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The Walrasian Output Beats the Market

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Abstract

We show that for any market-clearing price, average profits in a symmetric industry cannot exceed the individual profits from the Walrasian output. This immediately implies that a firm itself can guarantee to beat the market by producing the Walrasian output. This property clarifies and generalizes the conditions used in the literature to prove the success of Walrasian behavior from an evolutionary perspective.

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1 Introduction

The Walrasian Hypothesis, a central concept in Economics, states that economic agents take prices as given, that is, they behave as if their actions did not affect

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prices. In this paper, we present a novel property that relates profits in an industry to profits from the Walrasian quantity: In a symmetric industry, for any market-clearing price, average profits cannot exceed the profits from the Walrasian output.

This property can be rephrased as saying that a firm itself can guarantee profits above average profits by producing the Walrasian output. And from this perspective, it has far reaching implications for the theoretical foundations of the Walrasian Hypothesis.

Theoretical explanations for the Walrasian equilibria have a long tradition in Economics. They were first developed within the realm of the cooperative game theory, where a number of core equivalence theorems were found (see Hildenbrand, 1974); and then an approach based on non-cooperative game theory was developed (see Mas-Colell, 1980). More recently, attempts to ground the Walrasian Hypothesis on an evolutionary basis have appeared.

This paper contributes to the evolutionary foundations of the Walrasian equilibrium. The current state of the art presents a collection of particular results of static and dynamic nature. The static approach was initiated by Schaffer (1989), which proves that the Walrasian output is an Evolutionarily Stable Strategy in a duoply with constant marginal costs. The dynamic approach was pioneered by Vega-Redondo (1997), which proves that the Walrasian output is the unique Stochastically Stable Strategy for a particular imitation dynamic in an oligopoly with a general cost function.

The proof of these results relies in showing that the Walrasian quantity beats any other quantity in a pairwise comparison, that is, it offers higher profits when firms are restricted to play either of the two quantities.[1] In this paper we prove that this superiority extends to a general quantity profile: profits from the Walrasian output are higher than average profits. And once it is stated in these terms, it is straightforward

[1]See Section 2 in Schaffer (1989) and the proof of Lemma 1 in Vega-Redondo (1997).

that the evolutive advantage of the Walrasian quantity extends naturally to many dynamics, because what matters in evolutive terms are relative rather than absolute profits. Next Section proves the property and Section 3 concludes.

2 Beating the Market

Consider n firms competing for an homogenous product whose demand function is $P(Q)$. Assume for the moment that they all have constant marginal costs c. Consider a quantity profile $(q_1, ..., q_n)$ with aggregate output $Q = \sum_{j=1}^{n} q_j$. Average profits can be rewritten as follows

$$\pi^{AVG} = \frac{(P(Q) - c)\sum_{j=1}^{n} q_j}{n} = \pi^w + \frac{1}{n}(P(Q) - c)(Q - Q^w) \tag{1}$$

where Q^w is the aggregate Walrasian output and π^w denotes profits from the Walrasian output $q^w = \frac{Q^w}{n}$. Note that a downward-sloping demand function implies that $(P(Q) - c)(Q - Q^w) \leq 0$ and, therefore, that average profits are bounded above by π^w. Next theorem proves that this result extends to a general cost function $C(q)$ with the unique condition that a Walrasian equilibrium exists.

Theorem 1 *Average profits in the market never exceed the profits from the Walrasian output*

Proof. The Walrasian output q^w is given by the following condition:

$$P(nq^w)\, q - C(q) \leq P(nq^w)\, q^w - C(q^w), \qquad \forall q \geq 0 \tag{2}$$

The generalization of expression (1) is

$$\pi^{AVG} = \pi^w + \frac{1}{n}\left(P(Q)(Q - Q^w) - \sum_{j=1}^{n}(C(q_j) - C(q^w))\right)$$

In order to prove that the terms in large brackets are non-positive, we will show the following inequalities

$$P(Q)(Q - Q^w) \leq P(Q^w)(Q - Q^w) \leq \sum_{j=1}^{n}(C(q_j) - C(q^w))$$

The first one, $P(Q)(Q - Q^w) \leq P(Q^w)(Q - Q^w)$, results, as in the special case of constant marginal cost, from the downward-sloping nature of the demand function. The second inequality, $P(Q^w)(Q - Q^w) \leq \sum_{j=1}^{n} (C(q_j) - C(q^w))$, can be obtained by particularizing condition (2) for every component of the quantity profile and adding them all. ■

3 Discussion

The Theory of the Walrasian Equilibrium leaves unexplained where price-taking behavior comes from. Since first stated, a foundation for the Walrasian Hypothesis has been a long standing problem in Economics. Different approaches rely on different assumptions, although most of them are quite dependent of particular models of interactions among economic agents. The most recent approach, the evolutive foundation, also lacks of generality as it relies on imitation; for example, Vega-Redondo (1997) assumes that firms imitate the output of those which achieved the highest profit.[2] Also, it is technically demanding: it focuses on the limiting behavior of invariant distributions by using the counting mutations technique first developed by Freidlin and Wentzel, 1984 (e.g. Vega-Redondo, 1997) or the concept of recurrent set by Nöldeke and Samuelson, 1993 (e.g. Schipper, 2003).

This paper reveals a simple property of the Walrasian output that points to the heart of the evolutive argument in an elegant manner. The evolutive dynamics must be responsive to current differential payoffs, that is, it is relative rather than absolute performance what matters. Note that the Walrasian quantity assures a superiority in relative terms: it always fare better than the average. And this is exactly why previous papers have been successful in showing the evolutive dominance of the Walrasian output in some particular settings.

[2] Apesteguía et al (2007) extends Vega-Redondo's analysis to a different imitation rule proposed by Schlag (1998): imitate with a probability proportional to the payoffs difference.

In fact, this property helps extend the superiority of the Walrasian quantity to many other settings beyond the imitative dynamics. Think for example of the Replicator Dynamics, the foundation of evolutionary game theory (see Hofbauer and Sigmund, 1998). Under this dynamics, strategies faring better than average are favoured. Then, it trivially follows that the Walrasian quantity is selected as it fares better than average for every state of the population.

Finally, the dependence on the imitative assumption of the convergence result to the Walrasian output is so strong that experimental data have been interpreted in terms of imitative rules (see Apesteguía et al, 2007). However, using this property, we can construct simple non-imitative rules that lead a population of agents to the Walrasian outcome. Consider the following: "Delete from the strategy space the played action if it has performed worse than the average performance in the market". Given that in every period at least one action is deleted from the strategy space of at least one agent, the Walrasian output will be selected because it never gets deleted (see some simulations of this and related rules in Fernández-de-Córdoba and Navas, 2008). This suggests alternative explanations for previous experimental results.

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