Price Competition in the Market for Lemons

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Abstract

We consider a model of price competition among multiple sellers with asymmetric information. Information asymmetry is two-sided; each seller has perfect information of his own good, and the buyer has a private signal of the quality of the goods. Each seller chooses a price and makes a take-it-or-leave-it offer to the buyer. A seller with a high-quality good has an incentive to set a high price to signal the quality, while the competition among the sellers gives an incentive to lower the price. We show that in pure strategy, separating is impossible. Pooling is possible as long as there are sufficiently many high-quality goods in the market so that the total adverse selection does not occur. Hence, competition removes the capability of the price as a signaling device. We show a necessary and sufficient condition for the existence of pure-strategy pooling equilibria and characterize the set of equilibrium prices as the signal accuracy converges to zero. We also show the existence of a mixed-strategy equilibrium where only low-quality goods can be sold with a positive probability.

1 Introduction

In a market with uncertain quality of product, Akerlof (1970) showed that high quality goods are driven out of the market because of adverse selection. However, we see many examples in which goods with unobservable quality are traded in a market with different prices and qualities. One way to explain this phenomenon is that the price itself can function as a signaling device. If the expected average quality of goods increases sufficiently as the price increases, then it induces an increasing demand function, and there may be multiple equilibria among which high-quality goods are traded with positive probability. This intuition can be formalized with game-theoretical models based on the analysis of Perfect Bayesian Equilibria and several concepts of equilibrium refinement which gives restrictions on the off-the-equilibrium beliefs. In most of these models, the unique equilibrium is separating. This implies that the high-quality seller has an incentive to set a high price in order to signal the quality, and the buyers infer correctly that the high-price good comes from the high-quality seller. However, most of the models in the literature consider a monopolistic seller. When there are multiple sellers under price competition, they have an incentive to lower the price to attract consumers. With this downward-pricing motivation, the validity of signaling

by price is no longer obvious. In this paper, we would like to understand how the signaling role of the price is affected by price competition.

We consider a model of price competition among multiple sellers with asymmetric information. Each seller has an indivisible good, and the quality of the good is private information. Each seller sets a price simultaneously and makes a take-it-or-leave-it offer to the buyer. A representative buyer observes the prices. At the same time, the buyer receives an imperfect signal of each good which is correlated with the quality, but independent across the goods. To use the expression by Voorneveld and Weilbull (2007), the buyer receives "a scent of lemon" if the good is actually a lemon. Therefore, the information asymmetry is two-sided; each seller has perfect information of his own good, and the representative buyer has a private signal concerning the quality of the goods. The signals of the buyer are not observed by the sellers, meaning that the sellers cannot tell exactly what kind of impression the buyer obtains for each good. All that the sellers know is the conditional distribution of the signal given the quality, that is, the sellers can only guess what impression is more likely to be obtained by the buyer.

We show that, to the contrary of the monopolistic models, separating is impossible in pure strategy. Pooling is possible if there are sufficiently many high-quality goods in the market. Hence, under the downward pressure on the price caused by competition, the power of the price as a signaling device of the quality is lost. We show a necessary and sufficient condition for the existence of pure-strategy pooling equilibria and characterize the set of equilibrium prices, as the signal precision converges to zero. In contrast with the standard intuition from Bertrand price competition, we found that the equilibrium price does not necessarily fall down immediately to the competitive level as soon as there are two sellers. Instead, the set of pooling equilibrium prices is an interval which shrinks gradually as the number of sellers increases.

We also show the existence of a mixed-strategy equilibrium where only the low-quality goods can be sold with a positive probability. It is straightforward to show that there is at most one price which can be chosen by both types with positive probability. However, since the interpretation of mixed strategies needs some clarification, we must be careful about the implication for the validity of the price as a signal of quality, when we consider the mixed-strategy equilibrium.

Literature

Akerlof (1970) shows that adverse selection drives out high-quality goods from the market under asymmetric information. If there is no signaling device, such as advertisement in Milgrom and Roberts (1986) or warranty in Spence (1977), it is not obvious if high price itself can be a signal of high quality. Wilson (1979, 1980) shows that if the expected quality is an increasing function of the price, there may be multiple equilibria among which high-quality goods can be sold with positive probability. When consumers have correct beliefs inferred by pricing strategies, it induces a partially increasing demand function. This intuition is formalized by Bagwell and Riordan (1991), applying the concept of Intuitive Criterion by Cho and Kreps (1987). They show that the high-type seller has an incentive to overprice in order to signal the quality, and in equilibrium the buyer has

a correct inference. Other papers, such as Bagwell (1991), Overgaard (1993), Ellingsen (1997), and Bester and Ritzberger (2001) considered the price as a signaling device, but in all of these models, the price-setting seller is a monopolist.

Adriani and Deidda (2009a) consider price competition with signaling by price in a large market. They show that no pooling equilibrium survives the D1 criterion, because the high-type seller has a stronger incentive to deviate to a high price. According to the D1 criterion, the consumers correctly form the belief that the deviation comes from the high type. Then, they show that in the unique separating equilibrium, strong competition drives the high-type goods out of the market. Both goods are sold with different prices if the competition is weak. Laffont and Maskin (1987) consider duopoly. Cooper and Ross (1982) model free entry. Wolinsky (1983) and Bester (1993) consider search cost. Hartzendorf and Overgaard (2001) model price competition and advertisement signals. Daughety and Reinganum (2006) consider duopoly in a context of safety.

A two-sided information model is introduced by Voorneveld and Weibull (2007). They characterize both pooling and separating equilibria and find some discontinuites in the limit of signal precision. Adriani and Deidda (2009b) also consider a two-sided information model. They show that high quality goods are driven out under the assumption that the trade of the low quality goods is socially inefficient. Two-sided information is potentially connected with the information acquisition models. A seminal paper by Grossman and Stiglitz (1980) shows that price can be a signal of quality when consumers have an access to information by paying a cost of acquisition. Bester and Ritzberger (2001) suggested a model in which buyers have access to a perfect signal by paying a positive cost. They consider the case in which the cost converges to zero and show that the unique equilibrium which survives an extention of intuitive criterion is a partial separation.

2   The Model

There are n sellers; $i \in \{1, \cdots, n\}$. Each seller has an object with two possible qualities (types) $\theta_i \in \{L, H\} = \Theta$. Sellers' types are identically and independently distributed with a common prior, $\Pr[\theta_i = L] = \lambda$. A seller's valuation of the object is w_θ with $w_L < w_H$. There is a buyer whose valuation is v_θ with $v_L < v_H$. We assume $v_\theta > w_\theta$, i.e. there is a potential gain from trade regardless of the quality of the object. Let $\bar{v}$ be the expected valuation: $\bar{v} = \lambda v_L + (1 - \lambda) v_H$. Without loss of generality, we normalize $w_L = 0$ and $v_H = 1$.

The buyer does not observe the quality of the objects, but obtains a signal $q_i (\in \mathbb{R})$ which is correlated with θ_i but independent across goods. At time 0, nature chooses the state $(\theta_i)_{i=1}^n$ and signals $(q_i)_{i=1}^n$. At time 1, each seller observes his own quality and simultaneously chooses a price. At time 2, the buyer observes the prices $\mathbf{p} = (p_1, \cdots, p_n)$ and signals $\mathbf{q} = (q_1, \cdots, q_n)$, but not the qualities, and then chooses either to buy from a seller or not to buy. If the buyer buys a type-θ object at price p, then her payoff is $v_\theta - p$. If a seller sells a type-θ object at price p, his payoff is $p - w_\theta$. Otherwise, payoff is normalized as 0.

A pure strategy of seller i is $\left(p_i^L, p_i^H\right)$. A pure strategy of the buyer is $b : (\mathbb{R}_+)^n \times \mathbb{R}^n \to \{0, 1, ..., n\}$, where $b(\mathbf{p}, \mathbf{q}) = i$ means the buyer buys from seller i if $1 \le i \le n$, and $b(\mathbf{p}, \mathbf{q}) = 0$ means the buyer does not buy. A mixed strategy of seller i is represented by a Borel-measurable density function ρ_i where $\rho_i(p|\theta)$ denotes the probability that the seller i with type θ chooses price p. Let $\boldsymbol{\rho} = (\rho_1, \cdots, \rho_n)$ denote a mixed-strategy profile. A mixed strategy of the buyer is represented by $\beta : (\mathbb{R}_+)^n \times \mathbb{R}^n \to \Delta(\{0, 1, \cdots, n\})$.

We now define the perfect Bayesian equilibria. Let $\boldsymbol{\mu} = (\mu_1, ..., \mu_n)$ be the belief of the buyer where $\mu_i : \mathbb{R}_+ \times Q \to [0, 1]$, and $\mu_i(p_i, q_i)$ represents the probability that the buyer assigns for the object from the seller i to be type L, conditional on p_i and q_i.

Definition 1 (Perfect Bayesian Equilibrium) $(\boldsymbol{\rho}, \beta, \boldsymbol{\mu})$ *is a PBE if*

(i) For each i and θ, $\rho_i(\cdot|\theta)$ assigns a positive probability only to the prices which maximize seller i's expected payoff, given β and $\rho_j\ (j \ne i)$.

(ii) Given $\boldsymbol{\mu}$, β assigns a positive probability only to the sellers[1] which maximize the buyer's expected payoff.

(iii) $\boldsymbol{\mu}$ follows the Bayes rule whenever applicable.

Note that this definition does not give any restriction on the off-the-equilibrium beliefs. Several concepts have been proposed to give an appropriate restriction on the off-the-equilibrium beliefs. (We discuss this issue later.)

2.1 Signal precision

We parameterize the signal precision by a variable $\tau \in (0, \infty)$. Suppose that the signal consists of two parts: $q = v_\theta + \varepsilon$ where ε is normally distributed with mean 0 and precision τ.[2] Let $G(\cdot)$ and $g(\cdot)$ be the cdf and the pdf of ε. Similarly, let $F(\cdot|\theta, \tau)$ be the cdf of q, conditional on the type θ and precision τ. The unconditional cdf is denoted as $F(\cdot|\tau)$, hence $F(q|\tau) = \lambda F(q|L, \tau) + (1 - \lambda) F(q|H, \tau)$. Let $f(\cdot|\theta, \tau)$ and $f(\cdot|\tau)$ be the probability density functions accordingly. When $\tau = 0$, the signal is pure noise.

We assume that the signal structure, including the value of τ, is commonly known by the sellers and buyer. The following is a property of the signal distributions that we use later.

Lemma 1 *(MLRP) For $\forall \tau$, $\Pr[H|q]$ is increasing in q.*

Proof. Since $\Pr[H|q] = (1 - \lambda) f(q|H, \tau) / \{(1 - \lambda) f(q|H, \tau) + \lambda f(q|L, \tau)\}$, it suffices to show that $f(q|H, \tau) / f(q|L, \tau)$ is increasing in q. It is straightforward to show that $f(q|H, \tau) / f(q|L, \tau) = \exp(-\tau x/2)$ where $x = (q - v_H)^2 - (q - v_L)^2$. As x is decreasing in q, $\exp(-\tau x/2)$ is increasing in q. ∎

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[1] For convenience, when the buyer does not buy, we may describe it as "the buyer buys from seller 0."

[2] We use normal distribution for simplicity, but the economic implication in this paper does not depend on the specification of normally distributed signals. All we need here is MLRP and the limit properties.

Lemma 2 *(Limit of zero precision) For* $\forall q$, $\lim_{\tau \to 0} f(q|H, \tau) / f(q|L, \tau) = 1$.

Proof. As $\tau \to 0$, $\lim_{\tau \to 0} f(q|H, \tau) / f(q|L, \tau) = \exp(-\tau x/2) = 1$. ∎

3 Equilibrium analysis

In this section, we examine the set of equilibria. Since the sellers are ex-ante identical, we focus on the symmetric strategies. Moreover, at this point, we assume that the buyer believes any deviation comes from a low-type seller. Since this specification is the least favorable for the sellers, the set of equilibria is bigger than any other set of equilibria with some restrictions on off-the-equilibrium beliefs.

3.1 General signal precision

First, we suppose that there is no restriction on the signal precision; $\tau \in (0, \infty)$.

3.1.1 Pure-strategy equilibrium

In this subsection, we consider the case where all sellers use a symmetric pure strategy (p^L, p^H). We first show that no pure-strategy separating equilibrium exists, because of Bertrand-type price competition.

Proposition 1 *There is no symmetric, pure-strategy, separating equilibrium.*

Proof. Assume $p^L \neq p^H$. It is straightforward to show that $w_L \leq p^L \leq v_L$. (i) Suppose $p^L > w_L$. Then, a type-L seller can lower the price slightly and sell the good with probability one, conditional on that all other sellers also have type L. Since the increase in probability of selling the object is discontinuous and the decrease in the amount of profit is continuous, the product of those (and thus, the expected payoff also) increases discontinuously when the price decreases slightly. This induces a profitable deviation, hence there is no equilibrium with $w_L < p^L \leq v_L$. (ii) Suppose $p^L = w_L$. Then, the type-L seller's profit is zero in the equilibrium. First, suppose $p^H \leq v_H$. Then at this price the good is sold with a strictly positive probability and a type-L seller would make a positive profit by imitating this price. Now, suppose $p^H > v_H$. Then, a type-L seller can sell the object and make a positive profit by setting a price $p \in (w_L, v_L)$, since all other sellers have type H with a strictly positive probability. This is a profitable deviation, therefore there is no equilibrium such that $p^L = w_L$. ∎

Our result is contrastive to that of Adriani and Deidda (2009a). In their model, the unique equilibrium satisfying the D1 criterion is separating. The difference of the results stems from the difference of the market structures that we model. They consider a large market. If there are relatively many more buyers than low-type sellers, they can sell the goods with probability one, as the market size goes to infinity. This is a result of the law of large numbers – in weak

competition, the realized number of low-type sellers is smaller than that of buyers. As a result, Bertrand competition does not occur. In our model, we consider a small-sized market. If there are k different low-type sellers offering the best deal for the buyer, then each seller sells the good with probability $1/k$, even if the ex-ante probability of a low-type seller is small. As a result, the Bertrand price competition prevents any separating equilibrium from existing.

Now let us focus on pooling equilibria.

Lemma 3 *Suppose that p^* is a pooling equilibrium price. Then $p^* \in [w_H, v_H]$.*

Proof. Suppose $p^* > v_H$. Then the buyer never buys even if the realized signal is extremely good. A low-type seller would deviate then to a price in (w_L, v_L). Suppose $p^* < w_H$. Then a high-type seller makes a deficit and will deviate to a price higher than v_H. ∎

Facing the equilibrium price, the buyer decides which good to buy (or not to buy at all) conditional on the signals. Let $\varphi(q_i, \tau)$ be the expected valuation conditional on the signal q_i. Then,

$$\varphi(q_i, \tau) := \Pr[L|q_i] v_L + \Pr[H|q_i] v_H = \frac{v_L \lambda f(q_i|L, \tau) + v_H (1-\lambda) f(q_i|H, \tau)}{\lambda f(q_i|L, \tau) + (1-\lambda) f(q_i|H, \tau)}. \tag{1}$$

Conditional on the pooling price p^* and the realized signals $(q_1, \cdots, q_n)$, the buyer buys a good from the seller i, if $\varphi(q_i, \tau) \geq \max_j \{\varphi(q_j, \tau), p^*\}$. Hence, expected payoff of a type-θ seller in the pooling equilibrium is $\pi_\theta^* = (p^* - w_\theta) B_\theta(p^*)$ for $\theta \in \{L, H\}$, where $B_\theta(p^*)$ is the probability that a type-θ seller would sell the good at price p^*:

$$B_\theta(p^*) := \Pr\left[\varphi(q_i, \tau) \geq \max_j \{\varphi(q_j, \tau), p^*\} \middle| \theta_i = \theta\right].$$

If a type-θ seller deviates to a price p', the good is sold if $v_L - p' \geq 0$ and $v_L - p' \geq \varphi(q_j, \tau) - p^*$ for $\forall j \neq i$. An immediate observation is that there is no profitable deviation to a price $p' > v_L$ or $p' > p^*$. When $p' \leq \min\{v_L, p^*\}$, the probability that the good is sold is

$$\Pr\left[v_L - p' \geq \max_j \{\varphi(q_j, \tau)\} - p^*\right].$$

Note that this probability does not depend on θ_i anymore, because the signal q_i is no longer used to infer the quality of good i. Hence, the non-deviation condition is

$$(p^* - w_\theta) B_\theta(p^*) \geq \max_{p' \leq \min\{v_L, p^*\}} (p' - w_\theta) \Pr\left[v_L - p' \geq \max_j \{\varphi(q_j, \tau)\} - p^*\right]. \tag{2}$$

Proposition 2 *p^* is a pooling equilibrium price if and only if equation (2) is satisfied for both $\theta = L$ and H.*

Conjecture 1 *Let $P(\tau)$ be the set of pooling equilibrium prices. Then $\tau_1 > \tau_2$ implies $P(\tau_1) \subset P(\tau_2)$.*

It is not easy to specify analytically the set of prices which satisfy (2), for general $\tau \in (0, 1)$. However, as the signal precision τ converges to zero, we can describe the set of pooling equilibrium prices explicitly.

3.2 In the limit as the signal precision converges to zero

In this subsection, we consider the case where τ converges to zero. As we saw above, there is no pure-strategy separating equilibrium. First, let us focus on the pure-strategy pooling equilibria.

3.2.1 Pure strategy equilibria

Suppose that p^* is a pooling equilibrium price. Regardless the signal realization, the buyer believes that the quality is $\bar{v}$ when a seller posts the equilibrium price p^*. A seller cannot steal all consumers' demand by slightly lowering the price, because by doing so, the buyer believes that the good is low quality. As a result, Bertrand price competition does not occur here. Then a pooling equilibrium price can be strictly higher than the production cost of the low type. However, for the equilibrium to exist, the average valuation of the buyer should be sufficiently high. If the average quality is low, two things might happen. Firstly, the buyer's average valuation may be lower than the high-type seller's production cost. Then the high-type seller cannot make a positive profit, hence he would not pool at this price. Second, when the average quality is low, the belief of the buyer in the equilibrium attributes a high probability for the object to be low quality. Then, the deviation to a lower price is relatively attractive for the sellers because there is only a small space for the loss in the buyer's belief. As a result, for a price to support a pooling equilibrium, the average quality should be sufficiently high. The threshold is decreasing as a function of n, because when n becomes large, the gain in the share by lowering the price becomes large, thus deviation becomes more attractive.

Let us define the following interval for $n \geq 2$ as follows:

$$P_0 := \left[w_H, \min \left\{ \frac{1-\lambda}{1-(1/n)} (1-v_L), \bar{v} \right\} \right]. \tag{3}$$

Proposition 3 *(i) For any $p^* \in intP_0$, there exists $\tau'(>0)$ such that $\forall \tau \in (0, \tau')$, p^* is a pooling equilibrium price with signal precision τ. (ii) For any $p^* \notin P_0$, there exists $\tau'(>0)$ such that $\forall \tau \in (0, \tau')$, p^* is not a pooling equilibrium price with signal precision τ.*

Proof. By Lemma 3, $w_H \leq p^* \leq v_H$. By Lemma 2 and (1), $\lim_{\tau \to 0} \varphi(q_i, \tau) = \bar{v}$.

First, assume $p^* > \bar{v}$. For any signal realization q_i, the probability that the ex-post expected value of the good is higher than p^* converges to zero as τ approaches to zero, that is, $\lim_{\tau \to 0} \Pr[\varphi(q_i, \tau) \geq p^*] = 0$. Then the expected profit π_θ^* converges to zero as well. Then a low-type seller would deviate to a price $p' \in (w_L, v_L)$. Hence, the equilibrium price should satisfy $w_H \leq p^* \leq \bar{v}$. For such a price to exist, $w_H \leq \bar{v}$.

When $p^* \leq \bar{v}$, $\lim_{\tau \to 0} \Pr[\varphi(q_i, \tau) \geq p^*] = 1$. Then, $B_\theta(p^*)$ converges to $1/n$ by symmetry. Therefore, $\pi_\theta^* = (p^* - w_\theta)/n$. For the type-$\theta$ seller to not deviate, we need, by Proposition 2,

$$(p^* - w_\theta) \frac{1}{n} \geq (p' - w_\theta) \Pr[b(p', p^*, \mathbf{q}) = i] \quad \forall p', \ \forall \theta, \tag{4}$$

where $(p', p^*, \mathbf{q})$ means (with a slight abuse of notation) that seller i deviates to price p' while all other sellers choose p^*, i.e. $p_i = p'$ and $p_j = p^*$ for any $j \neq i$. Suppose that the buyer believes that the quality is low for any deviated prices, i.e. $\mu_i(p_i, q_i) = 1$ for any $p_i \neq p^*$.[3] Then $\Pr[b(p, p^*, \mathbf{q}) = i]$ is positive only if the expected payoff of buying from seller i is higher than or equal to buying from seller $j (\neq i)$, that is,

$$v_L - p \geq \overline{v} - p^*. \tag{5}$$

Note that if (5) holds, not buying is not a best response of the buyer, since $p^* \leq \overline{v}$. If the inequality (5) is strict, buying from seller i is strictly better than any other choice. Hence $\Pr[b(p, p^*, \mathbf{q}) = i] = 1$ for $p < p^* - (\overline{v} - v_L)$. If (5) holds with equality, $\Pr[b(p, p^*, \mathbf{q}) = i] = 1/n$. Therefore, the supremum of the right hand side of (4) is attained at $p = p^* - (\overline{v} - v_L)$. Hence (4) is equivalent to:

$$(p^* - w_\theta)\frac{1}{n} \geq p^* - (\overline{v} - v_L) - w_\theta \Leftrightarrow p^* \leq \frac{1-\lambda}{1-(1/n)}(v_H - v_L) + w_\theta \ \forall \theta. \tag{6}$$

Recall that p^* should be in the interval $[w_H, \overline{v}]$. For (6) to be satisfied for both $\theta \in \{L, H\}$, we need (8). The set of possible values of p^* is (3).

Now, suppose $p^* \in \text{int} P_0$. Then, for sufficiently small τ, sellers of both types are using a best response, because (4) is satisfied. The buyer's strategy is also optimal, because $p^* \leq \overline{v}$. The buyer's beliefs satisfy the Bayes Rule in the equilibrium. ∎

Corollary 1 *When the signal precision converges to zero, pure-strategy pooling equilibria exist if and only if*

$$w_H \leq \overline{v} \tag{7}$$

and

$$w_H \leq \frac{1-\lambda}{1-(1/n)}(1 - v_L). \tag{8}$$

Proof. For the interval P_0 to be non-empty, it is necessary and sufficient to have (7) and (8). ∎

The interval of pooling equilibrium prices (weakly) shrinks as the number of sellers increases. As n goes to infinity, whether the limit of the interval is empty or not depends on the valuation parameters.

Proposition 4 *(i) If $w_H \leq (1-\lambda)(1-v_L)$, then the limit set of pooling equilibrium prices, P_0, is non empty for all n. P_0 shrinks as n increases, and converges to $[w_H, (1-\lambda)(1-v_L)]$.*

(ii) If $(1-\lambda)(1-v_L) < w_H \leq \overline{v}$, then there exists an integer n_0 such that P_0 is non-empty if and only if $n \leq n_0$.

(iii) If $w_H > \overline{v}$, then there is no pure-strategy pooling equilibrium for any n.

Proof. (i) If $w_H \leq (1-\lambda)(1-v_L)$, then $w_H \leq (1-\lambda)(1-v_L) \leq 1-\lambda \leq 1-\lambda+\lambda v_L = \overline{v}$. The right-hand side of (8) is decreasing in n and converges to $(1-\lambda)(1-v_L)$ as n goes to infinity. (ii) $(1-\lambda)(1-v_L) < w_H$ implies that the interval (3) is empty when n is large. (iii) is obvious. ∎

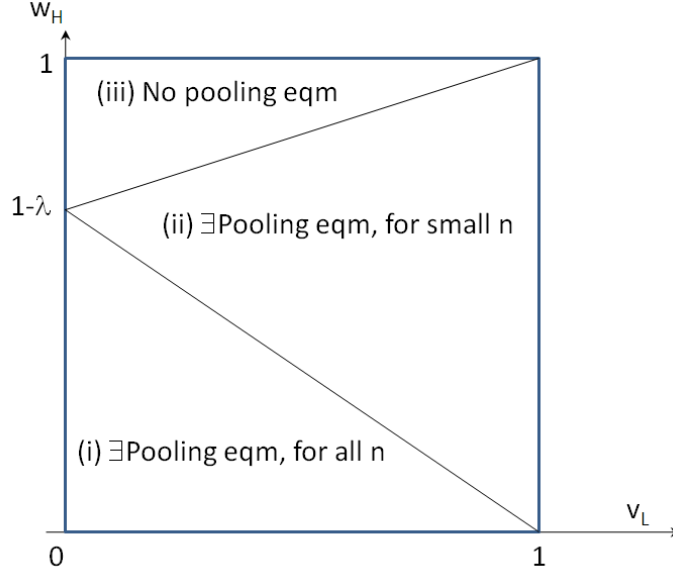[3] Later we need to consider IC or D1 refinement.

Figure 1: Existence of pure-strategy equilibria

This result implies that pooling equilibria exist when $v_L - w_H$ is big. This makes sense, because in the extreme case where $(v_L, w_H) = (1, 0)$, there is no difference in valuations between the two qualities both for sellers and buyers.

Now, suppose n is fixed. Then pooling equilibria exist for small λ. The incentive for type-H sellers to separate is weak when λ is small.

Proposition 5 *Suppose that τ converges to zero. For any (v_L, w_H), pooling equilibria exist if and only if λ is sufficiently small. More precisely, if and only if $\lambda \in [0, \bar{\lambda}]$ where (I) if $v_L < w_H/n$, then $\bar{\lambda} = (1 - w_H) / (1 - v_L)$, and (II) if $w_H/n < v_L$, then $\bar{\lambda} = 1 - w_H (1 - 1/n) / (1 - v_L)$.*

Proof. Remember the necessary and sufficient conditions in Proposition 3. (7) is equivalent to $\lambda \leq (1 - w_H) / (1 - v_L)$. (8) is equivalent to $\lambda \leq 1 - w_H (1 - 1/n) / (1 - v_L)$. It is straightforward to confirm that $v_L < w_H/n \Leftrightarrow (1 - w_H) / (1 - v_L) < 1 - w_H (1 - 1/n) / (1 - v_L)$. ■

3.2.2 Mixed strategy equilibria

Let $\rho_\theta(p)$ be the mixed strategy of a type-θ seller.[4] Let S_θ be the support of the mixed strategy. The buyer forms a belief for each good, and the belief is defined on each information set (p_i, q_i) by $\mu(p_i, q_i | \tau) := \Pr[\theta_i = L | p_i, q_i, \tau]$. Given the belief, the expected valuation of the buyer is defined as: $\varphi(p_i, q_i | \tau) := \mu(p_i, q_i | \tau) v_L + (1 - \mu(p_i, q_i | \tau)) v_H$.

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[4]Since we focus on symmetric strategies, we drop the indicator i from ρ.
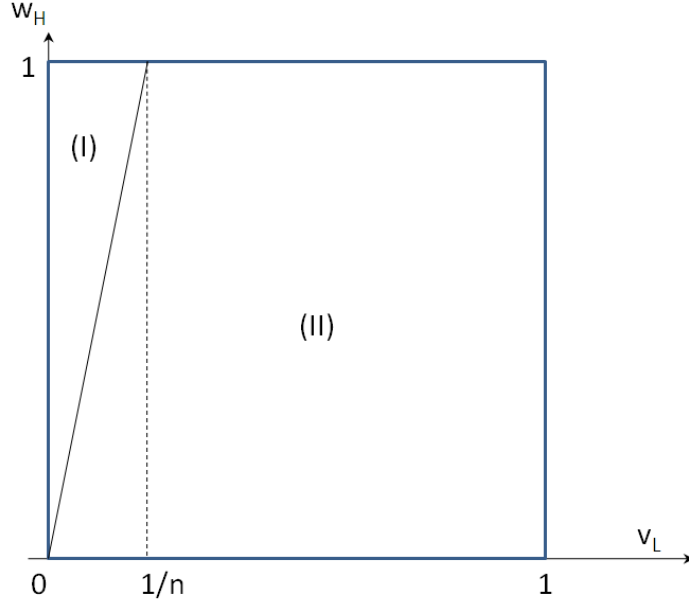
Figure 2: Two cases concerning the range of λ with which the pure-strategy equilibria exist

By consistency, the belief is pinned down uniquely if the price p_i is in the support of the mixed strategy of either type. For $p_i \in S_L \cup S_H$,

$$\mu(p_i, q_i|\tau) = \frac{\lambda \rho_L(p_i) f(q_i|L, \tau)}{\lambda \rho_L(p_i) f(q_i|L, \tau) + (1-\lambda) \rho_H(p_i) f(q_i|H, \tau)}.$$

Now, we consider the case where τ converges to zero. By Lemma 2,

$$\mu_0(p_i) := \lim_{\tau \to 0} \mu(p_i, q_i|\tau) = \frac{\lambda \rho_L(p_i)}{\lambda \rho_L(p_i) + (1-\lambda) \rho_H(p_i)},$$

and

$$\varphi_0(p_i) := \lim_{\tau \to 0} \varphi(p_i, q_i|\tau) = \frac{\lambda \rho_L(p_i) v_L + (1-\lambda) \rho_H(p_i) v_H}{\lambda \rho_L(p_i) + (1-\lambda) \rho_H(p_i)}.$$

Note that these limits do not depend on q_i. Let $B_\theta(p_i, \tau)$ be the probability that the good is sold when the seller i sets a price p_i and the other sellers follow the strategy Φ_θ. Then

$$B_\theta(p_i, \tau) = \Pr_{p_{-i}, \mathbf{q}} \left[\varphi(p_i, q_i|\tau) - p_i \geq \max_j \{\varphi(p_j, q_j|\tau) - p_j, 0\} \middle| \theta_i \right].$$

Remember that $B_\theta(p_i, \tau)$ depends on the type θ only through the fact that distribution of the signal q_i depends on θ. Since the limit φ_0 does not depend on q_i, $B_\theta(p_i)$ does not depend on θ in the limit as τ goes to zero. Therefore,

$$B_0(p_i) := \lim_{\tau \to 0} B_\theta(p_i, \tau) = \Pr_{p_{-i}} \left[\varphi_0(p_i) - p_i \geq \max_j \{\varphi_0(p_j) - p_j, 0\} \right].$$

The expected payoff of a type-θ seller should satisfy: $\pi_\theta := \max_p (p - w_\theta) B_0(p)$.

The following lemmas are useful.

Lemma 4 $B_0(p)$ *is weakly decreasing in* $p \in S_L \cup S_H$.

Proof. Take any $p \in S_L \cup S_H$. If there is $p' > p$ such that $B_0(p') > B_0(p)$, then the seller is strictly better off choosing p' instead of p. Then p cannot be in the support. ∎

Lemma 5 *If* $B_0(p) = B_0(p')$ *for* (p, p') *such that* $p < p'$ *and* $p \in S_L \cup S_H$, *then* $B_0(p) = B_0(p') = 0$.

Proof. If $B_0(p) = B_0(p') > 0$, then $(p - w_\theta) B_0(p) < (p' - w_\theta) B_0(p')$ for both θ. Contradiction with $p \in S_L \cup S_H$. ∎

Lemma 6 *There is no equilibrium in which the expected payoff of the type-L seller is zero.*

Proof. Suppose that $U_L = \max_p (p - w_L) B_0(p) = 0$. Then $B_0(p) = 0$ for $\forall p > w_L$. Since $\varphi_0(p) - p > 0$ for $\forall p \in (w_L, v_L)$, the low-type seller should choose the price w_L with probability one. Since type H never chooses a price lower than w_H, for any $p \in S_H$, $B_0(p) = 0$. ∎

Lemma 7 $\varphi_0(p) - p$ *is weakly decreasing in* p *for* $p \geq \inf S_L$.

Proof. Suppose that $\exists p_1, p_2$ such that $p_1 \in S_L$, $p_2 > p_1$, $\varphi_0(p_1) - p_1 < \varphi_0(p_2) - p_2$. Then the buyer is better off buying at p_2 than at p_1. Hence $B_0(p_2) \geq B_0(p_1)$. Since $p_1 \in S_L$, $\pi_L = (p_1 - w_L) B_0(p_1) > 0$ implies $B_0(p_1) > 0$. Then $\pi_L < (p_2 - w_L) B_0(p_2)$, contradiction to $\pi_L = \max_p (p - w_L) B_0(p)$. ∎

Now we show that pooling is possible at most at one price.

Proposition 6 *There is at most one price* p *in* $S_L \cap S_H$.

Proof. Suppose $p_1, p_2 \in S_L \cap S_H$. Then a type-$\theta$ seller should be indifferent between setting the price at p_1 and p_2. Hence,

$$B_0(p_1)(p_1 - w_\theta) = B_0(p_2)(p_2 - w_\theta) \text{ for } \theta \in \{L, H\}.$$

Hence $B_0(p_1)(w_H - w_L) = B_0(p_2)(w_H - w_L)$, thus $B_0(p_1) = B_0(p_2)$. If $\pi(p_1) > 0$, it implies $p_1 = p_2$. If $\pi(p_1) = 0$, then the expected profit of the seller is zero for both types. Contradiction. ∎

Now, suppose that $\pi_H > 0$. Then for $\forall p \in S_\theta$, $\pi_\theta = (p - w_\theta) B_0(p)$. Hence

$$B_0(p) = \frac{\pi_\theta}{p - w_\theta} \text{ for } p \in S_\theta.$$

Lemma 8 *If* $p^L \in S_L \backslash S_H$, $p^H \in S_H \backslash S_L$ *and* $p^L < p^H$, *then* $p^H - p^L \geq v_H - v_L$.

Proof. Suppose not. Then $\varphi_0(p^L) - p^L = v_L - p^L < v_H - p^H = \varphi_0(p^H) - p^H$. Contradiction with Lemma 7. ∎

3.3 Adverse selection in mixed strategy

Suppose that the sellers use a mixed strategy. Let $\Phi_\theta(p)$ be the cdf of the mixed strategy of type-θ seller.

First, suppose that $v_L < w_H$. Then there is a mixed-strategy equilibrium in which only low-type goods are sold.

Proposition 7 *Suppose $v_L < w_H$. If there is a mixed-strategy equilibrium in which only the low-type goods are sold with positive probability, then its cdf is*

$$\Phi_L(p) = \frac{1}{\lambda}\left(1 - (1-\lambda)\left(\frac{v_L - w_L}{p - w_L}\right)^{\frac{1}{n-1}}\right) \tag{9}$$

for $p \in \left[(1-\lambda)^{n-1}(v_L - w_L) + w_L, v_L\right].$

Proof. Remember that S_θ is the support of the mixed strategy of a type-θ seller. We first show that the closure of S_L is an interval with the highest value v_L. Suppose $p \in S_L$, $p < v_L$ and $\exists p' \in (p, v_L)$ with an open ball which has no intersection with S_L. Then by deviating from p to p', the low-type seller can increase the profit from selling without decreasing the probability of selling. This is a profitable deviation.

Now, suppose $S_H \subset (v_H, \infty)$. Then, given the strategy of other sellers, by setting a price p, a low-type seller can sell the good with probability $(1 - \lambda\Phi(p))^{n-1}$. Hence, the expected profit of setting the price p is $(1 - \lambda\Phi_L(p))^{n-1}(p - w_L)$. On the support S_L, the expected profit should be a constant. Let

$$\pi_L = (1 - \lambda\Phi_L(p))^{n-1}(p - w_L).$$

Since $\Phi_L(v_L) = 1$, $\pi_L = (1-\lambda)^{n-1}(v_L - w_L)$. Let $\hat{p} = \pi_L + w_L$. Then $1 = \pi_L/(\hat{p} - w_L) = (1 - \lambda\Phi_L(\hat{p}))^{n-1}$, which implies $\Phi_L(\hat{p}) = 0$. The mixed strategy is given by (9) for $p \in [\hat{p}, v_L]$. Now let us confirm that the low-type seller has no incentive to deviate. Deviating to a price $p > v_L$ is not profitable, because the good is not sold. Deviating to a price $p < \hat{p} = \pi_L + w_L$ is not profitable, because the profit of selling cannot exceed $\hat{p} - w_L = \pi_L$.

We assumed that the high-type goods are not sold in the equilibrium. If a high-type seller deviates to price p', the buyer believes that the quality is low. To be sold, p' should be smaller than v_L. But then by assumption, $p' < w_H$. This deviation cannot be profitable. ∎

When $n = 2$, $\hat{p} = (1-\lambda)v_L + \lambda w_L$. This seems to be related to a kind of bargaining. As n increases, the distribution of the mixed strategy becomes more skewed to the left.

4 Conclusion

We have examined a model of the market for lemons where multiple sellers are in price competition and the buyers obtain imperfect signals of the quality. It is shown that there no longer exists any

pure-strategy separating equilibrium. Downward pressure on the price-setting sellers caused by price competition removes the power of price as a signaling device of the quality. We characterize the conditions for the existence of pooling equilibria. When the precision of the signals converges to zero, we can explicitly describe the set of pooling equilibrium prices, which shrinks as the number of sellers increases. In contrast with standard Bertrand-type price competition, we found that the equilibrium price does not necessarily drop down to the competitive level as soon as there are two sellers.

There are various possibilities for the extension. We show the existence of mixed-strategy equilibria for certain cases. A complete characterization of the mixed-strategy equilibria seems to be challenging, but it will certainly allow us to have a deeper understanding of the pricing behavior. Also, we see multiplicity of equilibria. By giving restrictions on the off-the-equilibrium beliefs, we expect to be able to refine the set of equilibria. However, an immediate application of the concepts such as Intuitive Criterion or D1 seems to require some prudence when there are multiple sellers. Further research on the refinement of Perfect Bayesian Equilibria with multiple informed agents would be fruitful from a theoretical point of view. Last but not least, we assumed that the quality of the goods is distributed independently. However, in many examples which fit our model well, the quality may be correlated among different sellers. It would be interesting to see how our results could be generalized for the cases of correlated qualities.

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