A Spatial-Dependence Continuous-Time Model for Regional Unemployment Change in Germany

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Abstract

This paper analyzes patterns of regional labour market development in Germany over the period 2000-2003 by means of a spatial-dependence continuous-time model. (Spatial) panel data are routinely modelled in discrete time. However, there are compelling arguments for continuous time modelling of (spatial) panel data. Particularly, most social processes evolve in continuous time such that analysis in discrete time is an oversimplification, gives an incomplete representation of reality and leads to misinterpretation of estimation results. The most compelling reason for continuous time modelling is that, in contrast to discrete time modelling, it allows for adequate modelling of dynamic adjustment processes (see, for example, Special Issue 62:1, 2008, of *Statistica Neerlandica*). We introduce spatial dependence in a continuous time modelling framework and apply the integrated framework to regional labour market changes in Germany. The empirical results show substantial autoregressive effects for both unemployment and population change, as well as a negative effect of unemployment change on population change. The reverse effect is not significant. Neither are the effects of the changes of regional average wage and of the structure of the manufacturing sector on the changes of unemployment and population.

Keywords: unemployment change, population change, continuous time modelling, structural equation modelling, spatial dependence, panel data, disattenuation, measurement errors, Germany.

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1. Introduction

Socio-economic processes such as the development of unemployment are the outcomes of various decisions taken by different actors at different points in time. This basic feature gives rise to continuously evolving socio-economic dynamics, rather than to processes that change at discrete points in time only. The analyst, however, only observes the processes at discrete points in time (for example, yearly observations of regional unemployment). The typical approach in conventional (that is, discrete) time series modelling and panel data analysis is to ignore the continuous nature of the processes underlying discrete time observations. Consequently, discrete time series and discrete panel data analysis are simplifications of reality and may lead to bias in the mapping of dynamic adjustment processes of socio-economic phenomena and to misinterpretations of estimation results. Discrete time (DT) analysis is at best a simplified approximation of real-world processes in continuous time (CT) (Oud and Singer, 2008).

CT econometrics models the continuous nature of social processes by means of systems of differential equations. It departs from the assumption that different agents take different actions at different points in time. This assumption implies that there is no obvious time interval that can serve as a natural unit. This is in contrast to DT models (which are made up of systems of difference equations), which are necessarily formulated in relation to the data available (for example yearly or monthly data).

A DT model estimated on the basis of, for example, monthly data will be different from a model estimated on the basis of annual data. For CT approaches, however, the model is independent of the observation interval, and thus provides a common basis for accurate comparison of differently time-spaced models of the same process (Oud and Jansen, 2000). These features enable the analyst to obtain predictions and simulations for any time interval, rather than for the time interval inherent to the data, as in the case of DTmodelling.

CT modelling is particularly useful for the analysis of dynamic adjustment processes (Gandolfo, 1993). Whereas in DT models it may not be possible to obtain an estimate of the adjustment speed when the time lags are short compared to the observation period, in CT models it is in general possible to obtain an asymptotically unbiased estimate of it. Specifically, CT modelling makes it possible to determine at what pace an effect builds up over continuously increasing intervals, at which observation points or between which points the maximum impact of an effect occurs and at what pace it dies out. A CT model therefore allows a more satisfactory treatment of distributed-lag processes.

CT modelling has a long history in econometrics. Following the pioneering work by, amongst others, Bartlett (1946), Koopmans (1950) and Phillips (1959), CT modelling has become quite common in applied econometric work (for an overview, see Bergstrom, 1988). To our best knowledge, however, little attention has been paid to CT modelling in spatial econometrics.[2] The reverse also holds: In CT modelling no attention has been paid to spatial dependence nor, more generally, to dependence among units of observation.

[2] For an interesting application we refer to Piras et al (2007). Note that in spatial analyses the units of observation usually are discrete. There is, however, a growing interest in theoretical work on continuous

In this paper we introduce spatial dependence in a CT modelling framework to analyze the main determinants of regional unemployment change in Germany, viz. the changes of wages, population and industrial structure. This will be pursued in the framework of structural equations modelling (SEM). The German case is interesting and important, because inflexible wages are often considered to be one of the main causes of unemployment in Germany.

The paper is organized as follows. Section 2 presents the basic characteristics of CT modelling and points out the differences with respect to DT modelling. The regional unemployment change model for Germany is outlined in Section 3. In Section 4 the CT model with spatial dependence and its estimation procedure by means of a nonlinear SEM procedure are outlined, while in Section 5 estimation results are presented. Conclusions follow in Section 6.

2. Main characteristics of CT modelling

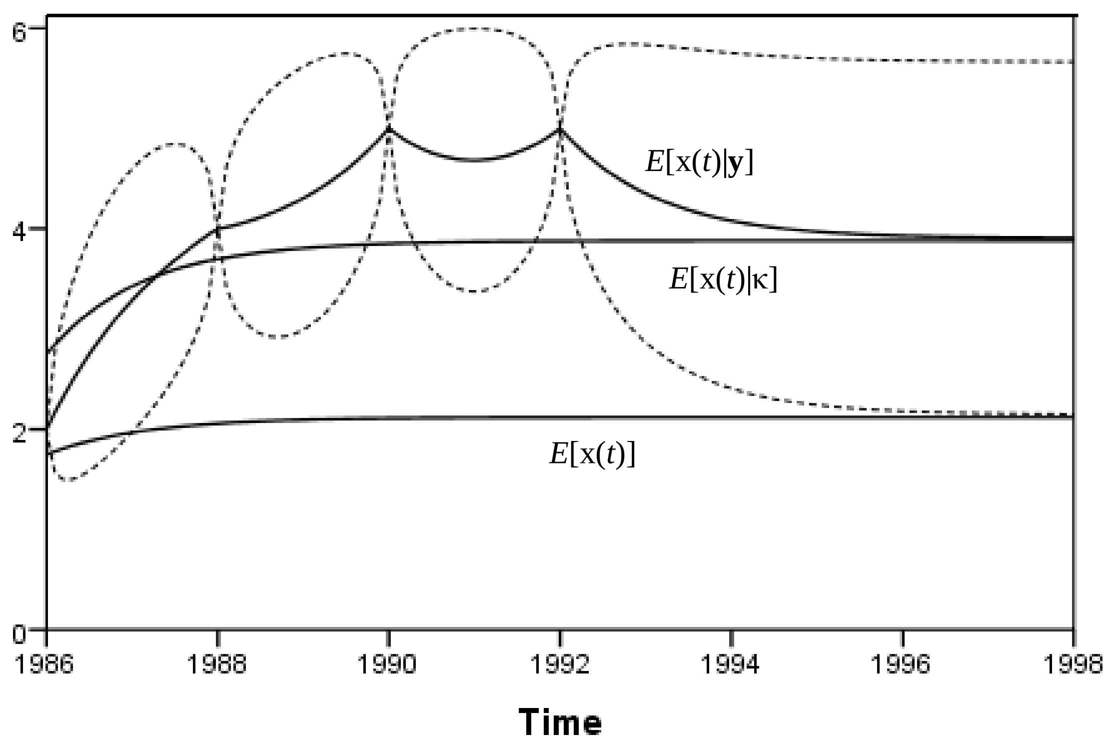

Figure 1. Estimated mean curve $E[\mathrm{x}(t)]$, subject-specific mean curve $E[\mathrm{x}(t)|\kappa]$ and sample trajectory curve $E[\mathrm{x}(t)|\mathbf{y}]$ (for the same subject as in $E[\mathrm{x}(t)|\kappa]$) in CT (confidence intervals for the sample trajectory curve in dotted lines).

space modelling (see, for example, Puu, 1997). It would be a great challenge in spatial modelling and spatial econometrics to explore the relationships between continuous time and continuous space modelling and to integrate both. See, amongst others Cressie 1993; Wackernagel 1995; Donaghy 2001).

As observed above, in CT the parameters estimated are independent of the observation intervals which implies that CT modeling make it possible to "fill out" the spaces between the discrete observation time points by model based estimates, including interpolations and extrapolations (predictions), of full developmental and effect curves. Figure 1 which is taken from Delsing and Oud (2008), illustrates this for the mean curve E[x(t)], which represents the estimated mean trajectory in the population, an individual subject specific mean curve E[x(t)|κ] (κ the random parameter with the subject specific value) and the trajectory E[x(t)|**y**] for the same individual subject. The latter, the conditional mean (conditional on the individual's data vector **y**) or state space smoother (Durbin & Koopman, 2001), represents the best estimate of the individual's sample trajectory through CT, which includes error components, on the basis of the total data vector **y.** The latter two curves, though both relating to the same individual subject, differ because they exploit different kinds of information. E[x(t)|**y**] is based on all measured information available for the subject, using the model only where data are lacking, while the subject-specific mean curve E[x(t)|κ] is the model expectation, conditioned on the single subject-specific parameter value (Oud & Singer, 2008).

Figure 1 shows that by E[x(t)|**y**] the gaps between measurements in 1986, 1988, 1990 and 1992 are filled out by interpolations and similarly for the prediction interval 1992-1998, while uncertainty of the interpolations and predictions are given by confidence intervals (dotted lines).[3] These confidence intervals typically go to zero at the observation points through 1992 but increase considerably in the prediction period after 1992. The confidence intervals of E[x(t)|**y**] show that the conditional mean is almost everywhere significantly above the mean curve E[x(t)], even in the prediction period, and for time points close to the the measurement points also significantly above the individual's subject-specific mean curve.

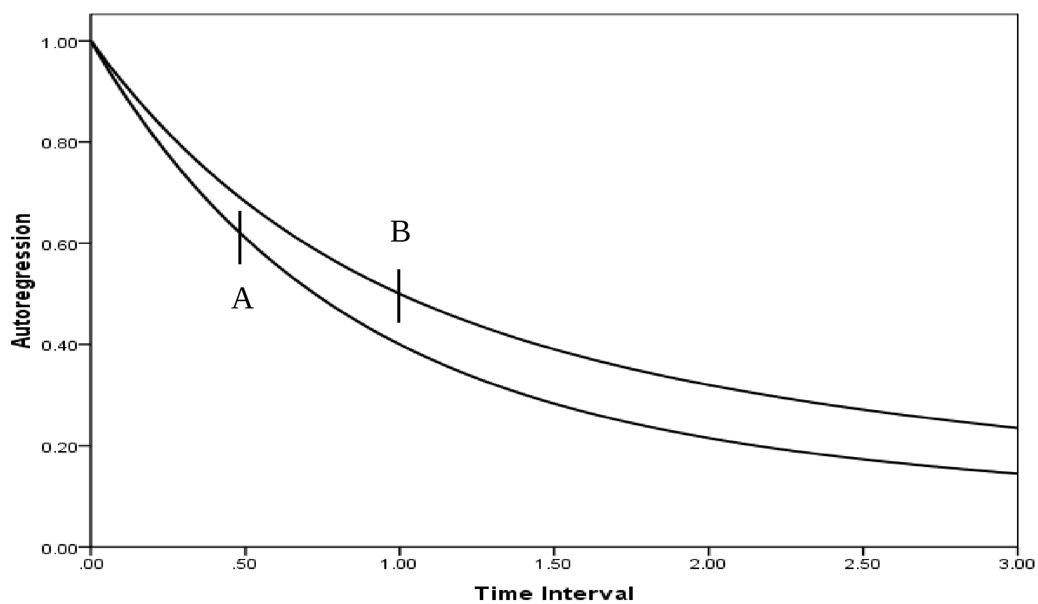

Figure 2: Two different autoregression functions in two different studies A and B.

[3]Since there are no error components in E[x(t)|κ] and E[x(t)], there are no confidence intervals for these curves.

In contrast to CT modelling which gives estimates for the entire interval 1986-1998, in DT modeling only estimates of the three types of means are obtained for the four observation time points 1986, 1988, 1990, 1992, and possibly the three prediction time points 1994, 1996, 1998. Clearly, the interpolations and predictions in CT are subject to uncertainty. However, the uncertainty is quantified in the model by means of the confidence intervals.[4]

DT modeling may be especially misleading in the case of unequal observation intervals, within the same study or when comparisons between different studies are made. This is shown in Figure 2, where (hypothetical) autoregression functions in CT of two studies, A en B, with two different observation intervals (0.50 in study A and 1.00 in study B) are depicted. Because CT autoregression function B exceeds A everywhere, the obvious conclusion is that the autoregression in study A is lower than in study B. Nevertheless, in DT at e.g. observation interval 0.50 (half-yearly observation interval) study A finds the autoregression value of 0.61 which is considerably larger than the value of 0.50 found at the interval 1.00 (yearly observation interval) in study B, which might lead to the erroneous conclusion that the autoregression in study A is larger than in study B. To obtain the correct answer CT modeling is required which makes it possible to compare the complete autoregression functions. An important corollary is that that CT modeling allows combining the data of several studies into one data set, test whether the underlying continuous time parameters are equal and, if so, to present one and the same autoregression function (Oud, 2001).

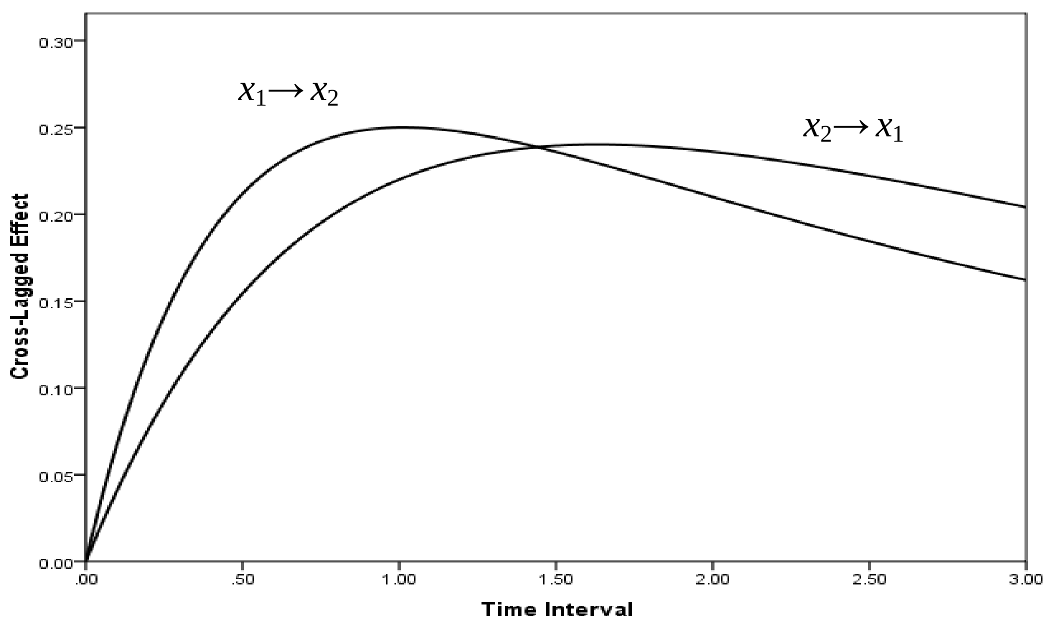

Figure 3: Cross-lagged effect functions for the reciprocal effects between x_1 and x_2

[4] For statistical details, see section 4.

Using equal observation intervals is no solution to the problems inherent to DT modeling discussed above. To see this, consider the two (hypothetical) CT reciprocal cross-lagged effect functions for variables x_1 and x_2 in Figure 3.[5] Unlike autoregression functions, which start at value 1, cross-lagged effect functions have starting value 0 (different variables cannot have any influence on each other over a time interval of length zero), increase until the maximum is reached (in Figure 3 the maxima 0.250 and 0.240 are reached at the time points 1.02 and 1.64, respectively), and next taper off to 0 (in a stable model). Figure 3 shows an example of two cross-lagged effect functions that cross at interval 1.44. Both have here the same value 0.239, but are different over all other intervals . Particularly, for observation intervals < 1.44 the effect of x_1 on x_2 is larger than the reverse effect while the opposite holds for intervals > 1.44. Observe that using one and the same interval in DT does not resolve the problem. For instance, using the observation interval 1.44 would lead to the false conclusion that the effect of x_1 on x_2 is equal to the effect of x_2 on x_1. Using only intervals < 1.44 would lead to the false inequality effect "effect of x_1 on x_2 > effect of x_2 on x_1" and using only intervals > 1.44 would lead to the opposite false inequality effect. Again, CT analysis, particularly estimating and displaying the full cross-lagged effect functions over the entire time axis is required to obtain the correct effects.

3. The Regional Unemployment Model

Following Elhorst (2003) and Blanchard and Katz (1992), we adopt a regional labour market model that relates regional unemployment rates (the result of job-matching) to regional labour supply, economic structure and wages. Elhorst points out that '*the regional unemployment rate both affects and is affected by regional factors of labour supply, labour demand, and wages*'. Therefore we adopt a simultaneous equations framework to study the reciprocal effects of regional unemployment development and regional labour supply development, as well as the impacts of the developments of economic structure and wages on both variables. The latter two variables are assumed exogenous. The rationale for considering the wage variable exogenous is based on the fact that in Germany, like in many other European countries, collective wage agreements are set at the national level on a sectoral basis rather than at the regional level. This means that contractual wages may be considered exogenous for a given region. This view is supported by a large literature in labour economics (see, for example, Lommerud et al. 2000; Correa López and Naylor 2004). The fact that wages are set nationally rather than regionally does of course not mean that average wages are largely equal across regions. Wage differentials occur due to differences in regional economic structure.

The rationale for taking economic structure as exogenous follows from the fact that this variable evolves slowly, such that changes only show up in the long run. Moreover, its evolution depends on a large set of factors and definitely not only on the regional wage structure. Since the time span considered in this paper is seven years only, we consider economic structure exogenous. Due to lack of data, economic structure is measured in this paper as the proportion of the workforce employed in manufacturing (for a similar approach, see Jones and Manning 1992).

[5] A cross-lagged effect function gives the effect of one variable on another as it develops across time.

Formally, the regional unemployment model comprises two equations. In the first equation, the unemployment development is explained by population development, as well as by development of average daily wages of fulltime workers, and of economic structure. The expected effect of population development on unemployment development is ambiguous, since demographic development has an impact on both labour supply (positive or zero impact on unemployment development) and labour demand (negative impact on unemployment development). On the supply side, population development – via natural growth or immigration – may lead to a larger workforce and to changes in its age structure. Changes in the age structure leading to a younger average population – due to higher birth rates – have been shown to lead to higher, and more persistent, unemployment rates (Elhorst 1995), because of a larger workforce. However, natural change of population is known to be a slow, long-run process rather than a short-run one. Consequently, it is not expected to have a significant effect on unemployment in the short panel considered in the present paper.

With regard to immigration, the expected effect of an additional migrant on unemployment is zero, if the migrant fills a job opening for which no one in the home region qualifies or if the migrant does not join the workforce. However, the impact is expected to be positive (increasing unemployment) when the effect of additional migrants is worked out through the accounting identity[6] (Elhorst 2003). On the demand side, negative effects of net immigration on unemployment development may be identified, for example, because of increased productivity induced by migrants with different skill endowments (Ghatak et al. 1996), or greater investments attracted by an increase of higher-skilled labour, or higher consumption levels due to a larger population. We expect the labour supply side to dominate the demand side (and hence a positive effect of net immigration on unemployment development; see, for example, Pissarides and McMaster 1990). Since in Germany there is outmigration from high unemployment regions in the East (unemployment going down) to low unemployment regions in the West (unemployment going up or remaining constant), migration is likely to induce some convergence of regional unemployment.

Because of the negative impact of wages on labour demand (they represent greater costs for firms), higher wages are expected to increase unemployment. A positive coefficient of wage development on unemployment development may then be expected (as, for example, in Hall 1972; Layard et al. 1991). With regard to the relationship between economic structure (regional specialization) and unemployment, we expect, for regions with a relatively dominant manufacturing sector and a low-skilled workforce (as in East Germany), that a decrease in the manufacturing sector will lead to an increase in the number of unemployed individuals. Workers laid off from the manufacturing sector are likely to experience difficulties in becoming re-employed, and will risk long-term unemployment and loss of skills. A negative effect of specialization in manufacturing on unemployment development may then be expected.

The second equation may be compared – with due differences – to the labour supply equation in the Blanchard and Katz (BK) model (Blanchard and Katz 1992). In this equation, population development is explained by the developments of unemployment, wages and manufacturing. We expect higher unemployment to increase out-migration and to lead to lower fertility rates. A negative effect of unemployment development on

[6] The accounting identity is a deterministic method for computing regional unemployment levels.

population development may then be expected. Inversely, higher wages will tend to attract individuals towards a region (in-migration). A positive value of the wage coefficient may be expected. With regard to regional economic structure, we note that, if decreasing specialization in manufacturing results in a greater number (or share) of long-term unemployed, a positive coefficient, though weak, may then be expected for the effect of change in specialization on population development (see, for example, Budd et al. 1987).
The model outlined above is formally presented in Section 4.

4. Specification and Estimation of the Spatial-Dependence Continuous-Time Model

Let $\mathbf{x}(t)$ be the n-dimensional (endogenous) state vector, $\mathbf{u}(t)$ the r-dimensional vector of (exogenous) fixed input variables,[7] $\boldsymbol{\kappa}$ the n-dimensional vector of subject (region-) specific deviations from the fixed mean intercept (that is, a vector of 'random subject effects' or 'unobserved unit heterogeneity') and $\mathbf{W}(t)$ the standard multivariate Wiener process. We consider the following spatial error model for regions $i = 1, 2, ..., N$:

$$\frac{\mathrm{d}\tilde{\mathbf{x}}(t)}{\mathrm{d}t} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}\tilde{\mathbf{u}}(t) + \tilde{\boldsymbol{\kappa}} + \frac{\mathrm{d}\tilde{\mathbf{z}}(t)}{\mathrm{d}t}, \tag{1}$$

$$\frac{\mathrm{d}\tilde{\mathbf{z}}(t)}{\mathrm{d}t} = \tilde{\mathbf{R}}\tilde{\mathrm{C}}\frac{\mathrm{d}\tilde{\mathbf{z}}(t)}{\mathrm{d}t} + \tilde{\mathbf{G}}\frac{\mathrm{d}\tilde{\mathbf{W}}(t)}{\mathrm{d}t}, \tag{2}$$

where $\tilde{\mathbf{x}}_{(Nn\times 1)} = \text{rowvec}\,\mathbf{X}_{(N\times n)}$ and $\tilde{\mathbf{u}}_{(Nr\times 1)} = \text{rowvec}\,\mathbf{U}_{(N\times r)}$ row-vectorize the data matrices $\mathbf{X}$ and $\mathbf{U}$ (for each region i there is a row with n values for the n state variables and a row with r values for the fixed input variables). Similarly for the random subject effects $\tilde{\boldsymbol{\kappa}}$. Furthermore, $\tilde{\mathbf{W}}$ has the same dimension $(Nn \times 1)$ as $\tilde{\mathbf{x}}$, $\tilde{\mathbf{A}} = \mathbf{I}_N \otimes \mathbf{A}$, $\tilde{\mathbf{B}} = \mathbf{I}_N \otimes \mathbf{B}$, $\tilde{\mathbf{G}} = \mathbf{I}_N \otimes \mathbf{G}$, where the drift matrix $\mathbf{A}$ contains the coefficients of the causal relationships among the state variables, $\mathbf{B}$ the coefficients of the effects of the fixed input variables on the state vector, and the lower triangular matrix $\mathbf{G}$ transforms the uncorrelated standard multivariate Wiener process $\mathbf{W}$ with variance t at time t into a process with variances possibly $\neq t$ at time t and correlations possibly $\neq 0$. Matrix $\mathbf{C}$ is the spatial $(N \times N)$ connectivity matrix. For the multivariate case we specify the $(Nn \times Nn)$ matrix $\tilde{\mathbf{C}} = \mathbf{C} \otimes \mathbf{I}_n$. Associated with $\tilde{\mathbf{C}}$ are the spatial dependence parameters. In the general case of a different spatial dependence parameter for each state variable, we have the $(n \times n)$ spatial parameter matrix $\mathbf{R}$, which for the N subjects becomes

[7] We assume that the state variables are latent variables, that is, they cannot be directly observed due to measurement error. Latent variables are measured by one or more indicators. For instance, the concept of socioeconomic status is usually measured by means of more than one indicator, for example income, education, and profession. In the present paper, we only deal with latent variables that are measured by one indicator, though with error. The introduction of latent variables requires the use of a measurement model relating the latent variables to their indicators, and a structural model which presents the relationships between the latent variables. For further details see Oud and Folmer (2008b).

$\tilde{\mathbf{R}} = \mathbf{I}_N \otimes \mathbf{R}$. In this paper, we assume one and the same spatial dependence parameter for the n state variables, that is, $\mathbf{R} = \rho \mathbf{I}_n$. This simplification safeguards the commutative property and has two advantages: (a) conventional standard procedures can be used to solve the stochastic differential equation implied by Equations (1) and (2); and (b) standard spatial econometric methods can be applied. Observe that Model (1)–(2) includes three parameter matrices to be estimated (**A**, **B**, and **G**) in addition to the spatial parameter ρ.

From Equations (1) and (2), we derive:

$$\frac{\mathrm{d}\tilde{\mathbf{x}}(t)}{\mathrm{d}t} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}\tilde{\mathbf{u}}(t) + \tilde{\boldsymbol{\kappa}} + (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathrm{C}})^{-1}\tilde{\mathbf{G}}\frac{\mathrm{d}\tilde{\mathbf{W}}(t)}{\mathrm{d}t}. \tag{3}$$

Equation (3) is solved over intervals $[t - \Delta t, t)$ of length Δt by:

$$\begin{aligned}\tilde{\mathbf{x}}(t) = \tilde{\mathbf{A}}_{\Delta t}\tilde{\mathbf{x}}(t-\Delta t) + \int_{t-\Delta t}^{t}\tilde{\mathbf{A}}_{t-s}\mathrm{d}s\,\tilde{\mathbf{B}}\tilde{\mathbf{u}}(t-\Delta t) + \int_{t-\Delta t}^{t}\tilde{\mathbf{A}}_{t-s}\mathrm{d}s\,\tilde{\boldsymbol{\kappa}} \\ + (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathrm{C}})^{-1}\int_{t-\Delta t}^{t}\tilde{\mathbf{A}}_{t-s}\tilde{\mathbf{G}}\mathrm{d}\tilde{\mathbf{W}}(s),\end{aligned} \tag{4}$$

where $\tilde{\mathbf{A}}_{\Delta t} = \mathbf{I}_N \otimes \mathbf{A}_{\Delta t}$, $\mathbf{A}_{\Delta t} = \mathrm{e}^{\mathbf{A}\Delta t}$, $\tilde{\mathbf{A}}_{t-s} = \mathbf{I}_N \otimes \mathbf{A}_{t-s}$, $\mathbf{A}_{t-s} = \mathrm{e}^{\mathbf{A}(t-s)}$, $\tilde{\mathbf{A}}_{t-s} = \mathbf{I}_N \otimes \mathbf{A}_{t-s}$, and where, for convenience sake, it is assumed that the input $\tilde{\mathbf{u}}(t)$ can be approximated by constants over the relevant intervals $[t - \Delta t, t)$ (for time-varying inputs, see Oud and Jansen, 2000). Observe the important role of the matrix exponential $\mathbf{A}_{\Delta t} = \mathrm{e}^{\mathbf{A}[t-(t-\Delta t)]} = \mathrm{e}^{\mathbf{A}\Delta t}$, as well as the matrix exponential $\mathbf{A}_{t-s} = \mathrm{e}^{\mathbf{A}(t-s)}$, which appears three times inside the integrals. Particularly, $\mathrm{e}^{\mathbf{A}\Delta t}$ gives the effect of $\mathbf{x}(t-\Delta t)$ over the whole interval Δt, while $\mathrm{e}^{\mathbf{A}(t-s)}$ inside the integrals accounts for the fact that input, subject, and noise effects enter continuously over the interval. These effects (from each time point s to t) must be ”summed” (via the integrals) to obtain the total effect. For an explicit expression of the integral $\int_{t-\Delta t}^{t}\mathbf{A}_{t-s}\mathrm{d}s$ we refer to Oud and Jansen (2000).

We write Equation (4) in compact form as follows:

$$\begin{aligned}&\tilde{\mathbf{x}}(t) = \tilde{\mathbf{A}}_{\Delta t}\tilde{\mathbf{x}}(t-\Delta t) + \tilde{\mathbf{B}}_{\Delta t}\tilde{\mathbf{u}}(t-\Delta t) + \tilde{\mathbf{H}}_{\Delta t}\tilde{\boldsymbol{\kappa}} + (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathrm{C}})^{-1}\tilde{\mathbf{w}}(t-\Delta t), \\ &\text{where } \tilde{\mathbf{B}}_{\Delta t} = \int_{t-\Delta t}^{t}\tilde{\mathbf{A}}_{t-s}\mathrm{d}s\,\tilde{\mathbf{B}}, \\ &\qquad\quad \tilde{\mathbf{H}}_{\Delta t} = \int_{t-\Delta t}^{t}\tilde{\mathbf{A}}_{t-s}\mathrm{d}s, \\ &\text{and } \tilde{\mathbf{w}}(t-\Delta t) = \int_{t-\Delta t}^{t}\tilde{\mathbf{A}}_{t-s}\tilde{\mathbf{G}}\mathrm{d}\tilde{\mathbf{W}}(s).\end{aligned} \tag{5}$$

For an explicit expression of the covariance matrix of $\tilde{\mathbf{w}}(t-\Delta t)$ we again refer to Oud and Jansen (2000).

We now turn to the estimation of the continuous-time parameters on the basis of discrete-time observation time points $t_i \in \{t_1, ..., t_T\}$. For this purpose, we specify, on the basis of Equation (5), the so-called exact discrete model (EDM) as follows (Oud and Jansen 2000):

$$\tilde{\mathbf{x}}_{t_i} = \tilde{\mathbf{A}}_{\Delta t_i}\tilde{\mathbf{x}}_{t_i-\Delta t_i} + \tilde{\mathbf{B}}_{\Delta t_i}\tilde{\mathbf{u}}_{t_i-\Delta t_i} + \tilde{\mathbf{H}}_{\Delta t_i}\tilde{\boldsymbol{\kappa}} + (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathrm{C}})^{-1}\tilde{\mathbf{w}}_{t_i-\Delta t_i}. \tag{6}$$

where the parameter matrices are as in (5). Equations (5) and (6) look very similar. However, whereas (5) is a CT model defined for all *t* in CT, DT model (6) is defined for the DT observation points $t_i \in \{t_1, ..., t_T\}$ only. The CT matrices in (5) impose nonlinear restrictions on the DT matrices in (6) in order to make them satisfy the CT model structure.

The EDM is called exact because the nonlinear restrictions it imposes at the DT points by solving the differential equation ensure that the parameters estimated are exactly equal to the parameters of the underlying differential equation model. This is in contrast to several alternative estimation procedures in the literature that approximate the CT parameter matrices in Equation (5) (see, for example, Singer 1990).

The CT parameters can be estimated by means of a nonlinear Structural Equation Model (SEM) procedure (using, for example, the Mx software by Neale et al. 2003). For that purpose, we formulate the SEM model by first defining the state, input, and error vectors $\vec{\mathbf{x}}$, $\vec{\mathbf{u}}$, and $\vec{\mathbf{w}}$, for successive observation time points t_i.

$$\begin{aligned}
\vec{\mathbf{x}} &= \left[\tilde{\mathbf{x}}'_{t_0}, ..., \tilde{\mathbf{x}}'_{t_{T-1}}\right]', \\
\vec{\mathbf{u}} &= \left[\tilde{\mathbf{u}}'_{t_0}, ..., \tilde{\mathbf{u}}'_{t_{T-1}}\right]', \\
\vec{\mathbf{w}} &= \left[\tilde{\mathbf{x}}'_{t_0} - E(\tilde{\mathbf{x}}'_{t_0}), \tilde{\mathbf{w}}'_{t_0}, ..., \tilde{\mathbf{w}}'_{t_{T-2}}\right]'.
\end{aligned} \tag{7}$$

Next, we write Equation (6) in comprehensive form as:

$$\vec{\mathbf{x}} = \vec{B}\vec{\mathbf{x}} + \vec{\Gamma}_{\mathrm{u}}\vec{\mathbf{u}} + \vec{\Gamma}_{\kappa}\tilde{\boldsymbol{\kappa}} + (\mathbf{I} - \vec{\mathbf{R}}\bar{\mathrm{C}})^{-1}\vec{\mathbf{w}}, \tag{8}$$

where we put all *T*-1 $(Nn \times Nn)$ matrices $\tilde{\mathbf{A}}_{\Delta t_i}$ at the appropriate places in the $(TNn \times TNn)$ matrix $\vec{B}$, the *T*-1 $(Nn \times Nr)$ matrices $\tilde{\mathbf{B}}_{\Delta t_i}$ in $(TNn \times TNr)$ matrix $\vec{\Gamma}_{\mathrm{u}}$, and *T*-1 $(Nn \times Nn)$ matrices $\tilde{\mathbf{H}}_{\Delta t_i}$ in the $(TNn \times Nn)$ matrix $\vec{\Gamma}_{\kappa}$. The block-diagonal $(TNn \times TNn)$ matrix $\bar{\mathrm{C}} = \mathbf{I}_T \otimes \tilde{\mathrm{C}}$ has (possibly asymmetric) blocks $\tilde{\mathrm{C}}$ on its diagonal. Because of the assumption $\mathbf{R} = \rho\mathbf{I}_n$, we can write $(\mathbf{I} - \vec{\mathbf{R}}\bar{\mathrm{C}})^{-1}$ as $(\mathbf{I} - \rho\bar{\mathrm{C}})^{-1}$ with *T* blocks $(\mathbf{I} - \rho\tilde{\mathrm{C}})^{-1}$ on its diagonal.[8]

[8] The notation in (8) is a combination of the standard notations in state-space modelling and structural-equation modelling. Although it would be possible to introduce a new notation, we prefer to apply the combined notation so as to facilitate access to the constituting literatures.

Specification of Equation (8) in terms of the spatially lagged variables $\vec{\mathbf{x}}_{\mathbf{C}} = \vec{\mathbf{C}}\vec{\mathbf{x}}$ and $\vec{\mathbf{u}}_{\mathbf{C}} = \vec{\mathrm{C}}_{\mathrm{u}}\vec{\mathbf{u}}$, gives:

$$\vec{\mathbf{x}} = \vec{B}\vec{\mathbf{x}} + \rho(\mathbf{I} - \vec{B})\vec{\mathbf{x}}_{\mathbf{C}} + \vec{\Gamma}_{\mathrm{u}}\vec{\mathbf{u}} - \rho\vec{\Gamma}_{\mathrm{u}}\vec{\mathbf{u}}_{\mathbf{C}} + \vec{\Gamma}_{\kappa}\tilde{\boldsymbol{\kappa}}_{\mathbf{C}} + \vec{\mathbf{w}}. \tag{9}$$

where $\vec{\mathbf{C}}\vec{\boldsymbol{\Gamma}}_{\mathrm{u}}\vec{\mathbf{u}} = \vec{\boldsymbol{\Gamma}}_{\mathrm{u}}\vec{\mathbf{C}}_{\mathrm{u}}\vec{\mathbf{u}}$ for $\vec{\mathbf{C}}_{\mathbf{u}} = \mathbf{I}_T \otimes \mathbf{C} \otimes \mathbf{I}_r$, and where the transformed unobserved heterogeneity $\tilde{\boldsymbol{\kappa}}_{\mathbf{C}}$ is related to the original $\tilde{\boldsymbol{\kappa}}$ in Equation (6) as follows: $\tilde{\boldsymbol{\kappa}}_{\mathbf{C}} = (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathbf{C}})\tilde{\boldsymbol{\kappa}}$. Note that in the derivation of Equation (9) we have made use of the commutative property several times.

Equation (9) can be specified as a latent variables SEM as follows:

$$\begin{aligned}
&\vec{\boldsymbol{\eta}} = \vec{B}\vec{\boldsymbol{\eta}} + \vec{\Gamma}\vec{\xi} + \vec{\zeta} \\
&\text{for} \\
&\vec{\boldsymbol{\eta}} = \vec{\mathbf{x}}, \vec{\xi} = \left[\vec{\mathbf{x}}_{\mathbf{C}}^{'}\ \vec{\mathbf{u}}^{'}\ \vec{\mathbf{u}}_{\mathbf{C}}^{'}\ \tilde{\boldsymbol{\kappa}}_{\mathbf{C}}^{'}\right]^{'}, \vec{\zeta} = \vec{\mathbf{w}}, \vec{\Gamma} = \left[\rho(\mathbf{I} - \vec{B}) \quad \vec{\Gamma}_{\mathrm{u}} \quad -\rho\vec{\Gamma}_{\mathrm{u}} \quad \vec{\Gamma}_{\kappa}\right],
\end{aligned} \tag{10}$$

which is conventionally written in variable form (rather than in terms of units of observation) as follows:

$$\boldsymbol{\eta} = B\boldsymbol{\eta} + \Gamma\boldsymbol{\xi} + \boldsymbol{\zeta}. \tag{11}$$

If a SEM contains latent variables (in addition to structural equations in (11)), measurement equations are required which specify how the latent variables are measured, that is, how the observed variables $\mathbf{y}$ are related to the latent variables $[\boldsymbol{\eta}' \ \ \boldsymbol{\xi}']'$:

$$\mathbf{y} = \Lambda\begin{bmatrix}\boldsymbol{\eta} \\ \boldsymbol{\xi}\end{bmatrix} + \boldsymbol{\varepsilon}. \tag{12}$$

Matrix Λ in (12) contains the loadings, while the measurement errors are given by $\boldsymbol{\varepsilon}$ (with covariance matrix Θ). The measurement model parameter matrices Λ and Θ are estimated simultaneously with the other parameter matrices of model (11). For reasons of interpretation and identification, it is customary to specify unifactorial observed variables only, which means that each observed variable in $\mathbf{y}$ has a loading on only one single latent variable in $[\boldsymbol{\eta}' \ \ \boldsymbol{\xi}']'$.

The vector $\vec{\mathbf{y}}$ and its spatially lagged counterpart $\vec{\mathbf{y}}_{\mathbf{C}}$ are defined analogously to $\vec{\mathbf{x}}$ and $\vec{\mathbf{x}}_{\mathbf{C}}$. Therefore, we assume that their loading matrices $\vec{\mathbf{L}}$ and measurement intercept vectors $\vec{\mathbf{d}}$ are equal. This gives the following measurement model for $\vec{\mathbf{x}}$ and $\vec{\mathbf{x}}_{\mathbf{C}}$.

$$\begin{aligned}
\vec{\mathbf{y}} &= \vec{\mathbf{L}}\vec{\mathbf{x}} + \vec{\mathbf{d}} + \vec{\mathbf{v}}, \\
\vec{\mathbf{y}}_{\mathbf{C}} &= \vec{\mathbf{L}}\vec{\mathbf{x}}_{\mathbf{C}} + \vec{\mathbf{d}} + \vec{\mathbf{v}}_{\mathbf{C}}.
\end{aligned} \tag{13}$$

We impose no equality constraints between the measurement error variances of $\vec{\mathbf{v}}$ and $\vec{\mathbf{v}}_{\mathbf{C}}$, since the measurement errors in $\vec{\mathbf{v}}_{\mathbf{C}}$ are linear combinations of the measurements errors in $\vec{\mathbf{v}}$ and therefore typically have lower variance.

If there is (as in the present case study) only one observed variable in $\vec{\mathbf{y}}$ for a given latent variable in $\vec{\mathbf{x}}$ as well as one observed variable in $\vec{\mathbf{y}}_{\mathbf{C}}$ for a given latent variable in $\vec{\mathbf{x}}_{\mathbf{C}}$, then $\vec{\mathbf{y}}$ and $\vec{\mathbf{x}}$ as well $\vec{\mathbf{y}}_{\mathbf{C}}$ and $\vec{\mathbf{x}}_{\mathbf{C}}$ are equal except for the measurement errors in $\vec{\mathbf{v}}$ and $\vec{\mathbf{v}}_{\mathbf{C}}$.[9] In such a case $\vec{\mathbf{L}} = \mathbf{I}$ and $\vec{\mathbf{d}} = \mathbf{0}$, so that no loadings or measurement intercepts are estimated. This model can typically be turned identified by specifying the measurement error variances for the repeated measurements of the same variable in $\vec{\mathbf{y}}$ or $\vec{\mathbf{y}}_{\mathbf{C}}$ at successive time points to be equal.[10]

Estimation of SEM models basically comes down to minimizing, in some metric, the distance between the theoretical variance-covariance or moment matrix of the observed variables (as determined by the model specifications) and the corresponding sample matrix. There exist various estimators for SEMs including maximum likelihood (ML). Oud and Folmer (2008b) show that in the case of ML estimation the standard SEM likelihood function for a spatial dependency model need to be augmented by the Jacobian correction term $\ln|\mathbf{I} - \rho\bar{\mathrm{C}}|$, where ln denotes the natural logarithm. The size of the Jacobian correction depends on the number of dependent variables. In a conventional spatial error model with only a single dependent variable and weights matrix $\mathbf{C}$, the correction is:

$$\ln|\mathbf{I} - \rho \mathrm{C}|. \tag{14}$$

In a multivariate model with equal spatial dependence parameter for n variables with $(Nn \times Nn)$ matrix $\tilde{\mathrm{C}}$, the Jacobian correction is:

$$\ln|\mathbf{I} - \rho\tilde{\mathrm{C}}| = n \ln|\mathbf{I} - \rho \mathrm{C}|. \tag{15}$$

In a longitudinal analysis with T observations and $\bar{\mathrm{C}}|$ of order $(TNn \times TNn)$, the correction is:

$$\ln|\mathbf{I} - \rho\bar{\mathrm{C}}| = Tn \ln|\mathbf{I} - \rho \mathrm{C}|. \tag{16}$$

Finally, if each of the n latent variables is measured by m indicators, a $(TNnm \times TNnm)$ matrix $\tilde{\bar{\mathrm{C}}}$ applies and the correction becomes (Oud and Folmer 2008b):

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[9] In the full model (13) (including both matrices $\vec{\mathbf{L}}$ and measurement intercept vectors $\vec{\mathbf{d}}$) more than one indicator per latent variable is specified.

[10] Of course, a measurement model would not be needed if there were only observed variables in the model in which case we have $\vec{\mathbf{y}} = \vec{\mathbf{x}}$ and $\vec{\mathbf{y}}_{\mathbf{C}} = \vec{\mathbf{x}}_{\mathbf{C}}$.

$$\ln|\mathbf{I}-\rho\tilde{\tilde{C}}| = Tnm \ln|\mathbf{I}-\rho C|. \tag{17}$$

We observe that nonlinear SEM programs like Mx (Neale et al. 1999) can be applied to estimate the Equations (11) and (12), including all linear and nonlinear restrictions implied by both the CT and the spatial dependence specification.

5. Empirical Results

The German regional labor market model outlined in section 2 can be summarized as follows. The model is made up of:

- The state variables unemployment change and population change. For each state variable, an autoregressive effect is expected. Moreover, feedback relationships are hypothesized between unemployment change and population change: a positive effect of population development on unemployment change, and a negative effect for the opposite direction from unemployment change to population change.
- The fixed input variables wage change and change of the manufacturing sector. We assume a positive wage change effect and a negative manufacturing change effect on unemployment change. For population change we assume positive effects of both input variables.
- A first-order spatial lag for each of the state variables. We assume the spatial dependence parameter for the state variables to be equal such that $\tilde{\mathbf{R}}$ in Equation (2) contains only a single spatial parameter ρ.

The German regional unemployment model as a CT is presented in Equation **¡Error! No se encuentra el origen de la referencia.**, where $ud(t)$ is unemployment change, $pd(t)$ population change, $wd(t)$ wage change, and $md(t)$ change of the manufacturing sector structure:

$$\begin{aligned}
\frac{\mathrm{d}ud(t)}{\mathrm{d}t} &= a_{11}ud(t) + a_{12}pd(t) + b_{11}wd(t) + b_{12}md(t) + b_1 + \kappa_1 + \frac{\mathrm{d}z_1(t)}{\mathrm{d}t}, \\
\frac{\mathrm{d}pd(t)}{\mathrm{d}t} &= a_{21}ud(t) + a_{22}pd(t) + b_{21}wd(t) + b_{22}md(t) + b_2 + \kappa_2 + \frac{\mathrm{d}z_2(t)}{\mathrm{d}t}.
\end{aligned} \tag{18}$$

Coefficients a_{11} and a_{22} represent the CT autoregressive effects of *ud* and *pd*, a_{12} and a_{21} the cross-effects of *pd* on *ud* and of *ud* on *pd*, respectively, whereas the effects of the input variables *wd* and *md* are given by b_{11} and b_{12} (on *ud*) and b_{21} and b_{22} (on *pd*). Finally, b_1 and b_2 are the intercepts, and κ_1 and κ_2 are the region-specific random intercepts. Because the random subject effects κ_1 and κ_2 represent deviations from the fixed intercepts b_1 and b_2, $\mathrm{E}(\kappa_1) = \mathrm{E}(\kappa_2) = 0$. The squared deviations show up in the model as the variances φ_{κ_1} and φ_{κ_2}; their covariance is $\varphi_{\kappa_1\kappa_2}$.

It should be observed that CT effects basically are the limits of the corresponding effects in DT for the observation interval going to zero. Due to the nonlinear relationship between CT and DT effects, the parameter values may differ between CT and DT. The infinitesimal 'cross-effects' a_{12} and a_{21} in CT can be interpreted similarly to the corresponding cross-lagged effect $a_{12\Delta t}$ and $a_{21\Delta t}$ in DT, but the values of a_{12} and a_{21} may differ considerably from $a_{12\Delta t}$ and $a_{21\Delta t}$, and even the signs may change when going from the DT coefficients to the CT coefficients time and vice versa. The reason is that the CT analysis accounts also for the autoregressive effects during the observation interval. In fact, the estimated cross-lagged effects are mixtures of the CT cross- and autoregressive effects. A variable with a high autoregressive effect, meaning that there is a strong tendency to sustain its value over time, tends also to retain the influence of other variables over a longer time interval than a variable with a low autoregressive effect. So, even a relatively small CT cross-effect can result in a relatively high cross-lagged effect in DT, if the variable influenced has a high autoregressive effect. But the converse can also be true: a relatively strong CT cross-effect having only small impact over a DT interval because of a rather low autoregressive effect in the dependent variable.

The CT autoregressive coefficients (direct feedback-effects) a_{11} and a_{22} additionally are to be interpreted differently, however, from the corresponding DT autoregressive coefficients $a_{12\Delta t}$ and $a_{21\Delta t}$. Suppose the cross-effects to be zero, then a CT autoregressive effect of 0 in $\mathbf{A}$ (no change) corresponds to a DT autoregression of 1 in $\mathbf{A}_{\Delta t}$, and a CT autoregressive effect of $-\infty$ in $\mathbf{A}$ (maximum negative feedback) to a DT autoregressive effect of 0 in $\mathbf{A}_{\Delta t}$. So, CT autoregressive effects in the range $(-\infty, 0)$ are transformed to DT autoregressive effects in the range $(0,1)$.

For the error components in Equation **¡Error! No se encuentra el origen de la referencia.**, Equation (2) applies with spatial dependence parameter ρ and the parameters g_{11}, g_{22} and g_{21} in matrix $\mathbf{G} = \begin{pmatrix} g_{11} & 0 \\ g_{21} & g_{22} \end{pmatrix}$. This matrix transforms the two independent standard Wiener processes

$$\frac{\mathrm{d}\mathbf{W}(t)}{\mathrm{d}t} = \begin{pmatrix} \dfrac{\mathrm{dW}_1(t)}{\mathrm{d}t} \\ \dfrac{\mathrm{dW}_2(t)}{\mathrm{d}t} \end{pmatrix}$$

into the correlated general Wiener processes for *ud* and *pd*.

Finally, we discuss the parameters relating to the initial time point when the process starts. First of all, there are the initial state means $\mu_{x_{1t_0}}$ and $\mu_{x_{2t_0}}$, their variances $\varphi_{x_{1t_0}}$, $\varphi_{x_{2t_0}}$, and covariance $\varphi_{x_{1t_0}x_{2t_0}}$. Because the initial means may differ in regions with different levels of *wd* and *md* at the initial point, one may regress $ud(t_0)$ and $pd(t_0)$ on $wd(t_0)$ and $md(t_0)$ (which gives the regression coefficients b_{11t_0}, b_{12t_0}, b_{21t_0} and b_{22t_0}) and subtract the regression means $b_{11t_0}\overline{wd(t_0)}+b_{12t_0}\overline{md(t_0)}$ and $b_{21t_0}\overline{wd(t_0)}+b_{22t_0}\overline{md(t_0)}$ to obtain the "pure" initial means. Moreover, conditional on the initial inputs, the initial conditional variances and covariance of the state variables can be calculated. Finally, as the region specific random effects κ_1 and κ_2 are assumed to influence the state variables before as well as after initial time point t_0, the four covariances between the initial state variables and the random region effects $\varphi_{\kappa_1 x_{1t_0}}$, $\varphi_{\kappa_1 x_{2t_0}}$, $\varphi_{\kappa_2 x_{1t_0}}$ and $\varphi_{\kappa_2 x_{2t_0}}$ are estimated, since they cannot, in general, be taken as zero.

We estimate two types of models: one without (I), and one with (II) measurement errors for the state variables, as well as for their spatially lagged counterparts. Compared to Model I, there are four additional parameters in Model II: the measurement error variances θ_{v1} and θ_{v2} for the observed state variables, and measurement error variances θ_{v1_C} and θ_{v2_C} for their spatially lagged counterparts (see Equation (13)).

Both Model I and Model II are estimated by ML. Since T = 4, n = 2 and m = 1 (a single indicator per latent variable), the Jacobian correction term added to the likelihood function in both models is $\ln|\mathbf{I}-\rho\tilde{\tilde{\mathrm{C}}}| = Tnm\ln|\mathbf{I}-\rho\mathrm{C}| = 8\ln|\mathbf{I}-\rho\mathrm{C}|$. We analyse unemployment change in 439 German labour market regions over the period 2000–2003 by means of the CT spatial-dependence modelling approach outlined above. All observed variables in the model (two endogenous (state) variables and two exogenous (input) variables) are defined as changes, that is differences between successive years. For computation reasons we divide the changes by 1,000. For instance, unemployment change in region *r* in 2000 is measured as (1/1000) * (unemployed in region *r* in 2000 − unemployed in region *r* in 1999).

The estimation results are given in Table 1. First of all, we refer to the spatial dependence parameter ρ, which is 0.375 in Model I and 0.378 in Model II. In both models, ρ is highly significant. Since it is rather restrictive, we relaxed the assumption of equal spatial dependence for *ud* and *pd* in the SEM model. This relaxation, however, did not lead to any significant improvement in model fit, as measured by the χ^2-difference test. We conclude that the assumption of the same spatial dependence ρ for both *ud* and *pd* is not contradicted by the data.

Table 1 ML parameter estimates and associated t-values for model I (without measurement errors) and model II (with measurement errors)

Par.	Model I		Model II	
	Coefficient	*t*	*Coefficient*	*t*
Spatial parameter				
ρ	0.375	16.30*	0.378	14.00*
Measurement error variances				
θ_{v1}			0.422	16.88*

Par.	Model I		Model II	
	Coefficient	*t*	*Coefficient*	*t*
$\theta_{\mathrm{v}2}$			0.081	1.09
$\theta_{v1_{\mathrm{C}}}$			0.015	0.82
$\theta_{\mathrm{v}2_{\mathrm{C}}}$			0.044	4.04*
State effects				
a_{11}	–1.664	–11.64*	–0.594	–6.83*
a_{12}	–0.009	–0.17	0.013	0.54
a_{21}	–0.252	–2.00*	–0.498	–4.64*
a_{22}	–1.363	–6.75*	–1.168	–5.01*
Input effects				
b_{11}	0.016	0.39	0.005	0.19
b_{12}	0.047	0.98	0.021	0.78
b_{21}	0.001	0.01	0.017	0.49
b_{22}	0.033	0.89	0.014	0.34
Fixed intercepts				
b_1	0.633	10.21*	0.463	11.87*
b_2	–0.136	–1.97*	–0.101	–1.65
Random intercept variance				
φ_{κ_2}	0.679	2.23*	0.439	1.64
Error parameters				
g_{11}	1.316	23.93*	0.156	1.61
g_{22}	1.196	19.93*	1.000	16.40*
g_{21}	–0.010	–0.18	–0.059	–0.31
Initial (reduced) state means				
$\mu_{x_{1_{t_0}}}$	–0.656	–13.39*	–0.647	–12.68*
$\mu_{x_{2_{t_0}}}$	–0.146	–2.24*	–0.148	–2.24*
Initial (conditional) state variances and covariance				
$\varphi_{x_{1_{t_0}}}$	0.419	14.96*	0.052	1.85
$\varphi_{x_{2_{t_0}}}$	1.270	14.76*	1.202	11.13*
$\varphi_{x_{1_{t_0}} x_{2_{t_0}}}$	–0.014	0.40	0.015	0.42
Initial time point regression coefficients				
b_{11t_0}	0.030	0.86	0.010	0.30
b_{12t_0}	–0.057	–1.63	–0.049	–1.44
b_{21t_0}	0.021	–0.51	–0.018	–0.42
b_{22t_0}	0.029	0.69	0.027	0.63

Par.	Model I		Model II	
	Coefficient	*t*	*Coefficient*	*t*
Covariances between random intercept and initial states				
$\varphi_{\kappa_2 x_{1_{t_0}}}$	–0.085	–2.24*	–0.091	–2.52*
$\varphi_{\kappa_2 x_{2_{t_0}}}$	0.940	5.25*	0.792	4.10*
Fit				
χ^2	770.2		650.7	
df	206		202	
RMSEA	0.081		0.071	

From the significance of the measurement error variance of *ud* (0.422) and of the spatially lagged *pd* (0.044), it follows that Model II is more adequate than Model I. This conclusion is supported by the significant improvement of model fit when the assumption of no measurement errors is dropped, as shown by the difference test ($\chi^2_{\text{dif}} = 770.2 - 650.7 = 119.5$ for *df* = 206 – 202 = 4). Finally, the RMSEA fit measure of Model II (0.071) is smaller than for Model I (0.08), and meets the criterion of a 'reasonable' fit (Jöreskog and Sörbom 1996, p. 124). We conclude that Model II is preferable to Model I. For the remainder of this paper, we only consider Model II.

In order to facilitate interpretation, we present in Table 4 the CT state effect matrices A, as well as the corresponding DT effect matrices $\mathbf{A}_{\Delta t}$ for observation interval $\Delta t = 1$ derived from the CT state effect matrices. (Observe that for other intervals than $\Delta t = 1$ the DT results would be totally different, as shown in Figures 4 and 5). Moreover, in Table 2 we present both the standardized (by the initial standard deviations of *ud* and *pd*) and unstandardized effects in **A** and $\mathbf{A}_{\Delta t}$ (in Table 1, only unstandardized effects are presented).[11]

Comparison of Models I and II in Table *2* illustrates the disattenuation effect due to explicitly accounting for measurement errors. With the exception of the unstandardized effect $ud \rightarrow pd$ the coefficients in Model II are larger than the corresponding coefficients in Model I, indicating that the latter are attenuated by measurement errors.

[11] Standardization prevents the effects from being dependent on the measurement scale unit of the variables involved.

Table 2 CT state effect matrices **A** and corresponding EDM autoregression matrices $\mathbf{A}_{\Delta t} = e^{\mathbf{A}\Delta t}$ for Model I (without measurement errors) and Model II (with measurement errors), over the one-year observation interval $\Delta t = 1$, in unstandardized and standardized form

Par.	Model I		Model II	
	A	$\mathbf{A}_{\Delta t=1}$	**A**	$\mathbf{A}_{\Delta t=1}$
Unstandardized				
ud	$a_{11} = -1.664^{*}$	$a_{11\Delta t=1} = 0.190$	$a_{11} = -0.594^{*}$	$a_{11\Delta t=1} = 0.550$
pd $\rightarrow$ *ud*	$a_{12} = -0.009$	$a_{12\Delta t=1} = -0.002$	$a_{12} = 0.013$	$a_{12\Delta t=1} = 0.006$
ud $\rightarrow$ *pd*	$a_{21} = -0.252^{*}$	$a_{21\Delta t=1} = -0.056$	$a_{21} = -0.498^{*}$	$a_{21\Delta t=1} = -0.209$
pd	$a_{22} = -1.363^{*}$	$a_{22\Delta t=1} = 0.256$	$a_{22} = -1.168^{*}$	$a_{22\Delta t=1} = 0.310$
Standardized				
ud	$a_{11} = -1.664^{*}$	$a_{11\Delta t=1} = 0.190$	$a_{11} = -0.594^{*}$	$a_{11\Delta t=1} = 0.550$
pd $\rightarrow$ *ud*	$a_{12} = -0.016$	$a_{12\Delta t=1} = -0.003$	$a_{12} = 0.063$	$a_{12\Delta t=1} = 0.027$
ud $\rightarrow$ *pd*	$a_{21} = -0.145^{*}$	$a_{21\Delta t=1} = -0.032$	$a_{21} = -0.104^{*}$	$a_{21\Delta t=1} = -0.044$
pd	$a_{22} = -1.363^{*}$	$a_{22\Delta t=1} = 0.256$	$a_{22} = -1.168^{*}$	$a_{22\Delta t=1} = 0.310$

From Table 1 and Table *2*, it follows that, in accordance with our expectations (see Section 3), both *ud* and *pd* show substantial autoregressive effects (–0.594 and –1.168 in **A**; 0.550 and 0.310 in $\mathbf{A}_{\Delta t}$). In both models the coefficients a_{11} and a_{22} are negative and significant, implying that the models are stable. Moreover, as hypothesized, *ud* has a negative effect on *pd* (standardized value of –0.104), which is highly significant. The cross-effect of *pd* on *ud*, however, though positive, is not significant (standardized value of 0.063). This result is in line with the hypothesized dual population effect on unemployment (see Section 2), in that supply factors (such as a potentially larger workforce) are counterbalanced by demand factors (such as an increased demand for goods).

Table 1 shows that neither of the two input variables (wage change and change of the structure of the manufacturing sector) has a significant effect on either of the two state variables. The insignificant effect of wage change on unemployment change is surprising. It could be due to the rigidity of wage setting in German (particularly wages are set nationally rather than regionally or at firm level), such that regional wages insufficiently reflect the regional unemployment structure. Its insignificant effect on population development could be due to the many constraints on labour mobility (such as, for example, inefficiencies in the housing market). Longer time lags (and hence longer time series) may be needed for significant effects to show up.

The fixed intercept of *ud* is positive, substantial (0.463) and highly significant, whereas the fixed intercept of *pd* is negative (–0.101) and not significant at the 5 per cent level. The random intercept variance and covariances for κ_1 $(\varphi_{\kappa_1}, \varphi_{\kappa_1 x_{1_{t_0}}}, \varphi_{\kappa_1 x_{2_{t_0}}})$ were not significant and have been left out of the final analysis because they affect the estimations of all other model parameters. However, because the covariances of κ_2 with the initial state variables, $\varphi_{\kappa_2 x_{1t_0}}$ and $\varphi_{\kappa_2 x_{2t_0}}$, are significant, the assumption of the presence of a random intercept for *pd* (variance $\varphi_{\kappa_2} = 0.439$) is supported by the data and therefore included in the model. From these results it follows that regions resemble each other much more with regard to unemployment change than population change. Particularly, for the latter random intercepts are needed to give each region its own expected development curve. For unemployment change on the other hand, the data support only *one* single expected curve towards which each region in Germany regresses.

With regard to the error variances of the structural equations, we find that g_{11} and g_{21} are insignificant. However, we do not impose restrictions of the type $g_{11} = 0$ and $g_{21} = 0$, since it is unrealistic to assume that the model explains all the variance in *ud* and all the covariance between *ud* and *pd*.

The initial state variance of *ud* (0.052 in Model II) is not significant. This outcome, together with the insignificance of its random intercept variance, means that regression for *ud* is not only towards the same expected curve for all regions, but also that the variance of the regions around this common expected curve is quite small. The initial state variance of *pd*, however, is significant, meaning that from the start in 2000 the regions show clear differences in population change.

The initial means of the state variables over the 439 German labour market regions show that both unemployment and population decrease at the beginning of 2000 (–0.647[12] and -0.148, respectively). The initial mean unemployment change is much larger (in absolute value) and has a higher *t*-value than the initial mean population change (t = –12.68 versus t = –2.24). It should be noted that these means have been reduced by subtracting the (insignificant) regression means $b_{11t_0}\overline{wd(t_0)} + b_{12t_0}\overline{md(t_0)}$ and $b_{21t_0}\overline{wd(t_0)} + b_{22t_0}\overline{md(t_0)}$ from the initial means (insignificant) regression effects. The uncorrected initial means are even larger in absolute value (–0.975 for *ud* and –0.288 for *pd*).

4.1 Autoregression Functions, Cross-Lagged Effect Functions, and Means Trajectories

The estimates of Model II will now be used to depict the autoregression functions of *ud* and *pd* (Figure 4), the standardized cross-lagged effect functions of $ud \rightarrow pd$ and $pd \rightarrow ud$ (Figure 5), and the means trajectories of *ud* and *pd* (Figure 6) in CT. Figure 4 shows the decay as indicated by the autoregressive effects of *ud* and *pd*. The decay is slower for *pd* than for *ud*. For *pd*, the decay is approximately 70 per cent after two years. For *ud*, the decay is approximately 90 per cent over the same period.

Because the model is asymptotically stable, both the autoregression functions and the cross-lagged effect functions go to zero. The standardized cross-lagged effect functions in Figure 5 show the effects in terms of standard deviation units of the dependent variable from a standard deviation unit increase in the explanatory variable. The cross-lagged effect functions start from zero, then reach a maximum ($pd \rightarrow ud$), and a minimum ($ud \rightarrow pd$), respectively, and finally die out towards zero. In both directions ($ud \rightarrow pd$ and $pd \rightarrow ud$), the effects are very small and in both cases the extreme values are reached

[12] Meaning a mean decrease of 647 unemployed persons per labor market region over the previous year.

after 1.15 years. Figure 5 shows that a standard deviation increase in unemployment change diminishes population change by 0.044 standard deviation, while a standard deviation increase in population change increases employment change by 0.027 standard deviation (standard deviations over regions). Observe that the effects die out rather slowly: in both cases, after four years, more than a quarter of the maximum impact is left.

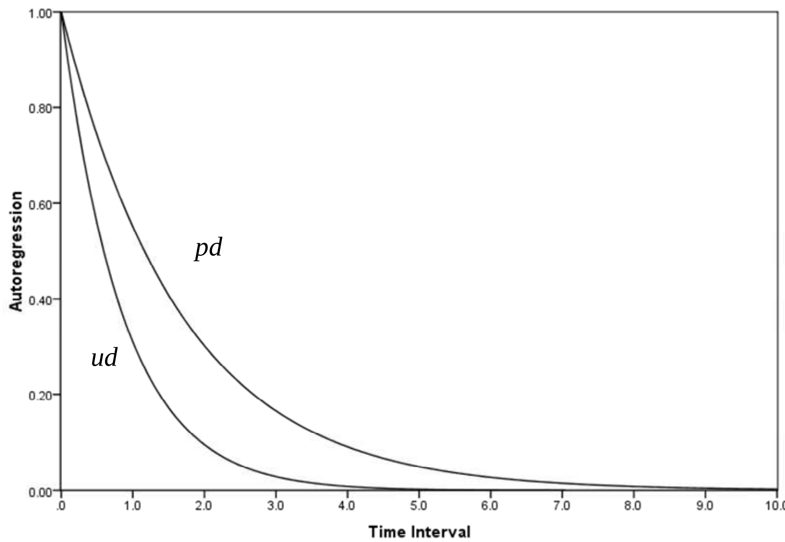

Figure 4 Autoregression functions based on Model II parameter estimates.

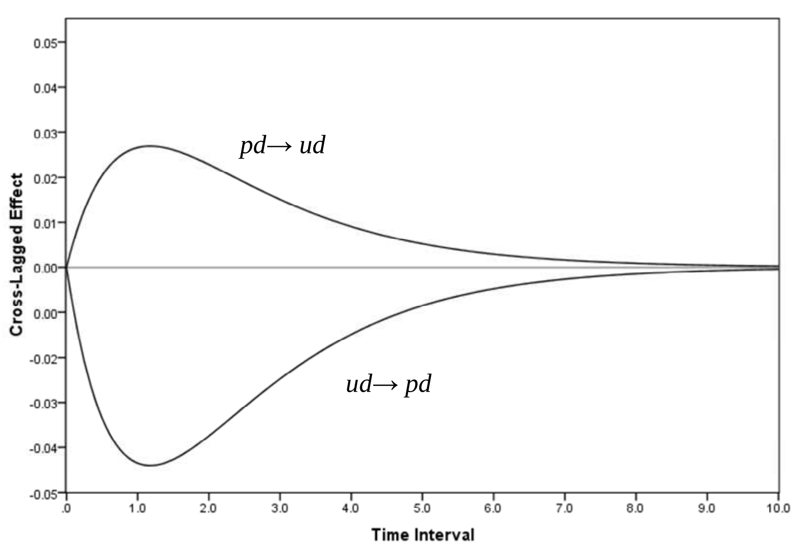

Figure 5 Standardized cross-lagged effect functions based on Model II parameter estimates

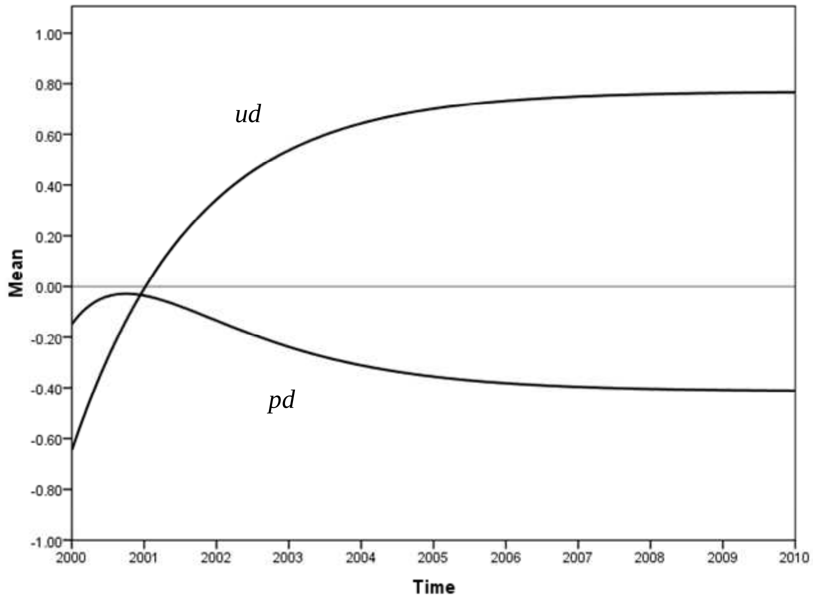

Figure 6 Means trajectories based on Model II parameter estimates

Figure 6 depicts the autonomous developments of the means of *ud* and *pd* , independent from input effects. They are given by (see Oud and Jansen 2000):

$$E[\mathbf{x}(t)] = \mathrm{e}^{\mathbf{A}(t-t_0)} E[\mathbf{x}(t_0)] + \mathbf{A}^{-1}[\mathrm{e}^{\mathbf{A}(t-t_0)} - \mathbf{I}]\mathbf{b}, \qquad (19)$$

where **b** includes the fixed intercepts. The mean development is driven by two components: the autoregression effect of the initial means $E[\mathbf{x}(t_0)]$ and the integrated effect of the intercepts **b** over the entire time period considered. Model II estimates are used for the specification of **A**, **b**, and $\mathbf{x}(t_0)$ in (19). Figure 6 shows that, over the observation period 2000–2003, shortly after 2001, the mean unemployment decrease turned into an unemployment increase which started levelling off after 2003. The mean population decrease diminished until shortly before 2001, and then the downward trend increased again. Both trajectories tend to a stable equilibrium position. This stable equilibrium position implies for *ud* a mean unemployment increase of 770.1 per region and, for *pd*, a mean population decrease of 414.4.

5. Conclusions

In this paper, continuous-time modelling, as introduced in econometrics in the 1950s by, amongst others, Koopmans (1950) and Phillips (1959), and in sociology by Simon (1952) and Coleman (1968), is used to analyse regional unemployment change in Germany on the basis of a data set for the period 2000–2003. For this purpose, we combine the continuous-time modelling approach developed by Oud and Jansen (2000) with the spatial dependence approach by Oud and Folmer (2008a).

Our results shed light on the determinants of regional unemployment and labour supply as measured by regional population development. We find that both unemployment change and population development change have substantial autoregressive effects. They show persistence over time, which, however, is weaker for unemployment change than for population change. Regarding cross-effects our results show that an increase of regional unemployment change leads to a decrease of local population change: increased unemployment change in a given region leads to increased out-migration. This result is consistent with theoretical expectations. On the other hand, we do not find a significant effect of increased population change (which usually generates pressure on the regional job market) on unemploymen tchange. While this finding is worth further investigation, we may – at this stage – attribute it to demand factors (such as increased economic activity stimulated by increased population change), which counterbalance the labour supply effect. Wage change and changes of specialization in manufacturing do not significantly impact on unemployment change nor population change. A possible explanation for this finding is that wage setting in Germany takes place at the national level and that there are only minor regional differences in wage changes in Germany. The non-significance of specialization in manufacturing may be explained by the fact that changes in economic structure rarely happen in the short run.

Secondly, we find that regions resemble each other much more with regard to unemployment change than with regard to population change . Particularly, regression of unemployment change is not only towards the same expected curve for all regions, but also the variance of the regions around the common expected curve is quite small. This result confirms the uniform change of regional unemployment in Germany.

Thirdly, we find that, for both unemployment change and population change, regional shocks are absorbed rather fast (50 per cent is absorbed within 14 months), and have generally a short lifespan. They are slightly longer for population change than for unemploymen tchange , most likely because of the many constraints to mobility, such as housing market imperfections. The reciprocal cross-lagged effects between unemployment and population change, though definitely small, are long-lasting. A peak is reached shortly after one year, and is reduced to 25 per cent after four years.

From a policy-making viewpoint, our findings suggest that wage change does not have a significant effect on regional unemployment change , which could be due to the fact that wages are set nationally on a sectoral basis in Germany. Therefore, locally set wages, which reflect regional unemployment change, might be considered as in instrument to reduce unemployment.

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