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PICKING THE WINNERS*

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Abstract

We analyze the problem of choosing the w contestants who will win a competition within a group of $n > w$ competitors when all jurors commonly observe who the w best contestants are, but they may be biased. We study conditions on the configuration of the jury so that it is possible to induce the jurors to always choose the best contestants, whoever they are. If the equilibrium concept is dominant strategies, the condition is very strong: there must be at least one juror who is totally impartial, and the planner must have some information about who this juror is. If the equilibrium concept is Nash (or subgame perfect) equilibria the condition is less demanding: for each pair of contestants, the planner must know at least one juror who is not biased in favor/against any of them. Furthermore, the latter condition is also necessary for any other equilibrium concept.

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J.E.L. Classification Numbers: C72, D71, D78.

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1 Introduction

A group of $n \geq 3$ contestants are involved in a competition. A jury must choose a set of $w < n$ contestants who will win the competition. All jurors know who the w best contestants are. We call this group "deserving winners". Each juror, however, may be biased in favor of/against some contestants (*i.e.*, the juror always prefers some contestants to be/not to be among the chosen winners, whatever the deserving winners are).

Examples of this situation are very common. Think, for instance, of the Olympic Games host selection. The candidate cities are the contestants and the members of the International Olympic Committee (IOC) are the jurors. The IOC has to choose the city where the next Olympic Games will be held (*i.e.*, $w = 1$). Suppose that all the members of the IOC know that, if the decision was taken purely on the quality and merit of the candidatures, then "city a" should be the chosen one. Some members of the IOC, however, might be biased in favor of/against certain candidatures. There are different reasons why this could happen, ranging from nationality to politics. The biased jurors will try to favor one candidature over another, regardless of which is the best one.

A different example is the hiring process in a department. The members of the department have to choose the w candidates who will be hired among a number of applicants. Suppose that all members of the department know who the best candidates are (they do not have private information). Although the optimal outcome is to hire the best candidates available, some professors may be biased in favor/against some candidates: *e.g.*, some professors may not want to hire candidates who are better at their jobs than they are, other professors may want to hire candidates who work in the same topics than them, even though they are not the best candidates, etc. A similar problem arises in the selection process for hiring civil servants: rather than administering a merit-based selection process for hiring the employees, some of the members of the commission might try to reward political allies with the posts.

In many cases, the problem is that only biased jurors have the relevant information about who are the best contestants. Consider an open tender to build a public infrastructure. Only expert engineers, who work in the subject area and probably have connections with some of the firms that tender for the contract, know which is the best design of the public construction. Similarly, in any good department, it is the incumbent professors who choose whom to hire. These are the only people at the university in a position to judge the abilities of candidates (the principal of the university might be trained as a historian, the dean as a mathematician, etc.).

This is the reason why fair jurors are sometimes useless: they are ignorant about the truth. Fortunately, the fact that the jurors are biased and look for their own interests does not necessarily imply that the decision of the jury will be unfair. Sometimes, when individuals pursue their self-interests, they promote the good of the society. This has been a main topic in economics since the days of Adam Smith and is the real point of the theory of implementation. Of course, in order to be possible to induce the jurors to choose the best contestants, there must be some limits on their self-interests (for instance, if all jurors are biased against the same contestant, he will never be chosen as one of the winners, no matter how good he is). This is precisely one of the main objectives of this paper: to provide restrictions on the configuration of the jury so that it is possible to induce the jurors to always choose the "deserving winners", whoever they are. For that we use the theory of implementation.

The socially optimal choice rule is E-implementable if, under the assumption that the jurors take their decisions according to the E equilibrium concept, there exists a mechanism that induces them to always choose the deserving winners, whoever they are. We study restrictions on the configuration of the jury so that the socially optimal choice rule is E-implementable. For that, we introduce the notion of treating a group of contestants fairly. If a juror treats contestants a and b fairly then, when comparing any two sets of winners which only differ in a and b, if a deserves to win but b does not, the juror prefers the set of winners that includes a rather than the set of winners that includes b.

We first provide necessary conditions for the E-implementability of the socially optimal choice rule. Proposition 1 states that, if the socially optimal choice rule is E-implementable in some equilibrium concept E then, for each pair of contestants, the planner must know at least one juror who treats them fairly. Whether this condition is fulfilled or not depends on the specific application being considered. It is important to note, however, the implications of its not being met. If the condition fails, the planner will not be able to induce the jurors to always chose the best contestants, no matter how the jurors behave.

We focus next on Nash implementation. Proposition 2 shows that the previous condition not only is necessary for the E-implementability of the socially optimal choice rule in any equilibrium concept E, but it is also sufficient when the equilibrium concept is Nash equilibria. The "canonical" mechanism for Nash implementation, however, does not work here. The reason is that there are situations where all jurors except one rank the same subset of contestants at the top of their preferences despite the fact that some of these contestants do not deserve to win; *i.e.*, the socially optimal choice rule does not satisfy the property of *no veto power* (see Maskin, 1999). Then, to prove Proposition 2, we propose a mechanism à la Maskin that does the job. Our mechanism is simpler than the "canonical" mechanism: each juror only has to announce a set of winners and an integer.[1] Since this mechanism also implements the socially optimal choice rule in subgame perfect equilibria, an immediate corollary is that the necessary condition stated in Proposition 1 is sufficient for subgame perfect implementation of the socially optimal choice rule as well.

We also study implementation in dominant strategies. The conditions for the implementability of the socially optimal choice rule are much stronger under this equilibrium concept. Proposition 3 shows that if the socially optimal choice rule is implementable in dominant strategies then there must be some juror who treats all contestants fairly. In addition, Proposition 4 states that the planner must have some information about who this juror is. These conditions are extremely demanding and the results can be viewed as impossibility results for the implementation of the socially optimal rule in dominant strategies.

Our results bear a resemblance to those in Amorós et al. (2002) and Amorós (2009), who also consider the problem of eliciting the "truth" from a group of partial jurors. In the model analyzed in these works, however, alternatives are rankings of all contestants instead of sets of winners. This makes the problem different from that studied in the present paper.[2] Our paper is

[1] Moore and Repullo (1990) provide necessary and sufficient conditions for the Nash implementability of any choice rule. It is difficult, however, to comprehend for a given choice rule if such conditions hold. Danilov (1992) also provides a necessary and sufficient condition for Nash implementation called *essential monotonicity*. This condition is not easy to check either (an immediate corollary of Proposition 2 is that φ satisfies *essential monotonicity*). Moreover, in contrast to the "canonical" mechanism for Nash implementation proposed by Maskin (1999) or the the mechanism proposed by Danilov (1992), in our mechanism the jurors do not have to announce the preferences of all jurors.

[2] The set of alternatives and the class of possible preferences for a juror are much larger in the problem of ranking a group of contestants than in the problem of choosing a subgroup of contestants.

also related to the literature on information transmission between multiple informed experts and an uninformed decision maker. Austen-Smith (1993) assumes that the decision maker gets advice from two biased and imperfectly informed experts, and compares simultaneous and sequential reporting. Krishna and Morgan (2001) analyze a situation in which two experts observe the same information, but they differ in their preferences, and show that if both experts are biased in the same direction there is no equilibrium in which full revelation occurs. Wolinsky (2002) analyzes a model where the experts share the same preferences, which differ from those of the decision maker, and possess different pieces of information. Gerardi et al. (2009) investigate how the decision maker can extract information from the experts by distorting the decisions that will be taken and show that when the number of informed agents become large one can extract the information at small cost (their focus, however, is not full implementation in the sense that they do not require that all equilibria implement the social choice rule). Finally, our paper is connected with the literature on strategic voting (*e.g.*, Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996; 1997; Duggan and Martinelli, 2001; Martinelli, 2002). The problem studied in these papers, however, is different from the problem studied here: the jurors only have to choose to convict or acquit a defendant.

The paper is organized as follows: Section 2 provides definitions, Section 3 states general necessary conditions for implementation, Section 4 analyzes Nash implementation, Section 5 studies implementation in dominant strategies, and Section 6 provides the conclusions.

2 The model

Consider a set $N = \{a, b, c, ...\}$ of $n \geq 3$ contestants in a competition. A group $J = \{1, 2, ...\}$ of jurors must choose a subset of w winners, where $0 < w < n$. Let 2^N_w denote the set of all subsets of N of size w. When referring to a subset of winners, if it is clear, we will write a for $\{a\}$, ab for $\{a, b\}$, etc. All jurors know who the w best contestants are. We call this group the **deserving winners**, $W_D \in 2^N_w$. The socially optimal outcome is that the deserving winners win.

Jurors have **preferences** defined over 2^N_w. The preferences of a juror may depend on who the deserving winners are. For example, if $N = \{a, b, c, d\}$ and $w = 2$, juror $i \in J$ may prefer ab to ac if the deserving winners are a

and b, but prefer ac to ab if the deserving winners are a and c. However, the preferences of a juror may also depend on "external factors". For instance, a juror might be "friend" of contestant d, so that this juror prefers d being one of the winners whoever the deserving winners are (*e.g.*, he might prefer bd to bc whoever the deserving winners are).[3]

Let $\Re$ be the class of preference relations defined over 2^N_w. Each juror $i \in J$ has a **preference function** $R_i : 2^N_w \longrightarrow \Re$ which associates with each set of deserving winners, $W_D \in 2^N_w$, a preference relation $R_i(W_D) \in \Re$. Let $P_i(W_D)$ denote the strict part of $R_i(W_D)$. Let $\mathcal{R}$ denote the class of all preference functions.

Let 2^N denote the set of all subsets of N. We say that **juror** i **treats the contestants in a set** $F_i \in 2^N$ **fairly** (with $|F_i| \geq 2$) if, for each pair $a, b \in F_i$, when comparing any two sets of winners which only differ in a and b, if a deserves to win but b does not, juror i prefers the set of winners that includes a rather than the set of winners that includes b.

Definition 1 *Let $F_i \in 2^N$ be the set of contestants that juror i treats fairly. The **preference function** $R_i \in \mathcal{R}$ is **admissible** at F_i if for each pair $a, b \in F_i$, each $W_D \in 2^N_w$, and each $W, \hat{W} \in 2^N_w$ with:*
(i) $a \in W$,
(ii) $b \in \hat{W}$,
(iii) $W \backslash \{a\} = \hat{W} \backslash \{b\}$
(iv) $a \in W_D$, and
(v) $b \notin W_D$,
we have $W \; P_i(W_D) \; \hat{W}$.

Let $\mathcal{R}(F_i) \subset \mathcal{R}$ denote the class of preference functions that are admissible at F_i. Next, we illustrate this notion.

Example 1 *Let $N = \{a, b, c, d\}$, $w = 3$, and $i \in J$. Then, $2^N_w = \{abc,\ abd,\ acd,\ bcd\}$. Let $F_i = \{a, b\}$. Any preference function that is admissible at F_i, $R_i \in \mathcal{R}(F_i)$, is such that if $W_D = acd$, then $acd \; P_i(W_D) \; bcd$. To see this, note that the only two contestants whose winner status changes between the two alternatives are a and b, both contestants are in F_i and, while a deserves to win, b does not. The concept of treating the contestants in a set fairly, however, does not imply any other restriction on the preference relation $R_i(W_D)$. For example, when comparing abc with acd, the only two contestants whose*

[3]Similarly, a juror could be an "enemy" of contestant d.

winner status changes are b and d. Since $d \notin F_i$, then for each $\hat{W}_D \in 2^N_w$, both rankings, abc $R_i(\hat{W}_D)$ acd and acd $R_i(\hat{W}_D)$ abc, are admissible (something similar happens when comparing any two sets of winners different from acd and bcd). Similarly, it is easy to see that, if $\tilde{W}_D = bcd$, then bcd $P_i(\tilde{W}_D)$ acd. Finally, if $\bar{W}_D \in \{abc, abd\}$, the preference relation $R_i(\bar{W}_D)$ need not fulfill any special requirement since then, each contestant in F_i deserves to win as well. Table I summarizes these restrictions.

$R_i : 2^N_w \longrightarrow \Re$

$W_D = abc$	$W_D = abd$	$W_D = acd$	$W_D = bcd$
No restriction on $R_i(W_D)$	No restriction $R_i(W_D)$	The only restriction on $R_i(W_D)$ is $acd\ P_i(W_D)\ bcd$	The only restriction on $R_i(W_D)$ is $bcd\ P_i(W_D)\ acd$

Table I. The case $n = 4$, $w = 3$, and $F_i = \{a, b\}$.

One could think of situations in which a juror treats fairly the contestants in two (or more) sets. For example, this could be the case of a sexist juror who always prefers a man rather than a woman, but who treats all women fairly (and treats all men fairly): *i.e.*, when comparing sets which only differ in female contestants (male contestants, respectively) he prefers the deserving winners to win. Suppose that juror i treats fairly the contestants in $F_i \in 2^N$ and treats fairly the contestants in $\hat{F}_i \in 2^N$, with $F_i \cap \hat{F}_i = \emptyset$. In terms of our model, it would be equivalent to having two different jurors $i, j \in J$, such that i treats fairly to the contestants in F_i, and j treats fairly the contestants in $\hat{F}_i$.[4]

Let $F \equiv (F_i)_{i \in J} \in (2^N)^{|J|}$ denote a profile of subsets of contestants that the jurors treat fairly (one subset for each juror). The planner may be uncertain about which contestants are treated fairly by each juror. Thus, there is a set $\mathcal{F} = \{F, \hat{F}, ...\}$ of profiles $F \equiv (F_i)_{i \in J} \in (2^N)^{|J|}$, $\hat{F} \equiv (\hat{F}_i)_{i \in J} \in (2^N)^{|J|}$,

[4]The fact that the sets F_i and $\hat{F}_i$ are disjoint can be interpreted as a requirement of transitivity: if i treats a and b fairly and i treats b and c fairly, then i should treat a and c fairly as well. We think that this assumption makes sense in many real situations (of course there are alternative ways to define fairness).

etc., that are admissible. The set $\mathcal{F}$ summarizes the planner's ignorance about the profile of subsets of contestants that the jurors treats fairly (he does not know whether the profile is F, or $\hat{F}$, etc.).

Given $\mathcal{F}$, a **state of the world** is a pair $(R, W_D) \in \mathcal{R}^{|J|} \times 2^N_w$ such that, for some $F \in \mathcal{F}$, $R \equiv (R_i)_{i\in J} \in \times_{i\in J}\mathcal{R}(F_i)$. Let $S(\mathcal{F})$ be the set of admissible states of the world when the set of possible profiles of subsets of contestants that the jurors treat fairly is $\mathcal{F}$:

$$S(\mathcal{F}) = \left\{(R, W_D) \in \mathcal{R}^{|J|} \times 2^N_w : \exists F \in \mathcal{F} \text{ s.t. } R \equiv (R_i)_{i\in J} \in \times_{i\in J}\mathcal{R}(F_i)\right\}$$

The **socially optimal choice rule** is the function $\varphi : S(\mathcal{F}) \to 2^N_w$ such that, for each $(R, W_D) \in S(\mathcal{F})$, $\varphi(R, W_D) = W_D$ (*i.e.*, for each admissible state, φ selects the deserving winners).

A **mechanism** is a pair $\Gamma \equiv (M, g)$, where $M = \times_{i\in J} M_i$, M_i is a message space for juror i, and $g : M \to 2^N_w$ is an outcome function.[5] Given a mechanism and a state of the world, the jurors must decide which messages to announce.

Let E be a game theoretic equilibrium concept. For each mechanism $\Gamma \equiv (M, g)$ and each state $(R, W_D) \in S(\mathcal{F})$, let $E(\Gamma, R, W_D) \subset M$ denote the set of profiles of messages that constitute an E-equilibrium of Γ when the state is (R, W_D). Let $\mathcal{E}$ be a class of equilibrium concepts E such that, for each pair $(R, W_D), (\hat{R}, \hat{W}_D) \in S(\mathcal{F})$ with $(R_i(W_D))_{i\in J} = (\hat{R}_i(\hat{W}_D))_{i\in J}$, $E(\Gamma, R, W_D) = E(\Gamma, \hat{R}, \hat{W}_D)$.[6] For example, $m \in M$ is a **dominant strategy equilibrium** of $\Gamma \equiv (M, g)$ at $(R, W_D) \in S(\mathcal{F})$ if, for each $i \in J$, each $\hat{m}_i \in M_i$, and each $\hat{m}_{-i} \in M_{-i}$, $g(m_i, \hat{m}_{-i})\ R_i(W_D)\ g(\hat{m}_i, \hat{m}_{-i})$. Similarly, $m \in M$ is a **Nash equilibrium** of $\Gamma \equiv (M, g)$ at $(R, W_D) \in S(\mathcal{F})$ if for each $i \in J$ and each $\hat{m}_i \in M_i$, $g(m)\ R_i(W_D)\ g(\hat{m}_i, m_{-i})$. Let $D(\Gamma, R, W_D) \subseteq M$ and $N(\Gamma, R, W_D) \subseteq M$ denote the sets of dominant strategy and Nash equilibria of Γ at (R, W_D), respectively.

We want our mechanisms to be such that, in each state, the deserving winners are selected at equilibrium.[7]

[5] This kind of mechanism is sometimes called "normal form mechanism" to distinguish it from "extensive form mechanisms" in which jurors make choices sequentially.

[6] For each $E \in \mathcal{E}$, if no juror changes his preferences from state (R, W_D) to state $(\hat{R}, \hat{W}_D)$, then the profiles of messages that constitute an $E-$equilibrium are the same in both states.

[7] Our notion of implementation is strong in that it requires that every equilibrium

Definition 2 *Let $E \in \mathcal{E}$. Given $\mathcal{F}$, the mechanism $\Gamma \equiv (M, g)$* ***E-implements φ*** *if, for each $(R, W_D) \in S(\mathcal{F})$:*

(i) there exists $m \in E(\Gamma, R, W_D)$ such that $g(m) = W_D$, and

(ii) if $m \in M$ is such that $g(m) \neq W_D$, then $m \notin E(\Gamma, R, W_D)$.

If such a mechanism exists, then φ is E-implementable.

3 Necessary conditions for implementation

There are situations in which implementation of φ is not possible, whatever equilibrium concept we consider. Suppose for example that $w = 1$ and that all jurors happen to prefer the same contestant a (no matter who deserves to win). It is clear that then the jurors would always choose a as the winner, whoever deserves to win and regardless of the mechanism they are faced with. For a mechanism to implement φ, such situations should not occur. Technically, that is equivalent to imposing restrictions on the set $\mathcal{F}$.

Our first result is that, given any equilibrium concept E, if φ is E-implementable then, for each pair of contestants, there is at least one juror who treats them fairly. The intuition is the following: if there are two contestants who are not both treated fairly by the same juror in some $F \in \mathcal{F}$, then there are two states $(R, W_D), (\hat{R}, \hat{W}_D) \in S(\mathcal{F})$ such that $W_D \neq \hat{W}_D$ but $R_i(W_D) = \hat{R}_i(\hat{W}_D)$ for each juror $i \in J$ (*i.e.*, the deserving winners are different but the preferences of the jurors are the same in both states); thus, for any mechanism Γ, the set of E-equilibria of Γ at (R, W_D) and the set of E-equilibria of Γ at $(\hat{R}, \hat{W}_D)$ coincide, and E-implementation is not possible.

Lemma 1 *Let $E \in \mathcal{E}$. If φ is E-implementable then for each pair of contestants, there is at least one juror who treats them fairly (i.e., for each $F \in \mathcal{F}$ and each $a, b \in N$ there is $i \in J$ such that $a, b \in F_i$).*

Proof. Let $F \equiv (F_i)_{i \in J}$. Suppose that there is $F \in \mathcal{F}$ and $a, b \in N$ such that, for each $i \in J$, either $a \notin F_i$ or $b \notin F_i$. Let $W_D, \hat{W}_D \in 2^N_w$ be such that (i) $a \in W_D$, (ii) $b \in \hat{W}_D$, and (iii) $W_D \backslash \{a\} = \hat{W}_D \backslash \{b\}$. Then there is $R \equiv (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ such that, for each $i \in J$, $R_i(W_D) = R_i(\hat{W}_D)$;

message profile induce the choice of the set of deserving winners (as opposed to requiring merely that for every state some equilibrium message profile induces the choice of the deserving winners). It would be interesting to study what could be gained by employing a weaker notion of implementation.

i.e., the preference relation of each juror i at state $(R, W_D) \in S(\mathcal{F})$ is the same as at state $(R, \hat{W}_D) \in S(\mathcal{F})$. Hence, given any solution concept $E \in \mathcal{E}$ and any $\Gamma \equiv (M, g)$, we have $E(\Gamma, R, W_D) = E(\Gamma, R, \hat{W}_D)$. If Γ E-implements φ, there exists $m \in E(\Gamma, R, W_D)$ such that $g(m) = W_D$. But then $m \in E(\Gamma, R, \hat{W}_D)$ and $g(m) \neq \hat{W}_D$. Thus, Γ does not E-implements φ after all. ■

Table II shows an example of the profile $R \equiv (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ defined in the proof of Lemma 1 for the case $N = \{a, b, c\}$, $J = \{1, 2, 3\}$, $w = 2$, and $F \in \mathcal{F}$ such that $F_1 = \{a, c\}$ and $F_2 = F_3 = \{b, c\}$ (higher alternatives in the table are preferred to lower alternatives). Note that, for each $i \in J$, the preference function $R_i \in \mathcal{R}$ represented in Table II is admissible at F_i, and $R_i(ac) = R_i(bc)$).[8]

	R_1			R_2			R_3		
$W_D =$	ab	ac	bc	ab	ac	bc	ab	ac	bc
	ac	ac	ac	ab	ac	ac	bc	bc	bc
Pref.	ab	bc	bc	ac	ab	ab	ab	ac	ac
	bc	ab	ab	bc	bc	bc	ac	ab	ab

Table II. An example in the proof of Lemma 1.

Suppose then that for each pair of contestants there is at least one juror who treats them fairly. Our next result is that this is not sufficient to guarantee that φ is implementable. If φ is E-implementable in some equilibrium concept $E \in \mathcal{E}$ then, for each pair of contestants, the planner knows at least one of the jurors who treats them fairly. In other words, no mechanism gives the jurors the incentive to reveal who treats whom fairly. The planner must have this information and any mechanism implementing φ must depend on it. Otherwise, the set of admissible states of the world is so large that the E-implementability of φ is prevented, whatever equilibrium concept E we consider.

[8] Given F_1, the only conditions that must satisfy a preference function of juror 1, $R_1 \in \mathcal{R}$, in order to be admissible are $ab \ P_1(ab) \ bc$ and $bc \ P_1(bc) \ ab$. Similarly, given F_2 and F_3, the only conditions that must satisfy a preference function of juror $i \in \{2, 3\}$, $R_i \in \mathcal{R}$, in order to be admissible are $ab \ P_i(ab) \ ac$ and $ac \ P_i(ac) \ ab$.

Proposition 1 *Let $E \in \mathcal{E}$. If φ is E-implementable, then for each pair of contestants, the planner knows at least one juror who treats them fairly (i.e., for each pair $a, b \in N$ there is $i \in J$ such that, for each $F \in \mathcal{F}$, $a, b \in F_i$).*

Proof. Let $E \in \mathcal{E}$. Suppose that φ is E-implementable by means of a mechanism $\Gamma \equiv (M, g)$. Suppose by contradiction and without loss of generality that there are $a, b \in N$ such that (i) for each $i \in J$, there is $F \in \mathcal{F}$ with $a \notin F_i$ or $b \notin F_i$, and (ii) for each pair of contestants different from (a, b), there is at least one juror who treats both contestants fairly in every $F \in \mathcal{F}$. By Lemma 1, there are $F, \hat{F} \in \mathcal{F}$ and $i, j \in J$ such that:

(i) $a, b \in F_i$,

(ii) either $a \notin F_j$ or $b \notin F_j$,

(iii) $a, b \in \hat{F}_j$, and

(iv) either $a \notin \hat{F}_i$ or $b \notin \hat{F}_i$.

Slightly abusing notation, let $\bar{R} \equiv (\bar{R}_i)_{i \in J} \in \Re^{|J|}$ be a profile of preference relations such that, when comparing any two sets of winners which only differ in two contestants, then:

(i) if the only two contestants that interchange their winner status between the two sets of winners are a and b, then juror i prefers the set of winners that includes a, while juror j prefers the set of winners that includes b, and

(ii) if the only two contestants that interchange their winner status between the two sets of winners are not a and b then, those jurors who treat these two contestants fairly in every profile in $\mathcal{F}$, prefer the set of winners that includes the contestant who comes first in the alphabetical order.

Let $W, \hat{W} \in 2^N_w$ be two sets of winners such that:

(i) $a \in W$ and $b \notin W$,

(ii) $a \notin \hat{W}$ and $b \in \hat{W}$, and

(iii) $W \backslash \{a\} = \hat{W} \backslash \{b\}$; in particular, if $w > 1$, the $w - 1$ first contestants in the alphabetical order in $N \backslash \{a, b\}$ are both in W and in $\hat{W}$.[9]

Note that there is a profile of preference functions that is admissible at F, $R \equiv (R_k)_{k \in J} \in \times_{k \in J} \mathcal{R}(F_k)$, such that, for each $k \in J$, $R_k(W) = \bar{R}_k$. Similarly, there is a profile of preference functions that is admissible at $\hat{F}$, $\hat{R} \equiv (\hat{R}_k)_{k \in J} \in \times_{k \in J} \mathcal{R}(\hat{F}_k)$, such that, for each $k \in J$, $\hat{R}_k(\hat{W}) = \bar{R}_k$. Thus, there are two states $(R, W), (\hat{R}, \hat{W}) \in S(\mathcal{F})$ such that $(R_k(W))_{k \in J} = (\hat{R}_k(\hat{W}))_{k \in J}$.

[9] For example, if $N = \{a, b, c, d, e, f\}$ and $w = 4$, then $W = \{a, c, d, e\}$ and $\hat{W} = \{b, c, d, e\}$.

Then, $E(\Gamma, R, W) = E(\Gamma, \hat{R}, \hat{W})$. Since Γ E-implements φ, there exists $m \in E(\Gamma, R, W)$ such that $g(m) = W$. But then, $m \in E(\Gamma, \hat{R}, \hat{W})$ and $g(m) \neq \hat{W}$, which contradicts the assumption that Γ E-implements φ. ■

Table III shows the profile of preference relations $\bar{R} \equiv (\bar{R}_i)_{i \in J} \in \Re^{|J|}$ in the proof of Proposition 1 for $N = \{a, b, c\}$, $J = \{1, 2, 3\}$, $w = 1$, and $\mathcal{F} = \{F, \hat{F}\}$ with $F_1 = \{a, b, c\}$, $F_2 = \{a, c\}$, $F_3 = \{b, c\}$, $\hat{F}_1 = \{a, c\}$, $\hat{F}_2 = \{a, b, c\}$, and $\hat{F}_3 = \{b, c\}$. In this case, $i = 1$, $j = 2$, $W = a$, and $\hat{W} = b$. Moreover, in every profile in $\mathcal{F}$, juror 1 treats a and c fairly, while juror 3 treats b and c fairly. Tables IV and V give examples of profiles of preference functions $R \equiv (R_k)_{k \in J} \in \times_{k \in J} \mathcal{R}(F_k)$ and $\hat{R} \equiv \left(\hat{R}_k\right)_{k \in J} \in \times_{k \in J} \mathcal{R}(\hat{F}_k)$ defined in the previous proof. Note that, for each $k \in J$, $R_k(a) = \hat{R}_k(b) = \bar{R}_k$. Therefore, for each equilibrium concept $E \in \mathcal{E}$ and each mechanism Γ, a profile of messages that is an E-equilibrium of Γ at (R, a) is also an E-equilibrium at $(\hat{R}, b)$.

$\bar{R}$

R_1	R_2	R_3
a	b	a
b	a	b
c	c	c

Table III. Example $\bar{R} \in \Re^{|J|}$ in proof of Prop. 1.

	R_1			R_2			R_3		
$W_D =$	a	b	c	a	b	c	a	b	c
	a	b	c	b	b	b	a	a	a
Pref.	b	a	a	a	a	c	b	b	c
	c	c	b	c	c	a	c	c	b

Table IV. Example R in proof of Prop. 1.

	$\hat{R}_1$			$\hat{R}_2$			$\hat{R}_3$		
$W_D =$	a	b	c	a	b	c	a	b	c
	a	a	b	a	b	c	a	a	a
Pref.	b	b	c	b	a	a	b	b	c
	c	c	a	c	c	b	c	c	b

Table V. Example $\hat{R}$ in proof of Prop. 1.

Since $n \geq 3$, except for the trivial case in which the planner knows that there is a juror who treats all contestants fairly, the necessary conditions formulated in Propositions 1 and 2 cannot be fulfilled if there are only two jurors. Thus, we need at least three jurors to be able to implement φ.

4 Implementation in Nash equilibria

The condition stated in Proposition 1 is not only necessary for the E-implementability of φ in any equilibrium concept $E \in \mathcal{E}$, but it is also sufficient when the equilibrium concept is Nash equilibrium (see Proposition 2).

If there are three or more agents, the "canonical" mechanism for Nash implementation proposed by Maskin (1999) implements any social choice rule that satisfies two properties.[10] The first property is called *monotonicity* and, in our model, it requires that if W is optimal in some state, then W must be also optimal in any other state where W does not fall in any jurors preference relation relative to any other $\hat{W}$. The second property is called *no veto power* and, in our model, it requires a set of winners to be optimal if all jurors but perhaps one rank it at the top of their preferences. It can be proved that if the condition stated in Proposition 1 is fulfilled, then φ satisfies *monotonicity.* Under the same condition, however, φ may not satisfy *no veto power.* To see this, let $N = \{a, b, c\}$, $J = \{1, 2, 3\}$, and $w = 1$. Suppose that $\mathcal{F} = \{F\}$, with $F_1 = \{a, c\}$, $F_2 = \{a, b\}$, and $F_3 = \{b, c\}$, so the necessary condition stated in Proposition 1 is fulfilled. Note that the profile of preference functions $\hat{R} \equiv (\hat{R}_i)_{i \in J}$ depicted in Table V is admissible in this situation; *i.e.*, $\hat{R} \equiv (\hat{R}_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$. Note also that a is the most preferred set of winners for two of the three jurors when the state is $(\hat{R}, b) \in S(\mathcal{F})$, but a is not socially optimal at that state.

[10]See also Repullo (1987) and Saijo (1988).

Since φ fails *no veto power*, the "canonical" mechanism for Nash implementation does not work in our setting. To prove Proposition 2, we propose a variation of that mechanism where each juror has to announce a set of winners and an integer between 1 and $|J|$. If all jurors send the same message $(W, z) \in 2^N_w \times \{1, 2, ..., |J|\}$, then W is chosen. If there is only one dissident $j \in J$ announcing $(W_j, z_j) \neq (W, z)$, then W_j is chosen if W_j and W only differ in contestants that j treats fairly. If more than two jurors disagree on their messages, then W_j is chosen, where $j \in J$ is such that $j = (\sum_{i \in J} z_i)(\text{mod}\,|J|)$.[11]

Proposition 2 *Suppose that there are at least three jurors. Suppose that for each pair of contestants, the planner knows at least one juror who treats them fairly (i.e., for each $a, b \in N$ there is $i \in J$ such that, for each $F \in \mathcal{F}$, $a, b \in F_i$). Then φ is Nash implementable.*

Proof. Let $\Gamma^N \equiv (M, g)$ be such that, for each $i \in J$, $M_i = 2^N_w \times \{1, 2, ..., |J|\}$, and for each $m \equiv (W_i, z_i)_{i \in J} \in M$, $g(m)$ is defined by the following three rules:

Rule 1. If, for each $i \in J$, $(W_i, z_i) = (W, z)$, then $g(m) = W$.

Rule 2. If there is $j \in J$ such that, for each $i \neq j$, $(W_i, z_i) = (W, z)$, but $(W_j, z_j) \neq (W, z)$, then

$$g(m) = \begin{cases} W_j; \text{ if, for each } F \in \mathcal{F}, \{W_j \cup W\} \backslash \{W_j \cap W\} \subseteq F_j \\ W; \text{ otherwise} \end{cases}$$

Rule 3. In all other cases $g(m) = W_j$ for $j \in J$ such that $j = (\sum_{i \in J} z_i)(\text{mod}\,|J|)$.

Claim 1. For each $(R, W_D) \in S(\mathcal{F})$ there exists $m \in N(\Gamma^N, R, W_D)$ such that $g(m) = W_D$.

Let $(R, W_D) \in S(\mathcal{F})$. Let $m = ((W_i, z_i))_{i \in J} \in M$ be such that, for each $i \in J$, $(W_i, z_i) = (W_D, 1)$. Then Rule 1 applies and $g(m) = W_D$. Furthermore, $m \in N(\Gamma^N, R, W_D)$. To see this, let $j \in J$ and $\hat{m}_j \equiv (\hat{W}_j, \hat{z}_j)$ be such that $g(\hat{m}_j, m_{-j}) = \hat{W}_j \neq W_D$. Then, by Rule 2, for each $F \in \mathcal{F}$, $\{\hat{W}_j \cup W_D\} \backslash \{\hat{W}_j \cap W_D\} \subseteq F_j$. Note that then there is a sequence $W^1, \ldots, W^s \in 2^N_w$ such that:

(1) $W^1 = W_D$,

(2) $W^s = \hat{W}_j$, and

[11] $\alpha = \beta\,\text{mod}(|J|)$ denotes that integers α and β are congruent modulo $|J|$.

(3) for each $q \in \{2, .., s\}$ there are $a^q, b^q \in N$ with (3.1) for each $F \in \mathcal{F}$, $a^q, b^q \in F_j$, (3.2) $a^q \in W^{q-1}$, (3.3) $b^q \in W^q$, (3.4) $W^{q-1} \backslash \{a\} = W^q \backslash \{b\}$, and (3.5) for each $r \neq q$, $a^q \neq b^r$.

The only difference between any two consecutive sets in the sequence, W^{q-1} and W^q, is that a^q is replaced by b^q; *i.e.*, $a^q \in W^{q-1}$, $b^q \in W^q$ and $W^{q-1} \backslash \{a\} = W^q \backslash \{b\}$. Moreover, since $W^1 = W_D$ and, for each $r \neq q$, $a^q \neq b^r$, then $a^q \in W_D$ and $b^q \notin W_D$. Thus, since, for each $F \in \mathcal{F}$ $a^q, b^q \in F_j$, $W_D = W^1 \; P_j(W_D) \; W^2 \; P_j(W_D) \ldots W^{s-1} \; P_j(W_D) \; W^s = \hat{W}_j$. Hence, juror j cannot improve his welfare by deviating from m.

Claim 2. For each $(R, W_D) \in S(\mathcal{F})$ and each $m \in M$ such that $g(m) \neq W_D$, $m \notin N(\Gamma^N, R, W_D)$.

Let $(R, W_D) \in S(\mathcal{F})$ and $m = ((W_i, z_i))_{i \in J} \in M$ be such that $g(m) = W \neq W_D$. Then, there exist $a, b \in N$ such that $a \in W_D$, $a \notin W$, $b \in W$, and $b \notin W_D$. Let $\hat{W} \in 2^N_w$ be such that $\hat{W} \backslash \{a\} = W \backslash \{b\}$ and $a \in \hat{W}$. Let $j \in J$ be a juror such that, for each $F \in \mathcal{F}$, $a, b \in F_j$. Then $\hat{W} \; P_j(W_D) \; W$.

Case 1. Rule 1 applies to m. Then, for each $i \in J$, $W_i = W$. Consider a unilateral deviation $\hat{m}_j = (\hat{W}, 1)$ by juror j. Then Rule 2 applies and, since for each $F \in \mathcal{F}$ $\{\hat{W} \cup W\} \backslash \{\hat{W} \cap W\} = \{a, b\} \subseteq F_j$, $g(\hat{m}_j, m_{-j}) = \hat{W}$. Since $\hat{W} \; P_j(W_D) \; W$, $m \notin N(\Gamma^N, R, W_D)$.

Case 2. Rule 2 applies to m.

Subcase 2.1. The dissident in m is juror j. If juror j is not announcing W in m, then the remaining jurors announce W in m, in which case j can improve his welfare by unilaterally deviating to $\hat{m}_j = (\hat{W}, 1)$ (as in Case 1). If juror j announces W in m, then the remaining jurors are all announcing some $\tilde{W} \neq W$ such that, for each $F \in \mathcal{F}$, $\{W \cup \tilde{W}\} \backslash \{W \cap \tilde{W}\} \subseteq F_j$. Note that then, for each $F \in \mathcal{F}$, $\{\hat{W} \cup \tilde{W}\} \backslash \{\hat{W} \cap \tilde{W}\} \subseteq F_j$. Consider a unilateral deviation $\hat{m}_j = (\hat{W}, 1)$ by juror j. Then Rule 2 applies and $g(\hat{m}_j, m_{-j}) = \hat{W}$. Since $\hat{W} \; P_j(W_D) \; W$, $m \notin N(\Gamma^N, R, W_D)$.

Subcase 2.2. The dissident in m is not juror j. Let $\hat{m}_j = (\hat{W}, \hat{z}_j)$ where, for each $i \neq j$, $\hat{z}_j \neq z_i$, and $j = (\hat{z}_j + \sum_{i \neq j} z_i)(\text{mod} \, |J|)$. Then Rule 3 applies to $(\hat{m}_j, m_{-j})$ and $g(\hat{m}_j, m_{-j}) = \hat{W}$. Thus, $m \notin N(\Gamma^N, R, W_D)$.

Case 3. Rule 3 applies to m. Then juror j can improve his welfare by deviating as in Subcase 2.2. Thus $m \notin N(\Gamma^N, R, W_D)$. ■

The mechanism à la Maskin (1999) proposed in the proof of Proposition 2 is quite abstract. This type of mechanisms have received criticism for being unnatural (see Jackson, 1992). Nevertheless, as argued by Serrano (2004), the main purpose of these mechanisms is the characterization of what can be implemented. As such, they are designed to handle a large number of social choice problems. In particular, our mechanism works whenever there are three or more jurors, for each pair of contestants there is at least one juror who treats them fairly, and the planner knows who these jurors are. Several situations are covered. Suppose for example that $N = \{a, b, c, d\}$, $J = \{1, 2, 3, 4\}$ and $\mathcal{F} = \{F\}$ where $F_1 = \{b, c, d\}$, $F_2 = \{a, c, d\}$, $F_3 = \{a, b, d\}$, and $F_4 = \{a, b, c\}$. Then the condition stated in Proposition 2 is fulfilled. Various situations are still possible: (i) contestants a, b, c and d are friends of jurors 1, 2, 3 and 4, respectively; (ii) contestants a, b, c and d are enemies of jurors 1, 2, 3 and 4, respectively; (iii) contestants a, b are friends of jurors 1 and 2, respectively, while contestants c and d could are enemies of jurors 3 and 4, respectively; etc. The mechanism proposed in the proof of Proposition 2 works in all these situations. This is precisely the reason why the mechanism is abstract. In some situations, the planner has more information about certain aspects of jurors' preferences. This information might allow him to disregard some of the previous cases and to design more realistic mechanisms.

It is also interesting to analyze implementation of φ via extensive form mechanisms. It is not possible to explain fully this approach here because it would take us too far. Roughly speaking, an extensive form mechanism is a dynamic mechanism in which agents make choices sequentially. Given $\mathcal{F}$, **$\boldsymbol{\varphi}$ is implementable in subgame perfect equilibria** if there exists an extensive form mechanism such that for each state $(R, W_D) \in S(\mathcal{F})$, the only subgame perfect equilibrium outcome is W_D.[12] Clearly, the necessary condition stated in Proposition 1 is also sufficient for the implementation of φ in subgame perfect equilibria if there are at least three jurors.[13]

[12]For each extensive form mechanism and each state of the world, a subgame perfect equilibrium induces a Nash equilibrium in every subgame. For the very positive results achieved on implementation with extensive form mechanisms under complete information see Moore and Repullo (1988) and Abreu and Sen (1990).

[13]Since the mechanism proposed in the proof of Proposition 2 is a one-shot-mechanism then, for each state, a profile of messages m is a Nash equilibrium if and only if it is a subgame perfect equilibria. Therefore, this mechanism also implements φ in subgame perfect equilibria.

5 Implementation in dominant strategies

Unlike what happens with Nash and subgame perfect implementation, the necessary condition stated in Proposition 1 is not sufficient for the implementability of φ in dominant strategies. Our next result shows that, if φ is implementable in dominant strategies, then there is some juror who treats all contestants fairly,

In our setting, the set of possible profiles of preference relations may not have a Cartesian product structure. Then, we cannot use the revelation principle for dominant strategies (Gibbard, 1973) to prove this result.[14] Instead, we show that, if no juror treats all contestants fairly, there exist some profiles of preference relations which, despite not being admissible, prevent any mechanism from implementing φ in dominant strategies.

Proposition 3 *If φ is implementable in dominant strategies then there is at least one juror who treats all contestants fairly (i.e., for each $F \in \mathcal{F}$ there is $i \in J$ such that $F_i = N$).*

Proof. Suppose, for simplicity, that $N = \{a, b, c\}$. Suppose that φ is implementable in dominant strategies by means of a mechanism $\Gamma^D \equiv (M, g)$. By Proposition 1, for each pair of contestants, there is $i \in J$ who have them in his set F_i for each $F \in \mathcal{F}$. Suppose by contradiction that there is $F \in \mathcal{F}$ such that, for each $i \in N$, $F_i \neq N$. Then, since $n \geq 3$, there are at least three jurors. Suppose, to simplify notation, that $J = \{1, 2, 3\}$, $F_1 = N \backslash \{a\}$, $F_2 = N \backslash \{b\}$, and $F_3 = N \backslash \{c\}$.

Case 1: $w = 1$**.** Consider the profile of admissible preference functions $R \equiv (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ depicted in Table VI. Note that each juror i prefers the contestant who is not in F_i to be chosen as winner, whoever the deserving winner is. Abusing notation, let us denote $R_1, \hat{R}_1 \in \Re$ the two following preference relations for juror 1: $R_1 = R_1(a) = R_1(b)$ and $\hat{R}_1 = R_1(c)$ (*i.e.*, $R_1, \hat{R}_1 \in \Re$ are such that $a \ P_1 \ b \ P_1 \ c$ and $a \ \hat{P}_1 \ c \ \hat{P}_1 \ b$). Similarly, let $R_2 = R_2(a)$, $\hat{R}_2 = R_2(b) = R_2(c)$, $R_3 = R_3(a) = R_3(c)$ and $\hat{R}_3 = R_3(b)$ (Table VII). Since $\Gamma^D \equiv (M, g)$ implements φ in dominant strategies then,

[14] The revelation principle for dominant strategies states that, under independent domains of preferences, if a social choice rule is implementable in dominant strategies then, for every agent, reporting the truth is a dominant strategy in the direct mechanism where each agent is simply asked to report his preference relation and the resulting outcome is the one prescribed by the social choice rule following the reports.

for each $i \in J$, there is a dominant strategy $m_i \in M_i$ for i when his preference relation is $R_i \in \Re$. Similarly, for each $i \in J$, there is a dominant strategy $\hat{m}_i \in M_i$ for i when his preference relation is $\hat{R}_i \in \Re$. Since $(R, a) \in S(\mathcal{F})$, $(m_1, m_2, m_3) \in D(\Gamma^D, R, a)$ and Γ^D implements φ in dominant strategies, then $g(m_1, m_2, m_3) = a$. Similarly, $g(m_1, \hat{m}_2, \hat{m}_3) = b$ and $g(\hat{m}_1, \hat{m}_2, m_3) = c$. Consider now the profile of messages $(m_1, \hat{m}_2, m_3) \in M$.[15] Note that: (i) since $\hat{m}_1$ is a dominant strategy for juror 1 when his preference relation is $\hat{R}_1$, $c = g(\hat{m}_1, \hat{m}_2, m_3)\ \hat{R}_1\ g(m_1, \hat{m}_2,\ m_3)$, and then $g(m_1, \hat{m}_2, m_3) \neq a$; (ii) since m_2 is a dominant strategy for juror 2 when his preference relation is R_2, $a = g(m_1, m_2, m_3)\ R_2\ g(m_1, \hat{m}_2, m_3)$, and then $g(m_1, \hat{m}_2, m_3) \neq b$; (iii) since $\hat{m}_3$ is a dominant strategy for juror 3 when his preference relation is $\hat{R}_3$, $b = g(m_1, \hat{m}_2, \hat{m}_3)\ \hat{R}_3\ g(m_1, \hat{m}_2, m_3)$, and then $g(m_1, \hat{m}_2, m_3) \neq c$. Therefore, no $W \in 2^N_w$ is such that $g(m_1, \hat{m}_2, m_3) = W$, which contradicts the definition of an outcome function.

Case 2: $w = 2$. Let $R \equiv (R_i)_{i \in J} \in \times_{i \in J} \mathcal{R}(F_i)$ be as depicted in Table VIII. Note that each juror i is biased against the contestant who is not in F_i (he always prefers that contestant not to be chosen as winner, whoever the deserving winner is). Abusing notation, let $R_1 = R_1(ab) = R_1(bc)$, $\hat{R}_1 = R_1(ac)$, $R_2 = R_2(ab)$, $\hat{R}_2 = R_2(ac) = R_2(bc)$, $R_3 = R_3(ab) = R_3(ac)$ and $\hat{R}_3 = R_3(bc)$ (Table IX). Since $\Gamma^D \equiv (M, g)$ implements φ in dominant strategies, for each $i \in J$, there is a dominant strategy $m_i \in M_i$ (respectively $\hat{m}_i \in M_i$) for i when his preference relation is $R_i \in \Re$ (respectively $\hat{R}_i \in \Re$). Since $(R, ab) \in S(\mathcal{F})$, $(m_1, m_2, m_3) \in D(\Gamma^D, R, ab)$, and Γ^D implements φ in dominant strategies, then $g(m_1, m_2, m_3) = ab$. Similarly, $g(m_1, \hat{m}_2, \hat{m}_3) = bc$ and $g(\hat{m}_1, \hat{m}_2, m_3) = ac$. Consider the profile $(m_1, \hat{m}_2, m_3) \in M$. Note that: (i) since $\hat{m}_3$ is a dominant strategy for juror 3 when his preference relation is $\hat{R}_3$, $bc = g(m_1, \hat{m}_2, \hat{m}_3)\ \hat{R}_3\ g(m_1,\ \hat{m}_2, m_3)$, and then $g(m_1, \hat{m}_2, m_3) \neq ab$; (ii) since m_2 is a dominant strategy for juror 2 when his preference relation is R_2, $ab = g(m_1, m_2, m_3)\ R_2\ g(m_1, \hat{m}_2, m_3)$, and then $g(m_1, \hat{m}_2, m_3) \neq ac$; (iii) since $\hat{m}_1$ is a dominant strategy for juror 1 when his preference relation

[15] The profile of preference relations $(R_1, \hat{R}_2, R_3) \in \Re^{|J|}$ does not correspond with any possible state of the world (and therefore the set of profiles of preference relations has not a Cartesian product structure). Suppose on the contrary that there is $(\tilde{R}_1, \tilde{R}_2, \tilde{R}_3) \in \times_{i \in J} \mathcal{R}(F_i)$ and $W_D \in 2^N_w$ such that $(\tilde{R}_1(W_D), \tilde{R}_2(W_D), \tilde{R}_3(W_D)) = (R_1, \hat{R}_2, R_3)$. Then: (i) since $b\ P_1\ c$ and $b, c \in F_1$, then $W_D \neq c$, (ii) since $c\ \hat{P}_2\ a$ and $a, c \in F_2$, then $W_D \neq a$, and (iii) since $a\ P_3\ b$ and $a, b \in F_3$, then $W_D \neq b$. Clearly, points (i), (ii), and (iii) are not compatible. The outcome function of mechanism Γ^D, however, must select some set of winners for the profile of messages $(m_1, \hat{m}_2, m_3)$.

is $\hat{R}_1$, $ac = g(\hat{m}_1, \hat{m}_2, m_3)\ \hat{R}_1\ g(m_1, \hat{m}_2, m_3)$, and then $g(m_1, \hat{m}_2, m_3) \neq bc$. Thus, no $W \in 2^N_w$ is such that $g(m_1, \hat{m}_2, m_3) = W$, which contradicts the definition of an outcome function. ■

	R_1			R_2			R_3		
$W_D =$	a	b	c	a	b	c	a	b	c
	a	a	a	b	b	b	c	c	c
Pref.	b	b	c	a	c	c	a	b	a
	c	c	b	c	a	a	b	a	b

Table VI. R in proof of Prop. 3, Case 1.

R_1	$\hat{R}_1$	R_2	$\hat{R}_2$	R_3	$\hat{R}_3$
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

Table VII. R_i and $\hat{R}_i$ in proof of Prop. 3, Case 1.

	R_1			R_2			R_3		
$W_D =$	ab	ac	bc	ab	ac	bc	ab	ac	bc
	bc	bc	bc	ac	ac	ac	ab	ab	ab
Pref.	ab	ac	ab	ab	bc	bc	ac	ac	bc
	ac	ab	ac	bc	ab	ab	bc	bc	ac

Table VIII. R in proof of Prop. 3, Case 2.

R_1	$\hat{R}_1$	R_2	$\hat{R}_2$	R_3	$\hat{R}_3$
bc	bc	ac	ac	ab	ab
ab	ac	ab	bc	ac	bc
ac	ab	bc	ab	bc	ac

Table IX. R_i and $\hat{R}_i$ in proof of Prop. 3, Case 2.

The necessary condition stated in Proposition 3 is not sufficient for implementation of φ in dominant strategies. Our next result shows that, if the planner has no idea about who treats all contestants fairly, then φ is not implementable in dominant strategies (*i.e.*, no mechanism gives the jurors the incentive to reveal who this juror is).

Proposition 4 *If the planner has no information about which juror treats all contestants fairly, then φ is not implementable in dominant strategies (i.e., if for each $i \in J$ there is some $F \in \mathcal{F}$ such that, for each $j \neq i$, $F_j \neq N$, then φ is not implementable in dominant strategies).*

Proof. Suppose for simplicity that $N = \{a, b, c\}$. Suppose that φ is implementable in dominant strategies. From Proposition 3, for each $F \in \mathcal{F}$ there is $i \in J$ such that $F_i = N$. Suppose that for each $i \in J$ there is $F \in \mathcal{F}$ such that, for each $j \neq k$, $F_j \neq N$. Then, since $n \geq 3$, there are at least three jurors. Suppose, to simplify notation, that $J = \{1, 2, 3\}$. From Proposition 1, for each pair of contestants, the planner knows at least one juror who treats them fairly. Suppose to simplify notation that, for each $F \in \mathcal{F}$, $a, b \in F_1$, $a, c \in F_2$, and $b, c \in F_3$. Then, there are $F, \hat{F}, \tilde{F} \in \mathcal{F}$ such that:

(i) $F_1 = \{a, b, c\}$, $F_2 = \{a, c\}$, and $F_3 = \{b, c\}$.

(ii) $\hat{F}_1 = \{a, b\}$, $\hat{F}_2 = \{a, b, c\}$, and $\hat{F}_3 = \{b, c\}$.

(iii) $\tilde{F}_1 = \{a, b\}$, $\tilde{F}_2 = \{a, c\}$, and $\tilde{F}_3 = \{a, b, c\}$.

Case 1: $w = 1$**.** Let $R \in \times_{i \in J} \mathcal{R}(F_i)$, $\hat{R} \in \times_{i \in J} \mathcal{R}(\hat{F}_i)$, and $\tilde{R} \in \times_{i \in J} \mathcal{R}(\tilde{F}_i)$ be the profiles of preference functions defined in Tables X, XI, and XII, respectively.[16] Abusing notation, for each $i \in J$, let $R_i, \bar{R}_i \in \Re$ be the following preference relations: $R_1 = R_1(a) = \hat{R}_1(c)$, $\bar{R}_1 = \tilde{R}_1(b)$, $R_2 = R_2(a)$, $\bar{R}_2 = \hat{R}_2(c) = \tilde{R}_2(b)$, $R_3 = R_3(a) = \tilde{R}_3(b)$, and $\bar{R}_3 = \hat{R}_3(c)$ (Table XIII). Let $\Gamma^D = (M, g)$ be a mechanism that implements φ in dominant strategies. Then, for each juror $i \in J$, there is a dominant strategy $m_i \in M_i$ (respectively $\bar{m}_i \in M_i$) for i when his preference relation is $R_i \in \Re$ (respectively $\bar{R}_i \in \Re$). Since $(R, a) \in S(\mathcal{F})$ and $(m_1, m_2, m_3) \in D(\Gamma^D, R, a)$, we have $g(m_1, m_2, m_3) = a$. Similarly, since $(\tilde{R}, b) \in S(\mathcal{F})$ and $(\bar{m}_1, \bar{m}_2, m_3) \in D(\Gamma^D, \tilde{R}, b)$, we have $g(\bar{m}_1, \bar{m}_2, m_3) = b$. Finally, since $(\hat{R}, c) \in S(\mathcal{F})$ and

[16] Notice that $R \in \times_{i \in J} \mathcal{R}(F_i)$ is compatible with a situation where juror 2 is biased in favor of b and juror 3 is biased against a. Similarly, $\hat{R} \in \times_{i \in J} \mathcal{R}(\hat{F}_i)$ is compatible with a situation where juror 1 is biased against c and juror 3 is biased in favor of a, while $\tilde{R} \in \times_{i \in J} \mathcal{R}(\tilde{F}_i)$ is compatible with a situation where juror 1 is biased in favor of c and juror 2 is biased against b.

$(m_1, \bar{m}_2, \bar{m}_3) \in D(\Gamma^D, \hat{R}, c)$, we have $g(m_1, \bar{m}_2, \bar{m}_3) = c$. Consider now the profile of messages $(m_1, \bar{m}_2, m_3) \in M$. Note that: (i) since $\bar{m}_3$ is a dominant strategy for juror 3 with preferences $\bar{R}_3$, $c = g(m_1, \bar{m}_2, \bar{m}_3)\ \bar{R}_3\ g(m_1, \bar{m}_2, m_3)$, and then $g(m_1, \bar{m}_2, m_3) \neq a$; (ii) since m_2 is a dominant strategy for juror 2 with preferences R_2, $a = g(m_1, m_2, m_3)\ R_2\ g(m_1, \bar{m}_2, m_3)$, and then $g(m_1, \bar{m}_2, m_3) \neq b$; (iii) since $\bar{m}_1$ is a dominant strategy for juror 1 with preferences $\bar{R}_1$, $b = g(\bar{m}_1, \bar{m}_2, m_3)\ \bar{R}_1\ g(m_1, \bar{m}_2, m_3)$, and then $g(m_1, \bar{m}_2, \bar{m}_3) \neq c$. Thus, there is no $W \in 2^N_w$ such that $g(m_1, \bar{m}_2, m_3) = W$, which contradicts the definition of an outcome function.

Case 2: $w = 2$**.** Let $R \in \times_{i \in J} \mathcal{R}(F_i)$, $\hat{R} \in \times_{i \in J} \mathcal{R}(\hat{F}_i)$, and $\tilde{R} \in \times_{i \in J} \mathcal{R}(\tilde{F}_i)$ be the profiles of preference functions defined in Tables XIV, XV, and XVI, respectively. Let $R_1, \bar{R}_1, R_2, \bar{R}_2, R_3, \bar{R}_3 \in \Re$ be the following preference relations: $R_1 = R_1(ab) = \hat{R}_1(ac)$, $\bar{R}_1 = \tilde{R}_1(bc)$, $R_2 = R_2(ab)$, $\bar{R}_2 = \hat{R}_2(ac) = \tilde{R}_2(bc)$, $R_3 = R_3(ab) = \tilde{R}_3(bc)$, and $\bar{R}_3 = \hat{R}_3(ac)$ (Table XVII). Let $\Gamma^D \equiv (M, g)$ be a mechanism that implements φ in dominant strategies. By an argument similar to that in Case 1, for each juror $i \in J$, there is a dominant strategy $m_i \in M_i$ (respectively $\bar{m}_i \in M_i$) for i when his preference relation is $R_i \in \Re$ (respectively $\bar{R}_i \in \Re$). Since $(R, ab) \in S(\mathcal{F})$ and $(m_1, m_2, m_3) \in D(\Gamma^D, R, ab)$, we have $g(m_1, m_2, m_3) = ab$. Similarly, since $(\hat{R}, ac) \in S(\mathcal{F})$ and $(m_1, \bar{m}_2, \bar{m}_3) \in D(\Gamma^D, \hat{R}, ac)$, we have $g(m_1, \bar{m}_2, \bar{m}_3) = ac$. Finally, since $(\tilde{R}, bc) \in S(\mathcal{F})$ and $(\bar{m}_1, \bar{m}_2, m_3) \in D(\Gamma^D, \tilde{R}, bc)$, we have $g(\bar{m}_1, \bar{m}_2, m_3) = bc$. Then, by an argument similar to that in Case 1, it can be shown that there is no $W \in 2^N_w$ such that $g(m_1, \bar{m}_2, m_3) = W$, which contradicts the definition of an outcome function. ■

	R_1			R_2			R_3		
$W_D =$	a	b	c	a	b	c	a	b	c
	a	b	c	b	b	b	b	b	c
Pref.	b	c	a	a	a	c	c	c	b
	c	a	b	c	c	a	a	a	a

Table X. R in proof of Prop. 4, Case 1.

	$\hat{R}_1$			$\hat{R}_2$			$\hat{R}_3$		
$W_D =$	a	b	c	a	b	c	a	b	c
	a	b	a	a	b	c	a	a	a
Pref.	b	a	b	b	c	a	b	b	c
	c	c	c	c	a	b	c	c	b

Table XI. $\hat{R}$ in proof of Prop. 4, Case 1.

	$\tilde{R}_1$			$\tilde{R}_2$			$\tilde{R}_3$		
$W_D =$	a	b	c	a	b	c	a	b	c
	c	c	c	a	c	c	a	b	c
Pref.	a	b	a	c	a	a	b	c	a
	b	a	b	b	b	b	c	a	b

Table XII. $\tilde{R}$ in proof of Prop. 4, Case 1.

R_1	$\bar{R}_1$	R_2	$\bar{R}_2$	R_3	$\bar{R}_3$
a	c	b	c	b	a
b	b	a	a	c	c
c	a	c	b	a	b

Table XIII. R_i and $\bar{R}_i$ in proof of Prop. 4, Case 1.

	R_1			R_2			R_3		
$W_D =$	ab	ac	bc	ab	ac	bc	ab	ac	bc
	ab	ac	bc	ac	ac	ac	bc	bc	bc
Pref.	ac	bc	ab	ab	ab	bc	ab	ac	ab
	bc	ab	ac	bc	bc	ab	ac	ab	ac

Table XIV. R in proof of Proposition 4, Case 2.

	$\hat{R}_1$			$\hat{R}_2$			$\hat{R}_3$		
$W_D =$	ab	ac	bc	ab	ac	bc	ab	ac	bc
	ab	ab	ab	ab	ac	bc	bc	bc	bc
Pref.	ac	ac	bc	ac	bc	ab	ab	ac	ab
	bc	bc	ac	bc	ab	ac	ac	ab	ac

Table XV. $\hat{R}$ in proof of Proposition 4, Case 2.

	$\tilde{R}_1$			$\tilde{R}_2$			$\tilde{R}_3$		
$W_D =$	ab	ac	bc	ab	ac	bc	ab	ac	bc
	ab	ab	ab	ac	ac	ac	ab	ac	bc
Pref.	ac	ac	bc	ab	ab	bc	ac	bc	ab
	bc	bc	ac	bc	bc	ab	bc	ab	ac

Table XVI. $\tilde{R}$ in proof of Proposition 4, Case 2.

R_1	$\bar{R}_1$	R_2	$\bar{R}_2$	R_3	$\bar{R}_3$
ab	ab	ac	ac	bc	bc
ac	bc	ab	bc	ab	ac
bc	ac	bc	ab	ac	ab

Table XVII. R_i and $\bar{R}_i$ in proof of Prop. 4, Case 2.

6 Conclusion

We have analyzed the problem of choosing the w best contestants who will win a competition within a group of $n > w$ competitors when the jurors may be partial. We have studied restrictions on the configuration of the jury so that it is possible to induce the jurors to always choose the best contestants, whoever they are (*i.e.*, φ is implementable). The necessary conditions for implementation in dominant strategies incorporate very strong informational requirements: there must be at least one juror who is totally impartial, and the planner must have some information about who this juror is. If the equilibrium concept is Nash (or subgame perfect) equilibria the necessary and sufficient conditions are less demanding: for each pair of contestants, the planner must know at least one juror who is not biased in favor/against any of them. As a matter of fact, the latter condition can be interpreted as the minimum degree of impartiality that we must require on the jury in order to guarantee that their decisions will correspond to socially optimal goals: this condition cannot be relaxed since it is necessary for the implementability of φ in any equilibrium concept.

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