Málaga Economic Theory Research Center Working Papers

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WP 2009-3
June 2009

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June 2, 2009

Abstract

We explore to what extent we can propose fixed negotiation rules as well as simple mechanisms (or protocols) that guarantee that political parties can form stable coalition-governments. We analyze the case where three parties can hold office in the form of two-party coalitions. We define the family of Weighted Rules, that select political agreements as a function of the bliss-points of the parties, and electoral results (Gamson's Law and equal-share among others are included). We show that every weighted rule yields a stable coalition. We make use of the theory of implementation to design a protocol (in the form of a mechanism) that guarantees that a stable coalition will govern. We find that no dominant-solvable mechanism can be used for this purpose, but there is a simultaneous-unanimity mechanism that implements it in Nash and strong Nash equilibrium.

Keywords: Coalition-government; Stability; Nash-implementation.
Jel: D71, D72.

*The author thanks Pablo Amorós, Carmen Beviá, Francis Bloch, Luis Corchón, Samuel Danthine, Dinko Dimitrov and Miguel Angel Meléndez for their helpful comments. Financial assistance from Ministerio de Ciencia e Innovación under the project SECO2008-03674/ECON, and Junta de Andalucía under the projects SEJ1645, SEJ522 is gratefully acknowledged.

1 Introduction

When no political party reaches a strict majority in elections, coalition governments are usually formed. Political parties are the main actors in the bargaining process that leads to different types of government formation. Coalition governments are the norm in many European governments as shown for instance in the empirical works of Diermeier et al. (2003) and Müller and Str∅m (2000). When a coalition of political parties governs, stability of such government is one of the main concerns of voters. In brief, stability requires that no (majoritarian) coalition has incentives to interfere with the governing coalition. In this paper, we explore whether there are fixed negotiation rules as well as simple mechanisms (set of rules or protocols) that guarantee that after the elections, parties can come up with a stable coalition-government. It is a novelty of this paper that we make use of the theory of implementation to analyze this coalition-government problem.

We study the case where three parties can hold office in the form of two-party coalitions.[1] Each party is identified with a bliss-point in a multidimensional policy space. Elections have already taken place. We assume that, when forming coalition-governments, parties are not only policy-motivated, they also derive some benefits from holding office. The policy adopted by a coalition is an agreement. Each agreement describes the policy that a coalition intends to pursue if it governs.

Under majority rule and a multidimensional policy space, no coalition can propose a policy that majority defeats every other policy (see Schofield, 1983; and Saari, 1997, for a formal proof of this statement). This observation was made by Plott (1967), who also suggested that there are some restrictions or constraints on the policies that a coalition can propose. Following Plott, if we account for the rule under which coalitions operate, it can be shown that there exist agreements that cannot be majority defeated.

In this paper, we propose some fixed negotiation rules under which coalitions can operate: the Weighted Rules. First, we define the weighted functions that distribute bargaining power among parties as a function of their votes (or seats in parliament). We account for a diverse distribution of bargaining power: from equal-share to proportional share. Second, we define

[1] The empirical evidence on Western Europe governments analyzed in Müller and Str∅m (2000) provides several examples within our scope (Germany and Luxemburg among others), where a three-party system and a two-party coalition government has been the norm during most of the time.

the family of Weighted Rules, that select political agreements as a function of bargaining powers and bliss-points of the parties. The agreements derived in this way are a weighted average of the bliss-points of the parties.

Our first result is that every weighted rule yields a unique stable coalition: the agreement of this coalition cannot be majority defeated by the agreements of other coalitions. For each weighted rule, the *stable function* selects the only coalition that is stable.

We then study the existence of mechanisms implementing the stable function. A mechanism specifies a space of messages (one for each party), and an outcome function (once each party announces a message, the outcome function selects a coalition). We show that there is no mechanism implementing the stable function in dominant strategies. We then analyze implementation in Nash equilibrium. We explore simple mechanisms where parties' messages are simultaneous (in this way we avoid a rule of order with the *formateur* as a player with first-move advantage). We study the simultaneous-unanimity mechanism where each party simultaneously announces a coalition that includes itself.[2] If two parties announce the same coalition, the outcome function selects such coalition. We show that this mechanism fails at implementing the stable function in Nash equilibrium. We then analyze an extended version of this mechanism where parties can announce any coalition (it only adds an extra element to the message space of each party). We show that this mechanism implements in Nash and strong Nash equilibrium the stable function. Furthermore, every equilibrium of this mechanism (Nash and strong Nash) has a natural interpretation since the two parties that form the stable coalition announce such coalition.[3]

The present paper is related to the literature on legislative bargaining games. This literature provides a *positive* analysis of legislatures where the decision making processes take the form of bargaining. Based on Baron and Ferejohn (1989), these models provide "structure-induced equilibrium" in contexts where the core is empty. Thus, predictions on the governing coalitions and legislative voting positions are derived as a result of different bargaining processes among the parties (see for instance Baron, 1991; Banks and Duggan, 2000, 2006; Jackson and Moselle, 2002). In contrast to this literature, our analysis is *normative* since we propose fixed negotiation rules

[2] The name of this game is due to von Neumann and Morgenstern (1944, section 57.3.1). See also Yi (2003) for a survey on games of coalition formation.

[3] We only consider *privacy-preserving mechanisms,* as defined by Hurwicz (1972), since we do not account for the information that a party has on the preferences of its partners.

as a way of escaping from the standard results of an empty core.

There are two other related papers that also describe some ways of escaping from the empty core (Kirchsteiger and Puppe, 1997 and Aragonès, 2007). In contrast to our proposal, these authors do not specify a concrete procedure describing how the coalitions of parties reach political agreements. Kirchsteiger and Puppe (1997) show that office-seeking incentives play a central role to guarantee stable coalition governments. They also account for policy-motivated incentives and show that these incentives guarantee stability when there are three parties. The policy-motivated incentives, as introduced by these authors are somehow *ad hoc*, since the preferences of the parties over coalitions depend on the distance between their political positions. We show that preferences of the parties, as represented by these authors, can be implicitly deduced when parties make agreements according to a weighted rule. Aragonès (2007) proposes a two-dimensional policy space where four political parties have symmetric political positions. She characterizes the stable government configurations (coalitions and political agreements) in terms of parties' reservation values, and intensity of preferences over issues.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 defines the weighted rules and derives the results on stability. Section 4 presents the results on implementation. Section 5 concludes.

2 The model

There are three political parties $N = \{1, 2, 3\}$. Let $\mathbb{R}^M$ be the policy space where $M \geq 2$ is the number of political issues. Each party $i \in N$ has a different bliss-point $x_i \in \mathbb{R}^M$ that specifies its ideal policy on each of the political issues.

Elections have already taken place. Each party's number of votes (or seats in parliament) are given by $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ where $\alpha_i > 0$ for every $i \in N$. A strict majority of votes is required to govern. No single party has a strict majority, but every two-party coalition is a minimal winning coalition. We just consider minimal winning coalitions.

A coalition is denoted by S, where $S \in \{\{1,2\}, \{1,3\}, \{2,3\}\}$. An agreement for coalition S is given by $x^S \in \mathbb{R}^M$, and it specifies the policy that coalition S will support if it governs. A *profile of agreements* x describes an agreement for each coalition, $x = (x^{12}, x^{13}, x^{23})$.

Preferences of party i over agreements are represented by

$$u_i(x^S) = \begin{cases} k_i - \left\|x^S - x_i\right\| & \text{when } i \in S \\ -\left\|x^S - x_i\right\| & \text{when } i \notin S \end{cases} \tag{1}$$

where $k_i > 0$ is party i's benefits from holding office, and $\|.\|$ is the Euclidean distance between the agreement of coalition S and the bliss-point of party i. We take k_i sufficiently large to guarantee that the most preferred agreement of each party is one where it is included.[4]

Given two different agreements $x^S, x^{S'}$, we say that party i strictly prefers coalition S to coalition S' when $u_i(x^S) > u_i(x^{S'})$ (when convenient we use notation $S \succ_i S'$). Each profile of agreements x, induces a profile of preferences over coalitions $\succ = (\succ_1, \succ_2, \succ_3)$, where each $\succ_i$ specifies party i's preferences over the three coalitions. For the sake of simplicity we do not consider indifferences.[5]

We pursue two properties that are key when forming coalition-governments: efficiency and stability.

We say that x^S is *efficient* when there is no $x' \in \mathbb{R}^M$ such that $u_i(x') > u_i(x^S)$ for all $i \in S$. Thus, if x^S is efficient, the parties included in coalition S cannot improve simultaneously by choosing another agreement. Every efficient agreement is located in the line between the bliss points of the parties. A *profile of agreements x is efficient* if every agreement in x is efficient.

Given a profile of agreements x and its induced preferences over coalitions $\succ$, we say that *coalition S is stable* if there is no other coalition S' such that $S' \succ_i S$ for all $i \in S'$. We say that the *profile of agreements x is stable* when there is some coalition S that is stable.

When S is stable, there is no other coalition S' such that its members strictly prefer agreement $x^{S'}$ to x^S. If S is stable, we say that S cannot be blocked by any other coalition. If x is not stable, however, every coalition is blocked by another coalition. In this case, the induced preferences over coalitions contain one of the following two circles: $\{1,2\} \succ_1 \{1,3\} \succ_3 \{2,3\} \succ_2 \{1,2\}$, or $\{1,3\} \succ_1 \{1,2\} \succ_2 \{2,3\} \succ_3 \{1,3\}$.[6]

[4] Our proposal is not within the scope of hedonic coalition structures (see Banerjee et al. 2001, Bogomolnaia and Jackson 2002) since preferences over those coalitions where a party is not included, depend on the agreement made by the other parties. On this point we differ from Aragonès (2007), Kirchsteiger and Puppe (1997), but we coincide with Rijt (2008).

[5] This is a minor assumption in a multidimensional policy space.

[6] There are however examples of preferences containing circles and where there is a

3 The weighted rules

In this section we propose a family of rules that select profiles of agreements as a function of the number of votes and bliss points of the parties.

The negotiation of agreements within coalitions can be made according to different bargaining weights. Several authors analyze bargaining rules from a theoretical, empirical, and experimental point of view. For instance, Carrol and Cox (2007) provide an empirical analysis that supports Gamson's Law (the bargaining power of each party is proportional to the party's contribution of seats to the coalition) as the criterion that parties follow when coalition-pacts are pre-electoral. As shown in the experiments proposed by Fréchette et al. (2005), post-electoral pacts are however characterized by other distributions of bargaining power such as the ability to form majoritarian coalitions. As we show next, our proposal of weighted rules defines a family of rules sufficiently extensive as to include different procedures that determine bargaining powers.

A weighted function distributes bargaining power among political parties. Formally, the *weighted function* f assigns to each party's number of votes (or seats), a fraction of power $f(\alpha_i) > 0$ such that $\sum\limits_{i \in N} f(\alpha_i) = 1$. Particular instances of weighted functions are the proportional function $f(\alpha_i) = \frac{\alpha_i}{\sum\limits_{i \in N} \alpha_i}$, or the equal-share function $f(\alpha_i) = \frac{1}{3}$. Each weighted function defines what we call a weighted rule.[7]

Definition: *The* **weighted rule** W_f *associated to a weighted function* f *selects, for each distribution of votes and parties' bliss points, a profile of agreements,* $W_f(\alpha, x_1, x_2, x_3) = (x_f^{12}, x_f^{13}, x_f^{23})$*, where each* x_f^S *satisfies:*

$$x_f^S = \frac{\sum\limits_{i \in S} f(\alpha_i) x_i}{\sum\limits_{i \in S} f(\alpha_i)}. \tag{2}$$

Thus, each x_f^S is a weighted average of the bliss points of the parties included in S. We refer to every x_f^S as a weighted agreement.

stable coalition.

[7]Note that every convex combination of the proportional and equal-share function also qualifies as a weighted function.

To provide some examples, if f is the proportional function, then $x_f^S = \frac{\sum\limits_{i\in S}\alpha_i x_i}{\sum\limits_{i\in S}\alpha_i}$, i.e., the weighted rule yields agreements according to Gamson's Law.[8] If f is the equal share function, then every agreement selected by the weighted rule is the midpoint of the parties' bliss-points, i.e., $x_f^S = \sum\limits_{i\in S}\frac{x_i}{2}$.

Let $S = \{i, j\}$. Substituting Expression (2) in the utility function of party i yields

$$u_i(x_f^S) = k_i - \left\| \frac{f(\alpha_i)x_i + f(\alpha_j)x_j}{f(\alpha_i)+f(\alpha_j)} - x_i \right\| ,$$

and simplifying

$$u_i(x_f^S) = k_i - \frac{f(\alpha_j)}{f(\alpha_i)+f(\alpha_j)} \left\| x_j - x_i \right\| . \qquad (3)$$

Hence, when agreements are selected according to a weighted rule, parties' preferences over those coalitions where they are included depend on the distance between the parties' bliss points.[9] Appendix A shows that Expression 3 can be also derived from the Generalized Nash Bargaining solution of a bargaining game within the parties in the coalition.

The following example illustrates a weighted rule.

Example 1: *Consider a scenario where the ideal political positions of the parties (x_1,x_2,x_3) are as represented in Figure 1. We propose a weighted function such that $f(\alpha_1) = f(\alpha_2) = .25$, $f(\alpha_3) = .5$. Then, we can represent the profile of agreements selected by the weighted rule (x_f^{12},x_f^{13},x_f^{23}). As it is shown in Figure 1, the lines from the bliss points to the agreements of the opposite side cross in a single point (this property satisfies regardless of the weighted rule that we consider).*

[8] This is in fact the norm in two-party races that operate under proportional representation (see for instance Austen-Smith and Banks, 1988; Grossman and Helpman, 1996).

[9] Preferences over coalitions as proposed by Kirchsteiger and Puppe (1997) are

$$u_i(S) = \begin{cases} g(\frac{\alpha_j}{\alpha_i+\alpha_j} d(x_j, x_i)) \text{ if } i \in S \\ 0 \text{ if } i \notin S \end{cases}$$

where d is a distance function, and g is strictly decreasing in d. These authors claim that, in their proposal, there is no explicit specification of the actual outcome of the bargaining process. We show, however, that their preferences intrinsically specify a procedure to select agreements.

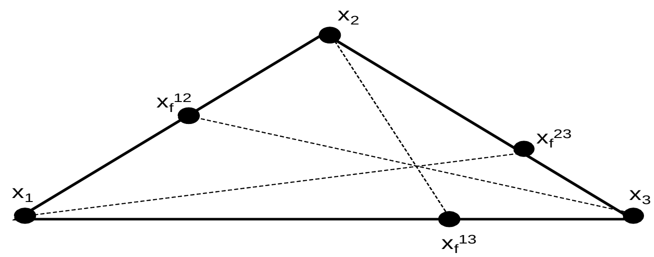

Figure 1: A profile of weighted agreements

For any given value of the parameters k_i that measure the benefits from holding office, we can derive the induced preferences over coalitions $\succ$ (ordered from top to bottom):

Party 1	Party 2	Party 3
12	12	23
13	23	13
23	13	12

In this example, coalition $S = \{1,2\}$ is stable (the two other coalitions are blocked by $\{1,2\}$).

It is easy to see that the weighted rules always yield efficient profiles of agreements since these agreements are located in the Pareto set (which corresponds to the line between the two parties' bliss-points). Moreover, as we show next, they also yield stable profiles of agreements.

Theorem 1 *Every weighted rule W_f yields stable profiles of agreements.*

Proof. Suppose to the contrary that there is a profile of agreements selected by a weighted rule that is not stable. Then, preferences over coalitions must contain a circle. Consider without loss of generality that $\{1,2\} \succ_1 \{1,3\} \succ_3 \{2,3\} \succ_2 \{1,2\}$. Then, by Expression (3):

$$\{1,2\} \succ_1 \{1,3\} \text{ implies } \|x_1 - x_2\| \tfrac{f(\alpha_2)}{f(\alpha_1)+f(\alpha_2)} < \|x_1 - x_3\| \tfrac{f(\alpha_3)}{f(\alpha_1)+f(\alpha_3)} \qquad (a)$$
$$\{2,3\} \succ_2 \{1,2\} \text{ implies } \|x_2 - x_3\| \tfrac{f(\alpha_3)}{f(\alpha_2)+f(\alpha_3)} < \|x_1 - x_2\| \tfrac{f(\alpha_1)}{f(\alpha_1)+f(\alpha_2)} \qquad (b)$$
$$\{1,3\} \succ_3 \{2,3\} \text{ implies } \|x_1 - x_3\| \tfrac{f(\alpha_1)}{f(\alpha_1)+f(\alpha_3)} < \|x_2 - x_3\| \tfrac{f(\alpha_2)}{f(\alpha_2)+f(\alpha_3)}. \qquad (c)$$

From (a), $\|x_1 - x_2\| < \|x_1 - x_3\| \frac{f(\alpha_3)(f(\alpha_1)+f(\alpha_2))}{f(\alpha_2)(f(\alpha_1)+f(\alpha_3))}$. Substituting it in (b), $\|x_2 - x_3\| \frac{f(\alpha_3)}{f(\alpha_2)+f(\alpha_3)} < \|x_1 - x_3\| \frac{f(\alpha_3)(f(\alpha_1)+f(\alpha_2))}{f(\alpha_2)(f(\alpha_1)+f(\alpha_3))} \frac{f(\alpha_1)}{f(\alpha_1)+f(\alpha_2)}$, that simplifying yields, $\|x_2 - x_3\| \frac{f(\alpha_2)}{f(\alpha_2)+f(\alpha_3)} < \|x_1 - x_3\| \frac{f(\alpha_1)}{f(\alpha_1)+f(\alpha_3)}$, which contradicts (c). Therefore, preferences do not contain a circle, and so, there is at least a coalition that is stable.[10] ■

As a consequence of Theorem 1, we can derive the following result.

Corollary 1: *Every profile of weighted agreements yields a unique stable coalition. Furthermore, if S is stable, then coalition S must be top-ranked for both parties in S and bottom-ranked for the remaining party.*

Proof. Suppose without loss of generality that $\{1,3\}$ is stable. By Theorem 1, every profile of agreements selected by a weighted rule is such that preferences over coalitions do not contain a circle. Three cases are possible.
Case 1: $\{1,3\} \succ_1 \{1,2\}$ and $\{1,3\} \succ_3 \{2,3\}$. It implies that $\{1,3\}$ is stable. Since neither can $\{2,3\}$ be top-ranked for party 1, nor can $\{1,2\}$ be top-ranked for party 3, it must be that $\{1,3\}$ is top-ranked for party 1 and party 3. In addition, by efficiency of the weighted agreements made by $\{1,2\}$ and $\{2,3\}$ respectively, coalition $\{1,3\}$ must be bottom-ranked for party 2.
Case 2: $\{1,2\} \succ_1 \{1,3\}$ and $\{1,2\} \succ_2 \{2,3\}$. Then, to guarantee that $\{1,3\}$ is stable, $\{1,3\}$ must be top ranked for party 2, a contradiction.
Case 3: $\{2,3\} \succ_2 \{1,2\}$ and $\{2,3\} \succ_3 \{1,3\}$. Then, to guarantee that $\{1,3\}$ is stable, $\{1,3\}$ must be top-ranked for party 2, a contradiction.
Finally, suppose that there are two stable coalitions S and S'. Then, both coalitions must be top-ranked for the party in the intersection $S \cap S'$, but it contradicts the fact that there are no indifferences. ■

We describe next all the profiles of preferences over coalitions where $\{1,3\}$ is stable (changing the identity of the parties we can derive every possible profile of preferences over coalitions).

1	2	3
<u>13</u>	12	<u>13</u>
12	23	23
23	<u>13</u>	12

1	2	3
<u>13</u>	23	<u>13</u>
12	12	12
23	<u>13</u>	23

1	2	3
<u>13</u>	23	<u>13</u>
12	12	23
23	<u>13</u>	12

1	2	3
<u>13</u>	12	<u>13</u>
23	23	23
12	<u>13</u>	12

[10] In a one-dimensional policy space Le Breton et al. (2008) show that Gamson's Law yields a stable coalition, here we generalize this particular result in two directions: multi-dimensional policy space and a wider family of weighted rules.

When describing these profiles, we do not only account for the results in Corollary 1, but also for the fact that every agreement is efficient. Thus, if $\{1,2\} \succ_2 \{2,3\}$ then by efficiency of the weighted agreement made by $\{2,3\}$, party 3's preferences cannot be such that $\{1,2\} \succ_3 \{2,3\}$.

The weighted rules induce a restricted domain of preferences over coalitions. In view of the above results, there are up to twelve different profiles of preferences over coalitions that can be induced by the weighted rules.

4 Implementing the stable function

In this section, we explore to what extent there are simple protocols (or practices) that lead to stable coalition-governments. Such protocols must work whatever the underlying information of parties concerning the preferences of its partners. The theory of implementation provides the proper tools to solve our question. Some definitions are needed to proceed.

The weighted rules induce a restricted domain of profiles of preferences that we denote by R. The *Stable Function* $\mathbb{S} : R \rightarrow \{\{1,2\}, \{1,3\}, \{2,3\}\}$ selects, for each admissible profile of preferences over coalitions $\succ \in R$, the stable coalition.

A mechanism is a pair (M, g) where $M = \underset{i \in N}{X} M_i$, each M_i is the message space for party i, and $g : M \rightarrow \{\{1,2\}, \{1,3\}, \{2,3\}\}$ is the outcome function that assigns to each profile of messages, a coalition.

An equilibrium concept specifies the strategic behavior of individuals faced with mechanism (M, g). We use the notions of *dominant strategies, Nash equilibrium,* and *strong Nash equilibrium* as equilibrium concepts. When the profile of preferences is $\succ$, a coalition generated as the outcome of an equilibrium of the mechanism (M, g) is called equilibrium coalition and is denoted by $E(M, g, \succ)$.

Definition: *The mechanism* (M, g) ***implements the Stable Function*** $\mathbb{S}$ *via an equilibrium concept when for all admissible profile of preferences* $\succ \in R$, the equilibrium coalition of (M, g) at $\succ$ is the stable coalition*, i.e.,* $E(M, g, \succ) = \mathbb{S}(\succ)$ for all $\succ \in R$.

When a mechanism implements the Stable Function then, in every realization of the preferences of parties over coalitions, the resulting outcome obtained through strategic behavior (on the part of the parties facing the mechanism) is a stable coalition.[11]

When a mechanism implements via two different equilibrium concepts, we refer to double-implementation.

We first show that there is no mechanism implementing the Stable Function in dominant strategies.

Theorem 2 *The Stable Function is not implementable in dominant strategies.*

Proof. According to Corollary 1, the domain of preferences R does not have a Cartesian product structure. Therefore, the standard revelation principle cannot be applied here.[12]

Each party i has four types of preference relations denoted by $\{\succ_i^1, \succ_i^2, \succ_i^3, \succ_i^4\}$. Consider the following three admissible profiles of preferences:

$(\succ_1^1, \succ_2^1, \succ_3^1)$

1	2	3
$\underline{13}$	12	$\underline{13}$
12	23	23
23	$\underline{13}$	12

$(\succ_1^2, \succ_2^1, \succ_3^2)$

1	2	3
$\underline{12}$	$\underline{12}$	23
13	23	13
23	13	$\underline{12}$

$(\succ_1^1, \succ_2^2, \succ_3^2)$

1	2	3
13	$\underline{23}$	$\underline{23}$
12	12	13
$\underline{23}$	13	12

If we suppose that there exists a mechanism (M, g) implementing $\mathbb{S}$ in dominant strategies, then each party i should have a dominant strategy for each type of preferences. Let m_i^1 be the dominant strategy for party i when party i's preferences are $\succ_i^1$, and let m_i^2 be the dominant strategy of party i when party i's preferences are $\succ_i^2$. According to the above described preferences, the outcome function g should satisfy:

$$\begin{aligned} g(m_1^1, m_2^1, m_3^1) &= \{1,3\} \\ g(m_1^2, m_2^1, m_3^2) &= \{1,2\} \\ g(m_1^1, m_2^2, m_3^2) &= \{2,3\}\,. \end{aligned} \quad (4)$$

[11] Bloch (1996) shows, in a setting of coalition formation with externalities, that every core stable structure can be obtained as equilibrium outcome of a sequential coalition formation game. Implementation of the core requires, additionally, that all equilibrium outcome of the game be a core stable structure (see Serrano 1995; Serrano and Vohra 1997).

[12] See also Amorós et al. (2002) for another restricted domain that has a non-Cartesian product structure, and where the proposed social choice function is not implementable in dominant strategies.

However, if we consider the following non admissible profile of preferences:[13]

$(\succ_1^1, \succ_2^1, \succ_3^2)$

1	2	3
13	12	23
12	23	13
23	13	12

where there is no stable coalition, the outcome function can select any coalition $g(m_1^1, m_2^1, m_3^2) \in S$. Suppose that $g(m_1^1, m_2^1, m_3^2) = \{1,2\}$, then given its preference relations, party 3 improves by deviating to m_3^1 since then, as specified by (4), the outcome is $\{1,3\}$ contradicting that m_3^2 is a dominant strategy for party 3 when its preferences are $\succ_3^2$. Suppose that $g(m_1^1, m_2^1, m_3^2) = \{1,3\}$, then given its preferences, party 2 improves by deviating to m_2^2 since then, as specified in (4), the outcome is $\{2,3\}$ contradicting that m_2^1 is a dominant strategy for party 2 when its preferences are $\succ_2^1$. Suppose that $g(m_1^1, m_2^1, m_3^2) = \{2,3\}$, then given its preferences, party 1 improves by deviating to m_1^2 since then, as specified in (4), the outcome is $\{1,2\}$ which contradicts that m_1^1 is a dominant strategy for party 1 when its preferences are $\succ_1^1$. This proves that no party has a dominant strategy, in contradiction with the assumption that the Stable Function is implementable in dominant strategies. ■

The direct mechanism is the general canonical mechanism where the message space of each party contains its profiles of preferences over coalitions, and the outcome function selects the stable coalition from the announced profiles of preferences. We find that the direct mechanism fails in our setting because it is not a dominant strategy for each party to truthfully reveal its preferences. Furthermore, there is no other mechanism implementing in dominant strategies the stable function.

We consider next implementation in Nash equilibrium. For that, we propose a simultaneous-move mechanism that avoids a rule of order among parties. Each party simultaneously announces a coalition that includes itself, i.e., $M_i = \{\{i,j\}, \{i,k\}\}$ for all $i \in N$. The outcome function is such that if two parties announce the same coalition, such coalition is formed. If every two parties announce a different coalition there is a lottery, denoted by L, that assigns equal probability to the three possible coalitions. This lottery

[13] Here we consider that parties do not know the domain of preferences.

represents the situation where there is no consensus and therefore, any coalition can finally govern. [14]

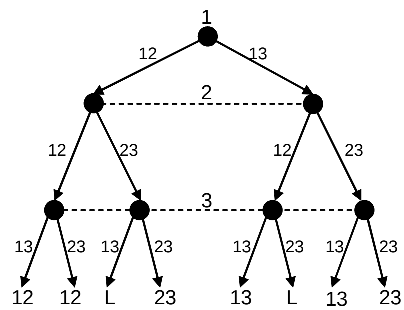

Figure 2: Simultaneous-unanimity mechanism

From a normative perspective, this mechanism is *simple* since it has a reduced message space, and *natural* since it can be interpreted in the context where it is applied. We call this mechanism the simultaneous-unanimity mechanism (based on the simultaneous-unanimity game of von Neumann and Morgenstern, 1944).

The messages where each party announces its top-ranked coalition are Nash equilibrium. We find, however, that the proposed mechanism has Nash equilibria which do not yield stable coalitions. Consider, for instance, the following profile of preferences where $\{1,2\}$ is stable:

1	2	3
12	12	23
13	23	13
23	13	12

The profile of messages $(m_1, m_2, m_3) = (\{1,3\}, \{2,3\}, \{2,3\})$ is a Nash equilibrium where $g(m) = \{2,3\}$. If party 1 or party 3 deviate, they may not improve, and if party 2 deviates, the outcome function selects the lottery that may not improve party 2. Thus, we conclude that the proposed mechanism fails at implementing the stable function in Nash equilibrium (even when the

[14] Preferences are measured according to von Neumann-Morgenstern utility functions.

lottery L is substituted by a concrete coalition).[15] In this example, party 2 accepts forming government with party 3 even when it is not its top-ranked option.

Next, we aim at finding the minimal extension of the previous mechanism that guarantees Nash-implementation. We enlarge each party's message space with an additional message so that $M_i = \{\{1,2\},\{1,3\},\{2,3\}\}$ for all $i \in N$. As in the previous mechanism, each party simultaneously announces a message. We call this proposal the enlarged-simultaneous-unanimity mechanism (see Figure 3).

There are basically two rules that describe the outcome function.
Rule 1: if two (or more) parties $\{i,j\}$ announce the same coalition $S = \{i,j\}$, then the outcome function proposes coalition S.
Rule 2: if two parties $\{i,j\}$ announce the same coalition $S \neq \{i,j\}$ and the remaining party announces another coalition S', the outcome function selects the coalition that is complement to the party in the intersection, i.e., $g(m) = N \backslash S \cap S'$. There are two exceptions to this rule: when party 1 and party 2 announce $S = \{2,3\}$, then $g(m) = m_3$, and when party 2 and party 3 announce $S = \{1,2\}$, then $g(m) = m_1$.

The mechanism preserves anonymity since every party has the same message space, and it is selected by the outcome function the same number of times.

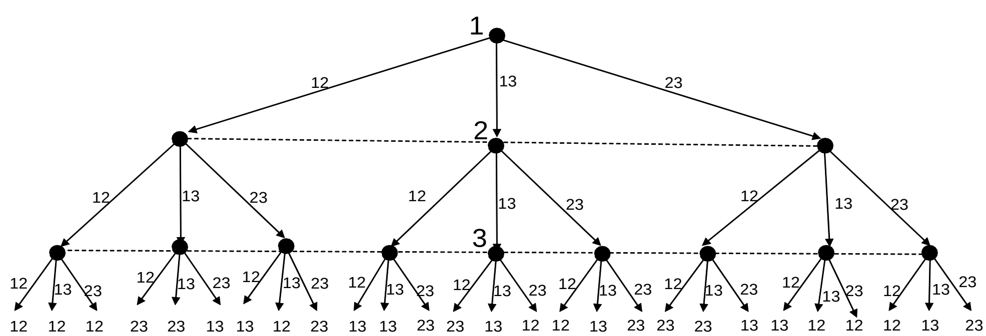

Figure 3: Enlarged-simultaneous-unanimity mechanism

The outcome function is designed in such a way that in every Nash equilibrium, the two parties that form the stable coalition announce such coalition,

[15] It can be shown that the simultaneous-unanimity mechanism implements the stable function in strong-Nash equilibrium and coalition-proof Nash equilibrium (see Bernheim et al. 1987 for a definition of the later equilibrium concept).

regardless of the message of the remaining party. Thus, for every other profile of messages, at least one of the political parties improves by deviating. Here comes our final result.

Theorem 3 *The enlarged-simultaneous-unanimity mechanism double implements the Stable Function in Nash and strong-Nash equilibrium.*

Proof. (See Appendix B). ■

The proposed mechanism is immune to deviations of coalitions since every Nash equilibrium of the game is also strong-Nash equilibrium.[16]

5 Conclusion

When agreements between political parties are binding, and are made according to a weighted rule, we escape from the standard results of an empty core. Gamsons's Law and equal share, among others, are procedures to reach agreements that qualify as weighted rules. As we have shown, every weighted rule yields a unique stable coalition. This result gives us the opportunity to analyze the problem of coalition governments from a normative perspective since the *stable coalition* becomes the socially desirable outcome.

The weighted agreements induce a restricted domain of preferences over coalitions. The fact that this admissible domain has a non-Cartesian product structure is the key to show that no dominant-solvable protocol can be designed to guarantee a stable coalition-government. This is an unexpected result since, as we have shown, the admissible domain of preferences over coalitions is particularly reduced.

Our personal view is that a mechanism (or protocol) applied to solve the problem of coalition governments must be sufficiently simple and natural. We find, however, that the simplest and most natural mechanism that we can propose in this line (the simultaneous-unanimity mechanism), does not implement in Nash equilibrium the stable function. The positive result comes from showing that there is a way of extending this mechanism such that the stable coalition is derived from every Nash and strong Nash equilibrium of the mechanism.[17]

[16] See Suh (1997) for a necessary and sufficient condition on double implementation in Nash and strong Nash equilibrium.

[17] As pointed out by Jackson (2001), once the preferences of the individuals are specified, the mechanism induces a game.

We then conclude that there are political scenarios where we can design procedures that reach efficient and stable coalition-governments. These procedures may correct those situations where parties cannot agree by themselves on a coalition to govern.

APPENDIX

Appendix A: The weighted rule and the Generalized Nash Bargaining Solution[18]

We show next that the utility that the parties in $S = \{i, j\}$ derive from a weighted agreement x_f^S coincides with the utility derived from the two-agents Generalized Nash Bargaining Solution of the game with bargaining set $\{(v_i, v_j) : v_i + v_j \leq \|x_i - x_j\|\}$, disagreement point $(u_i(x_j), u_j(x_i))$, and agents' weights $f(\alpha_i), f(\alpha_j)$.

The generalized Nash Bargaining solution to two-person bargaining problems is defined by maximizing over the bargaining set the product $v_i^{f(\alpha_i)} v_j^{f(\alpha_j)} = K$. Solving this problem, the slope of the bargaining set is tangent to the slope of $v_i = (\frac{K}{v_j^{f(\alpha_j)}})^{\frac{1}{f(\alpha_i)}}$. Thus, in the bargaining solution we have $\frac{v_i^* f(\alpha_j)}{v_j^* f(\alpha_i)} = 1$ and since $v_i^* + v_j^* = \|x_i - x_j\|$, we deduce that $v_i^* = \|x_i - x_j\| \frac{f(\alpha_i)}{f(\alpha_i)+f(\alpha_j)}$, $v_j^* = \|x_i - x_j\| \frac{f(\alpha_j)}{f(\alpha_i)+f(\alpha_j)}$. Each party utility in the disagreement point is given by $u_i(x_j) = k_i - \|x_j - x_i\|$ and $u_j(x_i) = k_j - \|x_i - x_j\|$. Thus, the utility derived by each party in the bargaining solution is $u_i = u_i(x_j) + v_i^*$ and $u_j = u_j(x_i) + v_j^*$, where substituting the values v_i^*, v_j^* yields $u_i = k_i - \frac{f(\alpha_j)}{f(\alpha_i)+f(\alpha_j)} \|x_i - x_j\|$, $u_j = k_j - \frac{f(\alpha_i)}{f(\alpha_i)+f(\alpha_j)} \|x_i - x_j\|$ (this coincides with Expression 3).[19]

Appendix B: Proof of Theorem 3

Proof. First, we show that $\mathbb{S}(\succ) \in E(M, g, \succ)$ for all $\succ \in R$. Given some $\succ \in R$, consider that $\mathbb{S}(\succ) = \{1, 3\}$. Then, by Corollary 1, $\{1, 3\}$ is top ranked for party 1 and party 3. Let $m = (\{1, 3\}, m_2, \{1, 3\})$ where $g(m) = \{1, 3\}$ for all $m_2 \in M_2$. Neither party 1, nor party 3 have incentives to deviate, and party 2 cannot modify the outcome. Thus, m is a Nash equilibrium and so, $\{1, 3\} \in E(m, g, \succ)$. The same reasoning applies when either $\mathbb{S}(\succ) = \{1, 2\}$ or $\mathbb{S}(\succ) = \{2, 3\}$.

Second, we show that $E(M, g, \succ) \in \mathbb{S}(\succ)$ for all $\succ \in R$. Consider the following Nash equilibria:

Let $m = (\{1, 2\}, \{1, 2\}, m_3)$ where $m_3 \in M_3$, then $g(m) = \{1, 2\}$. By Nash

[18] See Harsanyi and Selten (1972).

[19] Since every bilateral weighted agreement can be derived as a generalized Nash-bargaining solution, we wonder to what extent the weighted agreements have some non-cooperative foundations (see, for instance, Binmore et al. 1986).

equilibrium, $\{1,2\} \succ_1 \{1,3\}$ and $\{1,2\} \succ_2 \{2,3\}$. Thus, $\{1,2\}$ is top ranked for party 1 and party 2, which implies that $\{1,2\}$ is stable.
Let $m = (\{1,3\}, m_2, \{1,3\})$ where $m_2 \in M_2$, then $g(m) = \{1,3\}$. By Nash equilibrium, $\{1,3\} \succ_1 \{1,2\}$ and $\{1,3\} \succ_2 \{2,3\}$. Thus, $\{1,3\}$ is top ranked for party 1 and party 3, which implies that$\{1,3\}$ is stable.
Let $m = (m_1, \{2,3\}, \{2,3\})$ where $m_1 \in M_1$, then $g(m) = \{2,3\}$. By Nash equilibrium, $\{2,3\} \succ_2 \{1,2\}$ and $\{2,3\} \succ_3 \{1,3\}$. Thus, $\{2,3\}$ is top ranked for party 2 and party 3, and this implies that $\{2,3\}$ is stable.
Third, we show that the mechanism has no other Nash equilibrium.
Let $m = (\{1,2\}, \{1,3\}, \{1,2\})$ where $g(m) = \{2,3\}$. Since $g(\{1,2\}, \{1,2\}, \{1,2\}) = \{1,2\}$ and $g(\{2,3\}, \{1,3\}, \{1,2\}) = \{1,3\}$, to guarantee that m is a Nash equilibrium, $\{2,3\}$ must be top ranked for party 2 and party 3. Then, by Corollary 1, it must be the case that $\{2,3\}$ is bottom ranked for party 1. Then, party 1 improves by deviating since $g(\{2,3\}, \{1,3\}, \{1,2\}) = \{1,3\}$, a contradiction.
Let $m = (\{1,2\}, \{1,3\}, \{1,3\})$ where $g(m) = \{2,3\}$. Since $g(\{1,2\}, \{1,2\}, \{1,3\}) = \{1,2\}$ and $g(\{1,2\}, \{1,3\}, \{2,3\}) = \{1,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{2,3\}$ is top ranked for party 2 and party 3. Then, by Corollary 1, $\{2,3\}$ must be bottom ranked for party 1. In such case, party 1 can improve by deviating since $g(\{1,3\}, \{1,3\}, \{1,3\}) = \{1,3\}$, a contradiction.
Let $m = (\{1,2\}, \{1,3\}, \{1,3\})$ where $g(m) = \{2,3\}$. Since $g(\{1,2\}, \{1,2\}, \{1,3\}) = \{1,2\}$ and $g(\{1,2\}, \{1,3\}, \{2,3\}) = \{1,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{2,3\}$ is top ranked for party 2 and party 3. Then, by Corollary 1, $\{2,3\}$ must be bottom ranked for party 1. In this case, party 1 improves by deviating since $g(\{1,3\}, \{1,3\}, \{1,3\}) = \{1,3\}$, a contradiction.
Let $m = (\{1,2\}, \{2,3\}, \{1,2\})$ where $g(m) = \{1,3\}$. Since $g(\{1,3\}, \{2,3\}, \{1,2\}) = \{1,2\}$ and $g(\{1,3\}, \{2,3\}, \{2,3\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{1,3\}$ is top ranked for party 1 and party 3. Then, by Corollary 1, $\{1,3\}$ must be bottom ranked for party 2. In this case, party 2 improves by deviating since $g(\{1,2\}, \{1,2\}, \{1,2\}) = \{1,2\}$, a contradiction.
Let $m = (\{1,2\}, \{2,3\}, \{1,3\})$ where $g(m) = \{1,2\}$. Since $g(\{1,3\}, \{2,3\}, \{1,3\}) = \{1,3\}$ and $g(\{1,2\}, \{1,3\}, \{1,3\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{1,2\}$ is top ranked for party 1 and party 2. Then, by Corollary 1, $\{1,2\}$ must be bottom ranked for party 3. In this case, party 3 improves by deviating since

$g(\{1,2\},\{2,3\},\{2,3\}) = \{2,3\}$, a contradiction.
Let $m = (\{1,3\},\{1,2\},\{1,2\})$ where $g(m) = \{1,3\}$. Since $g(\{1,2\},\{1,2\},\{1,2\}) = \{1,2\}$ and $g(\{1,3\},\{1,2\},\{2,3\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{1,3\}$ is top ranked for party 1 and party 3. Then, by Corollary 1, $\{1,3\}$ is bottom ranked for party 2. In this case, party 2 improves by deviating since $g(\{1,3\},\{1,3\},\{2,3\}) = \{2,3\}$, a contradiction.
Let $m = (\{1,3\},\{1,2\},\{2,3\})$ where $g(m) = \{2,3\}$. Since $g(\{1,3\},\{1,3\},\{2,3\}) = \{1,2\}$ and $g(\{1,3\},\{1,2\},\{1,3\}) = \{1,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{2,3\}$ is top ranked for party 2 and party 3. Then, by Corollary 1, $\{2,3\}$ must be bottom ranked for party 1. In this case, party 1 improves by deviating since $g(\{1,2\},\{1,2\},\{2,3\}) = \{1,2\}$, a contradiction.
Let $m = (\{1,3\},\{1,3\},\{1,2\})$ where $g(m) = \{2,3\}$. Since $g(\{1,3\},\{2,3\},\{1,2\}) = \{1,2\}$ and $g(\{1,3\},\{1,3\},\{1,3\}) = \{1,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{2,3\}$ is top ranked for party 2 and party 3. Then, by Corollary 1, $\{2,3\}$ must be bottom ranked for party 1. In this case, party 1 improves by deviating since $g(\{2,3\},\{1,3\},\{1,2\}) = \{1,3\}$, a contradiction.
Let $m = (\{1,3\},\{1,3\},\{2,3\})$ where $g(m) = \{1,2\}$. Since $g(\{1,2\},\{1,3\},\{2,3\}) = \{1,3\}$ and $g(\{1,3\},\{2,3\},\{2,3\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{1,2\}$ is top ranked for party 1 and party 2. Then, by Corollary 1, $\{1,2\}$ must be bottom ranked for party 3. In this case, party 3 improves by deviating since $g(\{1,3\},\{1,3\},\{1,3\}) = \{1,3\}$, a contradiction.
Let $m = (\{1,3\},\{2,3\},\{1,2\})$ where $g(m) = \{1,2\}$. Since $g(\{1,2\},\{2,3\},\{1,2\}) = \{1,3\}$ and $g(\{1,3\},\{1,3\},\{1,2\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{1,2\}$ is top ranked for party 1 and party 2. Then, by Corollary 1, $\{1,2\}$ must be bottom ranked for party 3. In this case, party 3 can improve by deviating since $g(\{1,3\},\{2,3\},\{1,3\}) = \{1,3\}$, a contradiction.
Let $m = (\{2,3\},\{1,2\},\{1,2\})$ where $g(m) = \{2,3\}$. Since $g(\{2,3\},\{1,3\},\{1,2\}) = \{1,2\}$ and $g(\{2,3\},\{1,2\},\{2,3\}) = \{1,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{2,3\}$ is top ranked for party 2 and party 3. Then, by Corollary 1, $\{2,3\}$ must be bottom ranked for party 1. In this case, party 1 improves by deviating since $g(\{1,2\},\{1,2\},\{1,2\}) = \{1,2\}$, a contradiction.
Let $m = (\{2,3\},\{1,2\},\{2,3\})$ where $g(m) = \{1,3\}$. Since

$g(\{1,2\},\{1,2\},\{2,3\}) = \{1,2\}$ and $g(\{2,3\},\{1,2\},\{1,2\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{2,3\}$ is top ranked for party 2 and party 3. Then, by Corollary 1, $\{2,3\}$ must be bottom ranked for party 1. In this case, party 1 improves by deviating since $g(\{1,2\},\{1,2\},\{2,3\}) = \{1,2\}$, a contradiction.

Let $m = (\{2,3\},\{1,3\},\{1,2\})$ where $g(m) = \{1,3\}$ be a Nash equilibrium. If party 2 deviates, it can achieve either $g(\{2,3\},\{12\},\{1,2\}) = \{2,3\}$ or $g(\{2,3\},\{2,3\},\{1,2\}) = \{1,2\}$. Thus, it must be the case that $\{1,3\}$ is stable since otherwise, party 2 can improve by deviating. Then, by Corollary 1, $\{1,3\}$ must be top ranked for party 1 and party 3, and bottom ranked for party 2. It implies, however, that party 2 improves, a contradiction.

Let $m = (\{2,3\},\{1,3\},\{1,3\})$ where $g(m) = \{1,2\}$. Since $g(\{1,3\},\{1,3\},\{1,3\}) = \{1,3\}$ and $g(\{2,3\},\{1,2\},\{1,3\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{1,2\}$ is top ranked for party 1 and party 2. Then, by Corollary 1, $\{1,2\}$ must be bottom ranked for party 3. In this case, party 3 improves by deviating since $g(\{2,3\},\{1,3\},\{1,2\}) = \{1,3\}$. Hence, the described profile of messages cannot be a Nash equilibrium.

Let $m = (\{2,3\},\{1,3\},\{2,3\})$ where $g(m) = \{1,2\}$. Since $g(\{1,2\},\{1,3\},\{2,3\}) = \{1,3\}$ and $g(\{2,3\},\{2,3\},\{2,3\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{1,2\}$ is top ranked for party 1 and party 2. Then, by Corollary 1, $\{1,2\}$ must be bottom ranked for party 3. In this case, party 3 improves by deviating since $g(\{2,3\},\{1,3\},\{1,2\}) = \{1,3\}$, a contradiction.

Let $m = (\{2,3\},\{2,3\},\{1,2\})$ where $g(m) = \{1,2\}$. Since $g(\{1,3\},\{2,3\},\{1,2\}) = \{1,3\}$ and $g(\{2,3\},\{1,3\},\{1,2\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{1,2\}$ is top ranked for party 1 and party 2. Then, by Corollary 1, $\{1,2\}$ must be bottom ranked for party 3. In this case party 3 improves by deviating since $g(\{2,3\},\{2,3\},\{2,3\}) = \{2,3\}$, a contradiction.

Let $m = (\{2,3\},\{2,3\},\{1,3\})$ where $g(m) = \{1,3\}$. Since $g(\{1,2\},\{2,3\},\{1,3\}) = \{1,2\}$ and $g(\{2,3\},\{2,3\},\{2,3\}) = \{2,3\}$, to guarantee that m is a Nash equilibrium, it must be the case that $\{2,3\}$ is top ranked for party 2 and party 3. Then, by Corollary 1, $\{2,3\}$ must be bottom ranked for party 1. In this case, party 1 improves by deviating since $g(\{1,2\},\{2,3\},\{1,3\}) = \{1,2\}$, a contradiction. ■

References

[1] Amorós P., Corchón L.C. and Moreno B. (2002) The Scholarship Assignment Problem. *Games and Economic Behavior* 38, 1-18.

[2] Aragonès E. (2007) Government Formation in a Two-dimensional Policy Space. *International Journal of Game Theory* 35 (2) 151-184.

[3] Austen-Smith D. and Banks J. (1988) Elections, Coalitions, and Legislative Outcomes. *American Political Science Review* 82 (2), 405-422.

[4] Banerjee S., Konishi H. and Sönmez T. (2001) Core in a Simple Coalition Formation Game. *Social Choice and Welfare* 18, 135-153.

[5] Banks J. and Duggan J. (2000) A Bargaining Model of Collective Choice. *American Political Science Review* 94 (1), 73-88.

[6] Banks J. and Duggan J. (2006) A General Bargaining Model of Legislative Policy-making. *Quarterly Journal of Political Science* 1, 49-85.

[7] Baron D.P. (1991) A Spatial Bargaining Theory of Government Formation in Parliamentary Systems. *American Political Science Review* 85, 137-164.

[8] Baron D.P. and Ferejohn J. (1989) Bargaining in Legislatures. *American Political Science Review* 83 (4), 1181-2006.

[9] Bernheim B.D., Peleg B. and Whinston M.D. (1987) Coalition-Proof Nash Equilibrium. *Journal of Economic Theory* 42, 1-12.

[10] Binmore K., Rubinstein A. and Wolinsky A. (1986) The Nash Bargaining Solution in Economic Modelling. *Rand Journal of Economics* 17 (2), 176-188.

[11] Bloch F. (1996) Sequential Formation of Coalitions in Games with Externalities and Fixed Payoff Division. *Games and Economic Behavior* 14, 90-123.

[12] Bogomolnaia A. and Jackson M.O. (2002) The Stability of Hedonic coalition Structures. *Games and Economic Behavior* 38, 201-230.

[13] Carroll R. and Cox G.W. (2007) The Logic of Gamson's Law: Pre-election Coalitions and Portfolio Allocations. *American Journal of Political Science*, 51 (2), 300-313.

[14] Diermeier D., Eraslan H. and Merlo A. (2003) A Structural Model of Government Formation. *Econometrica*, 71 (1), 27-70.

[15] Fréchette F., Kagel J. and Morelli M. (2005) Gamson's Law versus Non-cooperative Bargaining Theory. *Games and Economic Behavior* 51 (2) 365-390.

[16] Grossman G.M. and Helpman E. (1996) Electoral competition and special interest politics. *Review of Economic Studies* 63, 265-286.

[17] Harsanyi J. C. and Selten R.A. (1972) Generalized Nash Solution for Two-Person Bargaining Games with Incomplete Information. *Management Science*, 18 (5), 80-106.

[18] Hurwicz L. (1972) On information decentralized systems. In: McGuire C.B., Radner R., *Decision and organization*. North Holland, Amsterdam.

[19] Jackson M.O. (2001) A Crash Course in Implementation Theory. *Social Choice and Welfare* 18, 655-708.

[20] Jackson M.O. and Moselle B. (2002) Coalition and Party Formation in a Legislative Voting Games. *Journal of Economic Theory* 103, 49-87.

[21] Kirchsteiger G. and Puppe C. (1997) On the Formation of Political Coalitions. *Journal of Institutional and Theoretical Economics* 153, 293-319.

[22] Le Breton M., Ortuño-Ortin I., and Weber S. (2008) Gamson's Law and Hedonic Games. *Social Choice and Welfare* 30 (1), 57-67.

[23] Müller W.C. and Str∅m K. (2000) *Coalition Governments in Western Europe*. Oxford University Press.

[24] Plott C.R. (1967) A Notion of Equilibrium and its possibility under Majority Rule, *American Economic Review* 57 (4) 787-806.

[25] Rijt J.W. van der (2008) An Alternative Model of the Formation of Political Coalitions, *Theory and Decision*, forthcoming.

[26] Saari D.G. (1997) The Generic Existence of a Core for q-rules. *Economic Theory* 9, 219-60.

[27] Schofield N. (1983) Generic Instability of Majority Rule, *Review of Economic Studies* 50, 695-705.

[28] Serrano R. (1995) A market to implement the core. *Journal of Economic Theory* 67, 285-294.

[29] Serrano R. and Vohra R. (1997) Non-cooperative Implementation of the Core. *Social Choice and Welfare* 14, 513-525.

[30] Suh, S. C. (1997) Double implementation in Nash and Strong Nash Equilibria, *Social Choice and Welfare* 14, 439-447.

[31] von Neumann J. and Morgenstern O. (1944) *Theory of Games and Economic Behavior*, Princeton: Princeton University.

[32] Yi S.S. (2003) Endogenous Formation of Economic Coalitions: A survey of the Partition Function Approach in C. Carraro (ed.) *The Endogenous Formation of Economic Coalitions*. Cheltenham: Edward Elgar, 80-124.