibilities to consider:

1.1 If $m_1 \neq 1$, then m_1 is chosen as winner. STOP.

1.2 If $m_1 = 1$, then go to Stage 2.

Stage 2: Juror 2 announces who the winner is, $m_2 \in N$. There are two possibilities to consider:

2.1 If $m_2 \neq 2$, then m_2 is chosen as winner. STOP.

2.2 If $m_2 = 2$, then go to Stage 3.

$\vdots$

Stage (n-1): Juror $n-1$ announces who the winner is, $m_{n-1} \in N$. There are two possibilities to consider:

(n-1).1 If $m_{n-1} \neq n-1$, then m_{n-1} is chosen as winner. STOP.

(n-1).2 If $m_{n-1} = n-1$, then go to Stage n.

Stage n: Juror n announces who the winner is, $m_n \in N$. Then m_n is chosen as winner. STOP.

Proposition 6 *Suppose that: (1) the number of contestants and jurors is $n \geq 4$, (2) each juror has only one (different) friend among the contestants and is fair with respect to the rest, and (3) $w = 1$. Then, the socially optimal rule can be implemented in subgame perfect equilibria via the extensive form mechanism 1.*

Proof. Suppose that $N = J = \{1, 2, 3, ..., n\}$. Suppose that for each juror $i \in J$ contestant $i \in N$ is his only friend and $F_i = N \backslash \{i\}$. Notice that then (1) for all $i \in N$ and all $W_t \neq \{i\}$ we have $\{i\} P_i(W_t) W_t$, and (2) for all $i \in N$, all $j \neq i$ and all $W_t \neq \{j\}$ we have $W_t P_i(W_t)\{j\}$ (*i.e.*, for each juror i, $\{i\}$ and W_t are the best and second best alternatives, respectively).

Claim 1. At Stage n, juror n announces $m_n = n$, whatever the true winner W_t is.

The proof of this claim is trivial since, for all W_t, $\{n\}$ is the most preferred alternative for juror n.

Claim 2. At Stage (n-1), if $W_t \neq \{n-1\}$ juror $n-1$ announces $m_{n-1} = W_t$.

To see this note that there is nothing that juror $n-1$ can do to be chosen as the winner. Therefore, his best option is to announce $m_{n-1} = W_t$ so that his second best alternative is selected (if $W_t = \{n\}$, juror $n-1$ would be indifferent between announcing $m_{n-1} = n$ or $\hat{m}_{n-1} = n-1$; in the latter case juror n would announce $m_n = n$ at Stage n and the true winner would be finally selected anyway).

Claim 3. At Stage (n-1), if $W_t = \{n-1\}$, juror $n-1$ announces $m_{n-1} = i^*$ for some $i^* \in N \backslash \{n-1\}$.

If $W_t = \{n-1\}$ the second best alternative for juror $n-1$ could be any $\{i^*\}$ with $i^* \in N \backslash \{n-1\}$. Obviously in this case juror $n-1$ will announce $m_{n-1} = i^*$ at Stage n-1 (as in the previous case, if $i^* = n$, juror $n-1$ would be indifferent between announcing $m_{n-1} = n$ or $\hat{m}_{n-1} = n-1$).

Claim 4. At Stage (n-2), if $W_t \neq \{n-1\}$, juror $n-2$ announces $m_{n-2} = W_t$.

If $W_t = n-2$ and juror $n-2$ announces $m_{n-2} = n-2$, the mechanism goes to Stage (n-1), juror $n-1$ announces $m_{n-1} = n-2$ and $n-2$ is chosen as winner (which is the best alternative for juror $n-2$). If $n-1 \neq W_t \neq n-2$, there is nothing that juror $n-2$ can do to be chosen as winner. Since W_t is his second best alternative, juror $n-2$ announces $m_{n-2} = W_t$ and the true winner is chosen.

Claim 5. At Stage (n-2), if $W_t = \{n-1\}$ and $i^* = n-2$, juror $n-2$ announces $m_{n-2} = n-2$.

If $W_t = \{n-1\}$ and juror $n-2$ announces $m_{n-2} = n-2$, the mechanism goes to Stage (n-1), juror $n-1$ announces $m_{n-1} = i^* = n-2$ and $n-2$ is chosen as winner (which is the best alternative for juror $n-2$).

Claim 6. At Stage (n-2), if $W_t = \{n-1\}$ and $i^* \neq n-2$, juror $n-2$ announces $m_{n-2} = W_t$.

In this case there is nothing that juror $n-2$ can do to be chosen as winner (if he announces $m_{n-2} = n-2$, the mechanism goes to Stage (n-1), juror $n-1$ announces $m_{n-1} = i^* \neq n-2$ and i^* is chosen as winner). Since W_t is his second best alternative, juror $n-2$ announces $m_{n-2} = W_t$ and the true winner is chosen.

Claim 7. At Stage (n-3), juror $n-3$ announces $m_{n-3} = W_t$.

Suppose first that $W_t \neq \{n-1\}$. By Claim 4, if the mechanism goes to Stage (n-2) juror $n-2$ will announce $m_{n-2} = W_t$ and the true winner will be chosen (not necessarily at Stage (n-2) in case that $W_t = \{n-2\}$). Since there is nothing that juror $n-3$ can do in order to be chosen as winner at Stage (n-3), his best option is to announce $m_{n-3} = W_t$ (if $W_t = \{n-3\}$, the mechanism will go to Stage (n-2) and $n-3$ will be selected as winner; if $W_t \neq \{n-3\}$, the true winner will be chosen at stage (n-3)).

Suppose now that $W_t = \{n-1\}$ and $i^* = n-2$. By Claim 5, if the mechanism goes to Stage (n-2) juror $n-2$ will announce $m_{n-2} = n-2$ and the mechanism will go to Stage (n-1). Then, by Claim 3, juror $n-1$ will announce $m_{n-1} = n-2$ and $n-2$ will be chosen as final winner. Since juror $n-3$ strictly prefers $W_t = \{n-1\}$ rather than $\{n-2\}$, he will announce $m_{n-3} = n-1$.

Finally, suppose that $W_t = \{n-1\}$ and $i^* \neq n-2$. By Claim 6, if the mechanism goes to Stage (n-2) juror $n-2$ will announce $m_{n-2} = W_t$. Then, by the same argument than in the case where $W_t \neq \{n-1\}$, the best option for juror $n-3$ is to announce $m_{n-3} = W_t$ (juror $n-3$ would be indifferent between announcing $m_{n-3} = W_t$ or $\hat{m}_{n-3} = n-3$; in the latter case the mechanism would go to the next stage and the true winner will be finally selected anyway).

Claim 8. If there are more than four jurors (*i.e.*, contestants), at any Stage (n-k) with $k \geq 4$ juror $n-k$ announces $m_{n-k} = W_t$.

To see this notice that juror $n-k$ cannot be selected as winner at Stage (n-k). By the previous claims, if the mechanism goes to Stage (n-k-1), then the true winner will be finally selected. Since $\{n-k\}$ and W_t are the best and second best alternatives for juror $n-k$, respectively, his best option is to announce $m_{n-k} = W_t$.

From Claims 1-8, for all $W_t \in 2^N_w$, there exist a subgame perfect equilibrium of the extensive form mechanism 1 such that juror 1 announces $m_1 = W_t$ and, in case that $W_t = \{1\}$, juror 2 announces $m_2 = \{W_t\}$. Obviously, this subgame perfect equilibrium always results in W_t, whatever it is. Moreover, although in some situations there can exist more than one subgame perfect equilibrium, it is easy to see that all of them result in W_t.[11] ■

The next example makes it clear why the extensive form mechanism 1 needs at least four jurors to work.

Example 2 *Let $N = J = \{1, 2, 3\}$. Suppose that $w = 1$. Suppose that for each juror $i \in J$ contestant $i \in N$ is his only friend and $F_1 = \{2, 3\}$, $F_2 = \{1, 3\}$ and $F_3 = \{1, 2\}$. Let $R^*_2 : 2^N_w \longrightarrow \Re$ be a preference function of juror 2 such that $\{2\}P^*_2(W_t)\{1\}P^*_2(W_t)\{3\}$ when $W_t = \{2\}$. Notice that this preference function is admissible. It is easy to see that, at any state (R, W_t) such that $R_2 = R^*_2$ and $W_t = \{2\}$, there is a subgame perfect equilibrium of the extensive form mechanism 1 where juror 1 announces $m_1 = 1$ and juror 2 announces $m_2 = 1$. Obviously, this equilibrium does not yield the true winner.*

[11] For example, if $N = J = \{1, 2, 3, 4\}$ and $W_t = \{2\}$, there is another subgame perfect equilibrium where juror 1 announces $m_1 = 1$, juror 2 announces $m_2 = 2$ and juror 3 announces $m_3 = 2$. If we assume that the jurors have intrinsic preferences for honesty in the sense that they dislike the idea of lying when it does not influence their welfare but instead goes against the intention of the social planner, then these type of equilibria would not exist (see Matsushima, 2008).

7 Conclusion

We have analyzed the problem of choosing the w best contestants who will win a competition within a group of $n > w$ competitors when the jurors may be partial. We have studied restrictions on the configuration of the jury so that it is possible to induce the jurors to always choose the best contestants, whoever they are (*i.e.*, the socially optimal rule is implementable). If the equilibrium concept used by the jurors is dominant strategies, the necessary and sufficient conditions incorporate informational requirements that are so strong that they actually prevent the implementability of the socially optimal rule. If we relax the equilibrium concept to Nash or subgame perfect equilibria the necessary and sufficient conditions are less demanding. As a matter of fact, the latter conditions can be interpreted as the minimum degree of impartiality that we must require on the jury in order to guarantee that their decisions will correspond to socially optimal goals: these conditions cannot be relaxed since they are necessary for the implementability of the socially optimal rule, whatever equilibrium concept the jurors use. The mechanism that we have proposed to prove our results in Nash and subgame perfect equilibria is necessarily quite abstract, as it is able to handle a large number of situations: the main purpose of this mechanism is the characterization of the necessary and sufficient conditions for the implementability of the socially optimal rule. More realistic mechanisms can be constructed when one deals with a specific application. We have studied one of these applications: the case where each juror has one and only one (different) friend among the contestants. We have proposed a simple and natural mechanism that implements in subgame perfect equilibria the socially optimal rule in this situation.

Appendix

The set of admissible profiles of preference relations may not have a Cartesian product structure

The following example shows that the set $\Re(F)$ may not have a Cartesian product structure.

Example 3 *Let* $N = \{a, b, c\}$ *and* $w = 1$. *Then* $2^N_w = \{\{a\}, \{b\}, \{c\}\}$. *Let* $J = \{1, 2, 3\}$ *and suppose that* $F = (F_i)_{i \in J}$ *is such that* $F_1 = \{a, b\}$,

$F_2 = \{a, c\}$, and $F_3 = \{b, c\}$. Consider the profile of preference functions $(R_1, R_2, R_3) \in \mathcal{R}^{|J|}$ defined in Table X. It is easy to check that $(R_1, R_2, R_3) \in \times_{i \in J} \mathcal{R}(F_i)$. Slightly abusing notation, let $\hat{R} = (\hat{R}_1, \hat{R}_2, \hat{R}_3)$, $\tilde{R} = (\tilde{R}_1, \tilde{R}_2, \tilde{R}_3)$, $\bar{R} = (\bar{R}_1, \bar{R}_2, \bar{R}_3) \in \Re^{|J|}$ be the profiles of preference relations defined in Table XI. The three profiles of preference relations are admissible; i.e., $\hat{R}, \tilde{R}, \bar{R} \in \Re(F)$. To see this note that the profile of preference functions defined in Table II is such that: (1) If $W_t = \{a\}$, $(R_1(W_t), R_2(W_t), R_3(W_t)) = (\hat{R}_1, \hat{R}_2, \hat{R}_3)$, (2) if $W_t = \{c\}$, $(R_1(W_t), R_2(W_t), R_3(W_t)) = (\tilde{R}_1, \tilde{R}_2, \tilde{R}_3)$, and (3) if $W_t = \{b\}$, $(R_1(W_t), R_2(W_t), R_3(W_t)) = (\bar{R}_1, \bar{R}_2, \bar{R}_3)$. The profile of preference relations $(\hat{R}_1, \tilde{R}_2, \bar{R}_3)$, however, is not admissible. Suppose on the contrary that there is $(R_1^*, R_2^*, R_3^*) \in \times_{i \in J} \mathcal{R}(F_i)$ and $W_t^* \in 2_w^N$ such that $(R_1^*(W_t^*), R_2^*(W_t^*), R_3^*(W_t^*)) = (\hat{R}_1, \tilde{R}_2, \bar{R}_3)$. Then:

(i) Since $\{a\}\hat{P}_1\{b\}$ and $a, b \in F_1$, then $W_t^* \neq \{b\}$.

(ii) Since $\{c\}\tilde{P}_2\{a\}$ and $a, c \in F_2$, then $W_t^* \neq \{a\}$.

(iii) Since $\{b\}\bar{P}_3\{c\}$ and $b, c \in F_3$, then $W_t^* \neq \{c\}$.

Clearly, points 1-3 above are not compatible and therefore $(\hat{R}_1, \tilde{R}_2, \bar{R}_3) \notin \Re(F)$.

Table X. Profile $(R_1, R_2, R_3) \in \times_{i \in J} \mathcal{R}(F_i)$ in Example 3.

	$R_1 : 2_w^N \longrightarrow \Re$			$R_2 : 2_w^N \longrightarrow \Re$			$R_3 : 2_w^N \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{a\}$	$\{b\}$	$\{b\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{c\}$
Preferences	$\{b\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$
	$\{c\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{a\}$

Table XI. Profiles $\hat{R}, \tilde{R}, \bar{R} \in \Re^{|J|}$ in Example 3.

$\hat{R}$			$\tilde{R}$			$\bar{R}$		
$\hat{R}_1$	$\hat{R}_2$	$\hat{R}_3$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$	$\bar{R}_1$	$\bar{R}_2$	$\bar{R}_3$
$\{a\}$	$\{a\}$	$\{b\}$	$\{b\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{c\}$	$\{b\}$
$\{b\}$	$\{c\}$	$\{c\}$	$\{a\}$	$\{a\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{c\}$
$\{c\}$	$\{b\}$	$\{a\}$	$\{c\}$	$\{b\}$	$\{a\}$	$\{c\}$	$\{b\}$	$\{a\}$

The impossibility of implementing the socially optimal rule via some "natural" mechanisms

It is not easy to define what a "natural" mechanism means. In our framework, however, the following can be seen as reasonable properties to be satisfied by a "natural" mechanism:

(1) Each juror only has to announce a subset of winners.

(2) Truth-telling is an equilibrium.

The justification for the first requirement is clear: large message spaces make it difficult for the jurors to understand the rules of the mechanism; since we have to choose a subset of winners, let's ask each of the jurors for this information directly. The second is a unanimity requirement which has twofold implications: (2.1) if all jurors agree on the announced W, then W is implemented and, (2.2) if there is only one dissident his announcement is implemented only if by doing so, if the rest of jurors were telling the truth (and given what we know about him), he would end up worse off.

Unfortunately, any mechanism satisfying these requirements fail to Nash implement the socially optimal rule, even though the necessary and sufficient conditions for Nash implementation stated in Propositions 1, 2 and 3 are fulfilled.

Example 4 *Let $N = \{a, b, c\}$ and $w = 1$. Then $2^N_w = \{\{a\}, \{b\}, \{c\}\}$. Let $J = \{1, 2, 3\}$ and suppose that the social planner knows that $F = (F_i)_{i \in J}$ is such that $F_1 = \{a, b\}$, $F_2 = \{a, c\}$, and $F_3 = \{b, c\}$. Then, the necessary and sufficient conditions for the Nash implementability of the socially optimal rule are fulfilled. Let $\Gamma = (M, g)$ be a mechanism such that:*

(1) For all $i \in J$, $M_i = \{\{a\}, \{b\}, \{c\}\}$.

(2) For all $m \in M$ such that $m_1 = m_2 = m_3$, then $g(m) = m_1$.

(3) For all $m \in M$ such that $m_i = m_j \neq m_k$ for some $i, j, k \in J$, then:[12]

$$g(m) = \begin{cases} m_k; \text{ if } m_i, m_k \in F_k \\ m_i; \text{ otherwise} \end{cases}$$

Note that we have not specify what happens when there are more than two jurors who disagree on their announcements. Let $m^* \in M$ be such that

[12] The third condition says that, if all jurors apart from k announce the same winner, then k is given the opportunity of choosing a different winner only if he is fair with respect to both winners (the one announced by him and the one announced by the rest). This is the only case in which the social planner can be sure that the dissident k is not deviating from a profile of truth-telling announcements in his own interest.

$m_1^* = \{a\}$, $m_2^* = \{b\}$, and $m_3^* = \{c\}$. Next we show that whatever the winner $g(m^*)$ is, mechanism Γ fails to Nash implement the socially optimal rule.

Claim 1. If $g(m^*) = \{a\}$, then Γ does not Nash implements the socially optimal rule.

Suppose that $g(m^*) = \{a\}$. Let $R = (R_1, R_2, R_3) \in \times_{i \in J} \mathcal{R}(F_i)$ be the profile of admissible preference functions defined in Table XII. The profile of messages m^* is a Nash equilibrium of Γ when the state of the world is $(R, \{b\}) \in S(F)$. To see this note that (i) any unilaterally deviation of juror 1 from m^* results in $\{c\}$, which is less preferred than $\{a\}$ by him at state $(R, \{b\})$ and, (ii) since $\{a\}$ is the most preferred alternative of jurors 2 and 3 at state $(R, \{b\})$, none of them has incentives to unilaterally deviate from m^*. Since $m^* \in N(\Gamma, R, \{b\})$ but $g(m^*) \neq \{b\}$, Γ does not Nash implement the socially optimal rule.

Claim 2. If $g(m^*) = \{b\}$, then Γ does not Nash implements the socially optimal rule.

Suppose that $g(m^*) = \{b\}$. Let $\hat{R} = (\hat{R}_1, \hat{R}_2, \hat{R}_3) \in \times_{i \in J} \mathcal{R}(F_i)$ be the profile of admissible preference functions defined in Table XIII. The profile of messages m^* is a Nash equilibrium of Γ when the state of the world is $(\hat{R}, \{a\}) \in S(F)$. To see this note that (i) any unilaterally deviation of juror 1 from m^* results in $\{c\}$, which is less preferred than $\{b\}$ by him at state $(\hat{R}, \{b\})$ and, (ii) since $\{b\}$ is the most preferred alternative of jurors 2 and 3 at state $(\hat{R}, \{b\})$, none of them has incentives to unilaterally deviate from m^*. Since $m^* \in N(\Gamma, \hat{R}, \{a\})$ but $g(m^*) \neq \{a\}$, Γ does not Nash implement the socially optimal rule.

Claim 3. If $g(m^*) = \{c\}$, then Γ does not Nash implements the socially optimal rule.

Suppose that $g(m^*) = \{c\}$. Let $\tilde{R} = (\tilde{R}_1, \tilde{R}_2, \tilde{R}_3) \in \times_{i \in J} \mathcal{R}(F_i)$ be the profile of admissible preference functions defined in Table XIV. The profile of messages m^* is a Nash equilibrium of Γ when the state of the world is $(\tilde{R}, \{b\}) \in S(F)$. To see this note that (i) since $\{c\}$ is the most preferred alternative of jurors 1 and 2 at state $(\tilde{R}, \{b\})$, none of them has incentives to unilaterally deviate from m^* and, (ii) any unilaterally deviation of juror 3 from m^* results in $\{a\}$, which is less preferred than $\{c\}$ by him at state $(\tilde{R}, \{b\})$. Since $m^* \in N(\Gamma, \tilde{R}, \{b\})$ but $g(m^*) \neq \{b\}$, Γ does not Nash implement the socially optimal rule.

Table XII. Profile $(R_1, R_2, R_3) \in \times_{i \in J} \mathcal{R}(F_i)$ in Claim 1 of Example 4.

	$R_1 : 2_w^N \longrightarrow \Re$			$R_2 : 2_w^N \longrightarrow \Re$			$R_3 : 2_w^N \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{a\}$	$\{b\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{a\}$	$\{a\}$	$\{a\}$
Preferences	$\{b\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{b\}$	$\{c\}$
	$\{c\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{c\}$	$\{c\}$	$\{b\}$

Table XIII. Profile $(\hat{R}_1, \hat{R}_2, \hat{R}_3) \in \times_{i \in J} \mathcal{R}(F_i)$ in Claim 2 of Example 4.

	$\hat{R}_1 : 2_w^N \longrightarrow \Re$			$\hat{R}_2 : 2_w^N \longrightarrow \Re$			$\hat{R}_3 : 2_w^N \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{a\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{c\}$
Preferences	$\{b\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{c\}$	$\{b\}$
	$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$

Table XIV. Profile $(\tilde{R}_1, \tilde{R}_2, \tilde{R}_3) \in \times_{i \in J} \mathcal{R}(F_i)$ in Claim 3 of Example 4.

	$\tilde{R}_1 : 2_w^N \longrightarrow \Re$			$\tilde{R}_2 : 2_w^N \longrightarrow \Re$			$\tilde{R}_3 : 2_w^N \longrightarrow \Re$		
$W_t =$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{b\}$	$\{c\}$
	$\{c\}$	$\{c\}$	$\{c\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$	$\{b\}$	$\{c\}$
Preferences	$\{a\}$	$\{b\}$	$\{b\}$	$\{c\}$	$\{a\}$	$\{a\}$	$\{c\}$	$\{c\}$	$\{b\}$
	$\{b\}$	$\{a\}$	$\{a\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{a\}$	$\{a\}$	$\{a\}$

One can construct similar examples to show that some “natural” extensive form mechanisms fail to implement the socially optimal rule in subgame perfect equilibria in the general framework.

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