Vintage Capital and the Declining Energy Intensity in the US Economy*

Raouf Boucekkine[a], Antonia Díaz[b] and Luis A. Puch[c†]

[a]CORE and IRES, Université catholique de Louvain
[b]Universidad Carlos III de Madrid
[c]Universidad Complutense, FEDEA and ICAE

November 2009
Very Preliminary. Do not quote

Abstract

Several authors have documented the low short-run elasticity of energy use to energy price changes. To be in conformity with this evidence existing theory considers capital goods that can be only costly reallocated from plants that use more energy intensive technologies to plants that use more energy saving technologies, together with some degree of irreversibility in investment. However the mechanisms in these models seem to conflict with the declining energy intensity observed for the US economy since the beginning of the 90s. In this paper, the results obtained with those plants models are reinterpreted in terms of a vintage capital model, and thus, obsolescence. The combined impact of embodied and disembodied technical change and its importance relative to that of energy price shocks and other influences is evaluated. In particular, the model suggests a role of increasing energy demand of emerging economies.

Keywords: Plants, Vintage Capital, Energy Price Shocks, News Shocks
JEL Classification: E22, E23

*We thank Omar Licandro, Ramón Marimón and Franck Portier for helpful comments, and Guzmán González-Torres for excellent research assistance. Díaz, and Boucekkine and Puch, thank respectively the Dirección General de Investigación, projects SEJ2004-00968 and SEJ2004-04579, for financial support. Boucekkine also thanks the Belgian Research Programmes PAI P4/01 and ARC 03/08-302. Puch also thanks Fundación FocusAbengoa, and to EUI for its hospitality while part of this research.

†Corresponding Author: Luis A. Puch, FEDEA, Jorge Juan 46, 28001 Madrid, Spain; E-mail: lpuch@fedea.es

1 Introduction

In time series data, energy use does not change much with energy price changes. However, energy use is responsive to international differences in energy prices in cross-section data across countries.

This paper studies the aggregate effects of rising prices of energy. We do this in two different macroeconomic models of energy use at the plant level. In the first model, building upon Atkeson and Kehoe (1999), capital cannot be reallocated from plants that use more energy intensive technologies to plants that use more energy saving technologies. In the second model, building upon Díaz et al. (2004), capital can be freely reallocated across plants but subject to reallocation costs. Both frameworks have proven to be useful to address the observed aggregate response to energy price shocks in the past.

Nevertheless, both theoretical approaches can be shown to be reduced forms of a fundamental vintage capital structure [cf. Benhabib and Rustichini 1991]. The explicit modeling of the underlying vintage structure will give us a deeper understanding of the effects of energy prices on the aggregates and the distribution of firms. The reason is that adopting an energy saving technology imposes different cost under alternative assumptions on the technology. Further, key results of existing frameworks rely on the behavior of unobservables that are no longer present once capital heterogeneity is related with its age. Thus, the response of energy use and energy expenditure to energy price changes can be related to an obsolescence cost.

Moreover, the models with reduced-form frictions do not do well to account for the response of energy aggregates since the mid nineties. Aggregation of a fully specified vintage structure gives a role to investment-specific technology shocks together with energy price shocks to account for recent observations. An alternative interpretation relates to increasing energy demand of emerging economies.

First, we illustrate the connections between the alternative assumptions on the technology. Then we show how to reinterpret both specifications of the models at the plant level in terms of vintage capital, and thus, obsolescence.

Our goal within this framework is to evaluate the response of the two models calibrated to US and EU data to alternative scenarios for energy price shocks in the years to come. These scenarios correspond to a highly persistent stationary process for the shock versus the occurrence of big energy price shocks with a small probability. We do find remarkable differences in the response of the capital to energy ratio between the two models under the alternative scenarios that can be related to obsolescence costs. Different initial conditions for the energy-saving technology between Europe and the US do also play a major role in these different responses.

2 The putty-clay model

This is a representative agent model. Individuals value the consumption of a commodity c (final good) and do not value leisure. The production of this final good requires labor and capital services. The amount of labor each period is normalized to one. This last commodity is produced in plants that use capital and energy to produce the services. The utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t). \tag{2.1}$$

2.1 The production of capital services

There is a constant returns to scale technology that uses capital and energy to produce capital services. Capital types are freely available. An amount $k(v)$ of capital of type $v \in V \subseteq \mathbf{R}_+$ requires a energy intensity of $\frac{k(v)}{v}$. In other words, the capital-energy ratio must be v. Therefore, the amount of services produced is

$$z_t(v) = f(v)\ \min\left\{\frac{k_t(v)}{v}, e_t(v)\right\}.$$

The function f satisfies $f' > 0$, $f'' < 0$, $f(0) = 0$ and $\lim_{x\to\infty} f'(x) = 0$. Notice that if $f(v) = F(v, 1)$, where F is a Cobb-Douglas with CRS the ratio $\frac{f(v)}{v}$ decreases with v. This implies that given the amount of capital, higher type plants produce a lower amount of capital services but use a lower amount of energy. Thus, the trade-off is lower energy requirement implies lower production for a given amount of capital.

The total amount of capital services is

$$z_t = \int_{v\in V} z_t(v)\ dv. \tag{2.2}$$

The total amount of energy used is

$$m_t = \int_{v\in V} e_t(v)\ dv. \tag{2.3}$$

Energy is purchased in a international market at price q_t, which follows a stochastic process (ARMA(1,1), usually). For simplicity, let us assume that domestic taxes on energy are zero.

2.2 The production of the final good and the resources constraint

Output is produced combining capital services and labor in a CRS technology, $G(z_t, l_t)$. Value added is

$$y_t = G(z_t, l_t) - q_t\ m_t. \tag{2.4}$$

All types of capital depreciate at the same rate δ and investment in each type of capital must be non-negative,

$$x_t(v) = k_{t+1}(v) - (1-\delta)\ k_t(v) \geq 0. \tag{2.5}$$

The aggregate resource constraint is given by

$$c_t + \int_{v \in V} x_t(v)\ dv = y_t. \tag{2.6}$$

2.3 The quasi-social planner

The problem is

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\
\text{s.t.} \quad & c_t + \textstyle\int_{v \in V} \left(k_{t+1}(v) - (1-\delta)\ k_t(v)\right)\ dv \leq z_t^{\theta} - q_t\ \int_{v \in V} e_t(v)\ dv, \\
& z_t \leq \textstyle\int_{v \in V} f(v)\ e_t(v)\ dv, \\
& 0 \leq e_t(v) \leq \tfrac{k_t(v)}{v}, \\
& k_{t+1}(v) - (1-\delta)\ k_t(v) \geq 0.
\end{aligned}$$

Proposition 1. *There exists $\widehat{v}_t \geq 0$ such that for all $v < \widehat{v}_t$, $e_t(v) = 0$. For all $v \geq \widehat{v}_t$, $e_t(v) = \frac{k_t(v)}{v}$.*

Proof: See Appendix 1. Only if energy requirement is sufficiently low the capital type v will be used in equilibrium.

Proposition 2. *There exists a unique type $\widetilde{v}_t$ that receives positive investment at period t, for all t.*

Proof: see Appendix 1.

2.4 Investment can be negative

Let us remove constraint $k_{t+1}(v) - (1-\delta)\ k_t(v)$ and replace it by $k_{t+1}(v) \geq 0$.

Proposition 3. *There is a unique type* $\widetilde{v}_t$ *that receives capital at each period.*

Proof: See Appendix 1. Notice then that in this case capital placed at that type is the total capital in the economy, K_t. Let us denote as A_t the type of capital that at each period receives all capital. Then the quasi-social planner problem can be written as

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\ s.t. \quad & c_t + K_{t+1} - (1-\delta)\ K_t \leq \left(\tfrac{f(A_t)}{A_t}\right)^{\theta} K_t^{\theta} - q_t\ \tfrac{K_t}{A_t} \end{aligned}$$

Notice that here the trade-off is: a better technology requires less energy per unit of capital used but has a lower productivity, $\left(\frac{f(A_t)}{A_t}\right)^{\theta}$ decreases with A_t.

3 The putty-putty model with energy saving capital

3.1 The household

The economy is populated by a continuum of infinitely lived households with measure one. Households are ex ante identical and maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t + \alpha \log(1-h_t)\right),$$
$$\beta \in (0,1)\,,\ \alpha > 0.$$

where c_t is consumption and h_t are hours worked at period t. Households are endowed with one unit of time in each period. Labor is indivisible: individuals either work a fixed number of hours h_0 or do not work at all. All households have equal number of shares of all plants.

We assume that agents are allowed to trade employment lotteries to diversify the idiosyncratic risk. Therefore, the economy behaves as if there were a representative individual with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t + \alpha \log(1-h_0)\pi_t\right), \tag{3.1}$$

where π_t denotes the probability of being employed at period t. Thus, at each period employment will be

$$h_t = \pi_t h_0.$$

3.2 Production of capital types and the accumulation of capital

Households accumulate capital of all types and own the technology to produce new types of capital. The production of new types needs of final goods,

$$V_{t+1} - \phi V_t,\ \phi \in (0,1). \tag{3.2}$$

Let us assume that the set of all possible types that are usable at period t is $V_t > 0$. This means that it is possible to use capital of type $v \in (0, V_t]$ at that period. Investment in capital of the different types is

$$x_t(v) = K_{t+1}(v) - (1-\delta)K_t(v). \tag{3.3}$$

Investment cannot be negative,

$$x_t(v) \geq 0, \text{ for all } t \text{ and all } v \in V_t. \tag{3.4}$$

Capital and the use of capital types are rented to plants. The timing goes as follows:

1. The price q_t is realized.
2. Production of the final good takes place in plants. Plants differ in the type of capital that is used. Each plant receives the idiosyncratic shock s_t. The plant decides whether

to operate or not. Production takes place. Households produce new types of capital and invest in capital of each type.

3. The number of plants that operate at period $t+1$ is decided. Each plant decide the type and the amount of capital to be used. The rental prices paid for each are $r_{t+1}^{k}(v)$, r_{t+1}^{v}, respectively.

3.3 Use of energy, capital, and the number of plants

Plants use capital, energy and labor. The technology is

$$y_t(v) = \begin{cases} (z+s_t)\, Bk_t^{\theta}(v)h(v,s_t) & \text{if } l_t(v) \geq \eta \text{ and } e_t \geq \gamma \frac{k_t(v)}{v}, \\ 0, & \text{otherwise.} \end{cases}$$

Capital of type v requires a use of energy equal to $\gamma \frac{k_t(v)}{v}$. $h(s_t, v)$ denotes the number of hours a plant is operated. It is either zero or h_0. The after tax price of energy is p_t and w_t is the hourly wage.

To analyze the decisions of the plants we need to proceed backwards. At period t the plant decides whether to operate or not and the labor market opens. Energy is used. At the previous period the plant has decided the amount of capital. A plant of type v operates if

$$z + s_t)Bk_t^{\theta}(v) \geq w_t \eta + p_t \gamma \frac{k_t(v)}{v}.$$

Thus, there exists a threshold level for the idiosyncratic shock, $\underline{s}_t(v)$ above which a plant of type v is operated. Now, we can define $n_t(v)$ as the fraction of plants of type v that operate,

$$n_t(v) = \int_{s(v)}^{\sigma} \frac{1}{2\sigma} ds = \frac{\sigma - \underline{s}_t(v)}{2\sigma}. \tag{3.5}$$

Profits of a plant of type v before the idiosyncratic shock is revealed,

$$ER_t(v) = (z + \sigma(1 - n_t(v)))\, n_t(v)\, Bk_t^{\theta}(v)\, h_0 - w_t \eta n_t(v)\, h_0 - p_t \gamma \frac{k_t(v)}{v} n_t(v)\, h_0.$$

Let us denote

$$B_t(v) = (z + \sigma(1 - n_t(v)))$$

Plants of each type decide the amount of capital they want. Since they decide before the energy price is known, they decide simultaneously the type to be used and the amount of capital that maximizes.

$$E\Pi_t(v) = E_{t-1}\left[ER_t(v) - r_t^k(v)\,k_t(v) - r_t^v v \diagup p_{t-1}\right]$$

Thus, capital used by the plant satisfies

$$\theta B_t(v) n_t(v) k_t^{\theta-1}(v) h_0 - p_t^e \gamma \frac{1}{v} n_t(v) h_0 = r_t^k(v), \tag{3.6}$$

Plants pay a rental price for the technology. Since at this time all types are regarded as substitutes, the rental price satisfies

$$p_t^e \gamma \frac{k_t(v)}{v^2} h_0 = r_t^v. \tag{3.7}$$

Conjecture 4. *All plants using the same type of capital use the same amount of capital,*

$$k_t(v) = \frac{K_t(v)}{m_t(v)}.$$

Moreover, $m_t(v)$ is such that $E\Pi_t(v)$ are zero in equilibrium,

$$E\Pi_t(v) = 0. \tag{3.8}$$

3.4 Definition of equilibrium

First, we need to define the household's budget constraint,

$$c_t + \int_{V_{t+1}} x_t(v)dv + (V_{t+1} - \phi V_t) \leq w_t \pi_t h_0 + \int_{V_t} r_t^v K_t(v)dv + r_t^v \int_{V_t} v m_t(v) + D_t + T_t, \quad (3.9)$$

where D_t stands for realized profits in period t paid to the household. Note that although expected profits are zero, realized profits are not.

$$D_t = \int_{V_t} \left[B_t(v) n_t(v) k_t^\theta(v) h_0 - w_t \eta n_t(v) h_0 - p_t \gamma \frac{k_t(v)}{v} n_t(v) h_0 - r_t^k(v) k_t(v) - r_t^v v \right] m_t(v) dv.$$

$$T = (q_t - p_t) \int_{V_t} E_t(v) dv, \quad (3.10)$$

$$E_t(v) = \gamma \frac{K_t(v)}{v m_t(v)} n_t(v) m_t(v) h_0 \quad (3.11)$$

Definition 5. *Many things and* $\pi_t \; h_0 = \int_{V_t} h_0 \; \eta \; n_t(v) \; m_t(v) \; dv.$

3.5 The quasi-social planner problem

The utility function can be written as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \alpha \log(1 - h_0) \int_{V_t} \eta n_t(v) m_t(v) dv \right]. \quad (3.12)$$

Production of all plants of type v,

$$Y_t(v) = B_t(v) n_t(v) K_t^\theta(v) m_t^{1-\theta}(v) h_0 \quad (3.13)$$

Employment is

$$\int_{V_t} h_0 \eta n_t(v) m_t(v) dv. \quad (3.14)$$

Value added produced by all plants of type v,

$$\int_{V_t} [Y_t(v) - p_t E_t(v)] \, dv \tag{3.15}$$

The aggregate resource constraint is

$$c_t + (V_{t+1} - \phi V_t) + \int_{V_{t+1}} x_t(v) dv + T \leq \int_{V_t} [Y_t(v) - p_t E_t(v)] \, dv \tag{3.16}$$

The quasi-social planner problem is to maximize (3.12) subject to (3.16), (3.3) (3.13) (3.11) (3.4) and $T = (q_t - p_t) \int_{V_t} E_t(v) dv$.

Lemma 6. *$n_t(v) = n_t$ for all v.*

Proposition 7. *At any period t investment is positive only in the highest efficiency type available, V_{t+1}.*

Proof: wait.

3.6 Investment can be negative

If constraint (3.4) is replaced by

$$k_t(v) \geq 0, \text{ for all } t \text{ and all } v \in V_t.$$

all capital is allocated to the newest type:

Proposition 8. *The solution of this problem is such that there exists a unique type, the one indexed by V_t that receives positive capital at each period t.*

Proof: wait.

Let us call A_t the newest type produced. The problem of the quasi-social planner can be written recursively as

$$V\left(K,A,q,\tau\right)=$$

$$\max_{c,X_k,X_a,m',n\in[0,1]}\left\{\log(c)+\alpha\log(1-h_0)mn\eta+\beta E\left[V\left(K',A',q',\tau\right)/q\right]\right\} \tag{3.17}$$

subject to

$$c+X_k+X_a\leq\left(z+\sigma(1-n)\right)nBK^{\theta}m^{1-\theta}h_0-(1+\tau)q\gamma\tfrac{K}{Am}mnh_0+T,$$

$$K'=X_k+(1-\delta)K,$$

$$A'=X_a+\phi A,$$

$$T=\tau q\gamma\tfrac{K}{A}nh_0,$$

$$q'=\psi(q).$$

The FOC's of the planner problem are:

$$\tfrac{\partial V}{\partial K'}=-u'(c)+$$

$$\beta E_q\left\{u'(c')\cdot\left[\theta(z+\sigma(1-n'))n'B\left(\tfrac{K'}{m'}\right)^{\theta-1}h_0-p\gamma\tfrac{1}{A'}n'h_0+(1-\delta)\right]\right\}=0. \tag{3.18}$$

$$\tfrac{\partial V}{\partial A'}=-u'(c)+$$

$$\beta E_q\left\{u'(c')\cdot\left[p\gamma\tfrac{K'}{(A')^2}n'h_0+(1-\delta)\right]\right\}=0. \tag{3.19}$$

$$\frac{\partial V}{\partial m'} = \beta E_q \left\{ \alpha \log(1-h_0) n' \eta + \right.$$

$$\left. u'(c') \cdot \left[(1-\theta)(z + \sigma(1-n')) n' B \left(\frac{K'}{m'}\right)^{\theta} h_0 \right] \right\} = 0. \tag{3.20}$$

$$\frac{\partial V}{\partial n} = \alpha \log(1-h_0) \eta m +$$

$$u'(c) \cdot \left[(z + \sigma - 2\sigma n) B K^{\theta} m^{1-\theta} h_0 - p\gamma \frac{K}{A} h_0 \right] \} \geq 0. \tag{3.21}$$

4 Augmented putty-putty versus putty-clay

The main difference is the technology. Adopting a energy saving technology has costs and benefits:

1. Benefits: It reduces energy expenditures and and a source of aggregate volatility.
2. Costs:
   (a) putty-clay: using same amount of capital produces less output since $\frac{f(v)}{v}$ decreases with v. (see first section)
   (b) augmented putty-putty: investing in more efficient technology reduces the amount of resources can be devoted to consumption and investment, $A_{t+1} - \phi A_t$.

5 Empirical implications

5.1 The Data

We build upon Atkeson and Kehoe (1999) and we proceed for an update of the database for the US. We apply the same methods to an aggregate of former EU15 countries, except Luxembourg.

Energy aggregates are obtained for nominal energy expenditure, real aggregate expenditure (energy use) and an energy deflator (energy price). Figures 1a and 1b illustrate on these features of the data for the US and EU experiences in recent years. It is remarkable the downward trend observed for European countries after the major oil price shocks.

Is it such a different story? Figure 2 depicts the residuals to an ARMA(1,1) process fitted to the energy price series for the US and the EU15 (REVISE). On the one hand, Figure 2a suggests that recent shocks in the US are less strong than those from the oil shocks era. On the other hand, Figure 2b shows a very different pattern for the EU15 and the US in 1991. The most remarkable difference occurs then just after the (First) Gulf War.

5.2 Calibration

Calibration is relatively standard. There are specific aspects related with the economies calibrated to the US or the EU. Namely

- energy price shocks processes
- share of energy expenditures in value added
- AMT investment on GDP (fix A/Y)
- elasticity of marginal adjustment costs on energy saving capital

5.3 Key feature

Both reduced-form and relying-on-unobservable models respond to large energy price shocks. See Figure 3.

Further, both of the models do not do well in accounting for recent observations. See Figures 4 and 5.

Next it is shown a vintage model provides a rationale for existing models, and quantitatively reacts to small and large shocks.

6 The vintage model

In such a framework, a plant is indexed by its vintage...

6.1 Technology and Preferences

New plants are built each period with one unit of capital. The production function at time t of a plant built at time z (hereafter, a plant of vintage z — or, for convenience, age $\tau = t - z \in T$)[1] is Cobb-Douglas

$$y_t(z) = A(1+\gamma)^t \left[Q(1+\lambda)^z u_t(z)\right]^\alpha h_t(z)^{1-\alpha}$$

with $0 < \alpha < 1$, where $y_t(z)$ is output of a plant of vintage z at time t, $h_t(z)$ is labor employed in such a plant, A is the level of disembodied technical knowledge which grows at rate $\gamma \geq 0$, Q is the level of embodied technical knowledge which grows at rate $\lambda \geq 0$ in vintage z, and $u_t(z)$ is an index of utilization of capital of the plant of vintage z at time t.

The output produced by a plant of vintage z at time t with one unit of capital requires $e_t(z)$ units of energy (a quasi-fixed factor). The energy requirement, $e_t(z)$, depends mainly on its utilization. Also, we assume there is energy saving technological progress at a rate $\gamma^e \geq 0$ in vintage z. Therefore,

$$e_t(z) \geq B(1+\gamma^e)^{-z} u_t(z)^\mu$$

where $\mu > 1$. (Indeed, the energy requirement increases with utilization and decreases for newer vintages at a rate γ^e). Therefore, from one unit of capital at utilization $u_t(\tau)$ we have

$$y_t(s,z) = \begin{cases} y_t(s,z) & \text{if } e_t(z) \geq B(1+\gamma^e)^{-z} u_t(z)^\mu \\ 0 & \text{otherwise.} \end{cases} \tag{6.1}$$

where $\mu > 1$, and s denotes a possible idiosyncratic shock that we assume equal for all plants of vintage z. As in Gilchrist and Williams idiosyncratic uncertainty should serve to obtain a

[1] In general, output of a plant of age τ is described by $y_t(\tau)$, $y_t : T \to [0, \infty)$. We further restrict this assumption for aggregation purposes [cf. Benhabib and Rustichini (*JET,* 91)]

well-defined aggregate production function despite the Leontief of the microeconomic energy choice.

The parametric assumption ... ... and the existence of a fixed cost should be imposed for a finite optimal lifetime of a vintage. Thus, we abstract from scrapping. Further, once T is specified, investment can be described as

[MORE...]

The economy is populated by a continuum of infinitely lived households with measure one. Households are ex ante identical and maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t + \varphi \log(1 - H_t)\right),$$
$$\beta \in (0,1)\,,\; \varphi > 0.$$

where C_t is consumption and H_t are hours worked at period t. Households are endowed with one unit of time in each period. All households have equal number of shares of all plants.

6.2 Capacity utilization and value added

The profits of a plant of vintage z at time t are

$$\pi_t(z) = y_t(z) - w_t h_t(z) - p_t^e e_t(z),$$

net of the cost of one unit of capital (relate with (dis-)embodied investment-specific technical progress). The plant chooses $h_t(z)$ and $u_t(z)$ to maximize profits. This implies, on the one hand, that marginal productivity of labor equals wage in each period. From this condition it follows that labor productivity is equal across vintages and then

$$h_t(z) = \frac{u_t(z)}{u_t(t)}\,(1+\lambda)^{-(t-z)} h_t(t),$$

and that employment in a new plant is

$$h_t(t) = \left(\frac{w_t}{AQ^\alpha (1-\alpha)} (1+g)^{-t} \right)^{-\frac{1}{\alpha}} (1+g)^{-t} u_t(t)$$

where $1+g = [(1+\gamma)(1+\lambda)^\alpha]^{1/(1-\alpha)}$.

On the other hand, optimal utilization is such that the marginal productivity of a vintage equals its marginal cost. Therefore,

$$u_t(z) = u_t(t)(1+\delta)^{-(t-z)} \tag{6.2}$$

where $1+\delta = [(1+\lambda)(1+\gamma^e)]^{1/(1-\alpha)}$, and correspondingly, utilization in a new plant is

$$u_t(t) = \left(\frac{\alpha AQ^\alpha}{\mu \chi p_t^e (1+\gamma^e)^{-t}} \right)^{\frac{1}{\mu-1}} \left(\frac{w_t}{AQ^\alpha (1-\alpha)} (1+g)^{-t} \right)^{-\frac{\theta}{\mu\alpha}}$$

where $\theta \equiv \mu(1-\alpha)/(\mu-1)$. It is apparent then that utilization decreases with the energy price as well as with the wage rate, and increases due to technical progress (embodied, both technological and energy saving, and disembodied). More importantly, utilization (and employment) decreases as plant ages the higher are embodied and energy saving technical progress. This decline in the optimal utilization is linked to the age of capital and thus, to the arrival of new and more productive capital goods that makes old capital goods to become obsolete. This indicator of the utilization of vintages can be interpreted as a use-related depreciation and should play an important role in the pace of energy-price related replacement of capital.

So a change in p_t^e does not affect all vintages in the same proportion since technological and energy saving technical progresses are embodied.

[WRITE PROFITS]

6.3 Aggregation

Aggregation closely follows Solow (1960) vintage model. Aggregate gross production at time t is the sum of production in all plants surviving at time t. If we assume that $T = \infty$

$$Y_t = \sum_{\tau=0}^{\infty} I(t-\tau) A(1+\gamma)^t \left[Q(1+\lambda)^{t-\tau} u_t(t-\tau)\right]^{\alpha} h_t(t-\tau)^{1-\alpha}$$

where $I(t-\tau)$ is the number of plants of vintage z. (Further, to introduce (standard) exponential depreciation it can be assumed that $I(t-\tau) \equiv (1-\omega_\tau)\tilde{I}(t-\tau) = (1+\omega)^{-\tau}\tilde{I}(t-\tau)$

Correspondingly, aggregate employment is the sum of employment at all plants surviving at time t,

$$\begin{aligned} H_t &= \sum_{\tau=0}^{\infty} I(t-\tau) h_t(t-\tau) \\ &= \left(\frac{w_t}{AQ^{\alpha}(1-\alpha)}(1+g)^{-t}\right)^{-\frac{1}{\alpha}} (1+g)^{-t} \sum_{\tau=0}^{\infty} u_t(t-\tau) I(t-\tau)(1+\lambda)^{-\tau} \end{aligned}$$

so that it can be written

$$w_t = (1-\alpha) AQ^{\alpha}(1+g)^{(1-\alpha)t} \tilde{K}_t^{\alpha} H_t^{-\alpha} \tag{6.3}$$

where

$$\tilde{K}_t = \sum_{\tau=0}^{\infty} u_t(t-\tau) I(t-\tau)(1+\lambda)^{-\tau} \tag{6.4}$$

is the replacement value of the stock of capital. Further, plugging (6.2) into (6.4)

$$\tilde{K}_t = u_t(t) \sum_{\tau=0}^{\infty} I(t-\tau)(1+\delta)^{-\tau}(1+\lambda)^{-\tau} \equiv u_t K_t,$$

and therefore

$$K_{t+1} = I_t + (1-d_\tau) K_t \tag{6.5}$$

where now we have renamed $\tilde{I}(t-\tau) \equiv I(t-\tau)$, and d_τ includes both the physical depreciation rate of capital, ω, as well as the use-related depreciation rate, which captures the decline of capital utilization when plants age.

$$(1 - d_\tau) = ((1+\omega)(1+\delta)(1+\lambda))^{-1}$$

Indeed, in terms of $\tilde{K}_t$, u_t shows up with investment and in d_τ. Therefore,

$$\tilde{K}_{t+1} = u_t I_t + (1 - \tilde{d}_\tau)\tilde{K}_t \tag{6.6}$$

where $(1 - \tilde{d}_\tau) = \frac{u_t}{u_{t-1}} ((1+\omega)(1+\delta)(1+\lambda))^{-1}$

This is the replacement value of capital. It is more proper to write capital services. For this purpose equation (6.3) can be alternatively written as

$$w_t = (1-\alpha)A(1+\gamma)^t \left(Q(1+\lambda)^z \sum_{\tau=0}^{\infty} u_t(t) I(t-\tau)(1+\lambda)(1+\gamma^e)^{-\tau} \right)^{\alpha} H_t^{-\alpha}$$

(which resembles the tech at the plant level), so that aggregate gross production can be written

$$\hat{Y}_t = A(1+\gamma)^t \hat{K}_t^{\alpha} H_t^{1-\alpha}$$

provided $\hat{K}_t$ is defined

$$\hat{K}_t = u_t(t) q_t \sum_{\tau=0}^{\infty} I(t-\tau)(1+\lambda)^{-\tau}(1+\delta)^{-\tau} \equiv u_t q_t K_t,$$

where $q_t = Q(1+\lambda)^t$ and therefore, the law of motion for K_t is as (6.5) above, and

$$\hat{K}_{t+1} = u_t q_t I_t + (1 - \tilde{d}_\tau)\hat{K}_t$$

where now $(1 - \tilde{d}_\tau) = \frac{q_t}{q_{t-1}} \frac{u_t}{u_{t-1}} ((1+\omega)(1+\delta)(1+\lambda))^{-1}$, Finally, the total amount of energy used is

$$E_t = \sum_{\tau=0}^{\infty} I(t-\tau) e_t(t-\tau)$$

It turns out from optimal utilization that $p_t^e E_t = \alpha \frac{Y_t}{\mu} + ...$ Furthermore, aggregate energy consumption can be characterized according to law of motion

$$\hat{E}_{t+1} = u_t^{\mu} q_t^e I_t + (1 - \tilde{d}_{\tau}^e)\hat{E}_t$$

where now $(1 - \tilde{d}_{\tau}^e) = ..,$

[MORE...]

Balanced Growth

C_t, Y_t, and I_t grow at g. K_t also. w_t grows at g also, and therefore for U_t to be constant either $\gamma^e = 0$ or p^e exhibits stochastic growth at $(1 + \gamma^e)$.

[ADD DETAILS]

6.4 Discussion

We simulate the aggregate model with capacity utilization and embodied technical progress with two shocks. One shock are the innovations to realized energy price shock process according to

$$\log p_{t+1} = (1-\rho)\log \overline{p} + \rho \log p_t + \phi \epsilon_t + \epsilon_{t+1}, \tag{6.7}$$

The other is an investment-specific technology shock which is identified with the relative price of investment. The relative price corresponds to the ratio of the chain weighted NIPA deflators for durable consumption and private investment over non-durable consumption. Our baseline estimates are based upon the innovations to the realized growth rate of relative price (ν_t) according to

$$\log \nu_{t+1} = (1 - \rho_\nu) \log \overline{\nu} + \rho_\nu \log \nu_t + \eta \varepsilon_t + \varepsilon_{t+1}, \tag{6.8}$$

For newer vintages, for a given size of the energy price shock, aggregate capacity utilization together with an investment-specific technology shock act through the model so as to amplify actual energy price shocks. Figure 6 illustrates on this result.

[MORE...]

7 Related literature

1. Blanchard & Galí (07)

   Macroeconomic Effects of oil price shocks

   - there is a pre/post-84 role of shocks in 6 var VAR: shocks to (broader) PPI index of crude materials (rather than direct oil price shocks) tracks GDP and emp movts before 84, and not after..

2. Gilchrist and Williams (00, 04)

   Utilization channel, idio uncertainty – BC properties

3. Licandro and Wycherley (2007)

   - Adoption model: two tecnologies

References

Atkeson, Andrew and Patrick Kehoe, 1999, Models of Energy Use: Putty-Putty Versus Putty-Clay, *American Economic Review* 89(4), 1028-1043.

Benhabib, Jess and Aldo Rustichini, 1991, Vintage capital, investment and growth, *Journal of Economic Theory* 55, 323-339.

Boucekkine, Raouf, Fernando del Rio and Blanca Martinez, 2007, Technological progress, obsolescence and depreciation, *Oxford Economic Papers*, forthcoming.

Díaz, Antonia and Luis A. Puch, 2004, Costly Capital Reallocation and Energy Use, *Review of Economic Dynamics* 7(2), 494-518.

Licandro, Omar and Michael Wycherley, 2007, Has Information Technology Reduced the Impact of Oil Shocks?, mimeo.

Gilchrist, Simon and John C. Williams, Putty-Clay and Investment: A Business Cycle Analysis, *Journal of Political Economy,* October 2000, 108 (5), 928960.

Solow, Robert M., Investment and Technological Progress, in Kennetth J. Arrow, Amuel Karlin, and Patrick Suppes, eds., *Mathematical methods in the social sciences 1959,* Stanford, CA: Stanford University Press, 1960.

A Appendix 1

The quasi-social planner's problem is

$$
\begin{array}{lll}
\max & E_0 \sum\limits_{t=0}^{\infty} \beta^t u(c_t) & \\
\text{s.t.} & c_t + \int_{v\in V} \left(k_{t+1}(v) - (1-\delta)\ k_t(v)\right)\ dv \leq z_t^{\theta} - p_t\ \int_{v\in V} e_t(v)\ dv, & (\lambda_t^c) \\
 & z_t \leq \int_{v\in V} f(v)\ e_t(v)\ dv, & (\lambda_t^z) \\
 & 0 \leq e_t(v) \leq \frac{k_t(v)}{v}, & (\psi_t(v),\ \mu_t(v)) \\
 & k_{t+1}(v) - (1-\delta)\ k_t(v) \geq 0. & \left(\lambda_t^k(v)\right)
\end{array}
$$

The FOC's are:

$$\frac{\partial \mathfrak{L}}{\partial c_t}: \quad \beta^t u'(c_t) - \lambda_t^c = 0, \tag{A.1}$$

$$\frac{\partial \mathfrak{L}}{\partial z_t}: \quad \lambda_t^c \cdot \theta z_t^{\theta-1} - \lambda_t^z = 0, \tag{A.2}$$

$$\frac{\partial \mathfrak{L}}{\partial e_t(v)}: \quad \psi_t(v) - \mu_t(v) + \lambda_t^z \cdot f(v) - p_t \cdot \lambda_t^c = 0, \text{ for all } v \in V, \tag{A.3}$$

$$\frac{\partial \mathfrak{L}}{\partial k_t(v)}: \quad \lambda_t^k(v) - \lambda_t^c + E_t\left\{\frac{1}{v}\mu_{t+1}(v) - (1-\delta)\cdot\left(\lambda_{t+1}^k(v) - \lambda_{t+1}^c\right)\right\} = 0. \tag{A.4}$$

Proof of Proposition 1. There exists $\widehat{v}_t \geq 0$ such that for all $v < \widehat{v}_t$, $e_t(v) = 0$ and for all $v \geq \widehat{v}_t$, $e_t(v) = \frac{k_t(v)}{v}$.

Proof: Let us assume for that some $b \geq 0$ $\psi_t(b) - \mu_t(b) = 0$. This implies that $\lambda_t^z \cdot f(b) - p_t \cdot \lambda_t^c = 0$. For any $v > b$ since f is increasing function $\lambda_t^z \cdot f(v) - p_t \cdot \lambda_t^c > 0$. This implies that for the FOC to hold, $\psi_t(v) - \mu_t(v) < 0$ which amounts to $\psi_t(v) < \mu_t(v)$. This last inequality implies $e_t(v) = \frac{k_t(v)}{v}$. The opposite occurs for $v < b$. Since $f(0) = 0$, then there exists $\widehat{v}_t > 0$ such that only for $v \geq \widehat{v}_t$, we have that $e_t(v) = \frac{k_t(v)}{v}$. For any $v < \widehat{v}_t$, $e_t(v) = 0$. □

Corollary 9. *Then the multiplier $\mu_t(v)$ satisfies*

$$\mu_t(v) = \max\left\{\lambda_t^z \cdot f(v) - p_t \cdot \lambda_t^c, 0\right\}.$$

Lemma 10. *The factor $\frac{1}{v}\mu_t(v)$ strictly decreases for all $v \geq \widehat{v}_t$. It is zero for any $v < \widehat{v}_t$.*

Proof. It follows from the properties of the function f and Proposition 1. □

Proof of Proposition 2. There exists there, at most, a unique $\widetilde{v}_t$ for which $x_t(v) > 0$, $\lambda_t^k(v) = 0$. For any other v investment is zero.

Proof: By induction, the expression (A.4) can be written as

$$\lambda_t^k(v) = \lambda_t^c - \sum_{i=1}^{\infty}(1-\delta)^{i-1} \cdot E_t\left(\frac{1}{v} \cdot \mu_{t+i}(v)\right),$$

The factor $H = \sum_{i=1}^{\infty}(1-\delta)^{i-1} \cdot E_t\left(\frac{1}{v} \cdot \mu_{t+i}(v)\right)$ is either zero or strictly decreasing with v. Notice that for v sufficiently large, the factor H is arbitrarily close to zero and $\lambda_t^k(v)$ is positive. For v very close to zero the factor H is zero and $\lambda_t^k(v)$ is positive.

Suppose that there exists $b \in V$ such that $\lambda_t^k(v) = 0$. For any $v > b$ the level of investment is zero since $H(v)$ is lower than $H(b)$. There exists $\underline{v} \leq b$ above which $H(\underline{v})$ is positive. But $H(\underline{v})$ has to be greater than $H(b)$ which implies $\lambda_t^k(v) < 0$ for all v, $\underline{v} \leq v < b$. Thus, $x_t(v) = 0$, for all v, $\underline{v} \leq v < b$. For any $v < \underline{v}$, $H(v) = 0$ and then, $\lambda_t^k(v) > 0$. Thus, if there exists some b such that $\lambda_t^k(b) = 0$, it has to be unique. □

A.1 Investment can be negative

Lemma 11. *If for some $b \in V$ $\lambda_t^k(v) = 0$, which amounts to $k_{t+1}(b) > 0$ then,* $E_t \left\{ \frac{1}{v}\mu_{t+1}(v) \right\} > 0$.

Proof. Suppose that $E_t \left\{ \frac{1}{v}\mu_{t+1}(v) \right\} = 0$. By definition of $\mu_{t+1}(v)$ this implies that $\mu_{t+1}(v) = 0$ for any realization of the price and that, hence, this capital will not be used in any state of nature. This contradicts the maximization principle. Thus, $E_t \left\{ \frac{1}{v}\mu_{t+1}(v) \right\} > 0$. □

Proof of Proposition 3. The FOC is written as

$$\lambda_t^k(v) = \lambda_t^c - E_t \left\{ \frac{1}{v}\mu_{t+1}(v) + (1-\delta) \cdot \lambda_{t+1}^c \right\} \geq 0.$$

We know that there exists at least some b for which $k_{t+1}(b) > 0$. Then, using the previous Lemma,

$$\lambda_t^c - E_t \left\{ \frac{1}{v}\mu_{t+1}(v) + (1-\delta) \cdot \lambda_{t+1}^c \right\} = 0 \Rightarrow \lambda_t^c - E_t \left\{ (1-\delta)\lambda_{t+1}^c \right\} = E_t \left\{ \frac{1}{v}\mu_{t+1}(v) \right\} > 0.$$

Now suppose that there exist b_1 and b_2 that receive positive capital. This implies that

$$E_t \left\{ \frac{1}{b_1}\mu_{t+1}(b_1) \right\} = E_t \left\{ \frac{1}{b_2}\mu_{t+1}(b_2) \right\},$$

which only can happen if

$$E_t \left\{ \frac{1}{b_1}\mu_{t+1}(b_1) \right\} = E_t \left\{ \frac{1}{b_2}\mu_{t+1}(b_2) \right\} = 0,$$

which contradicts the previous Lemma. **Thus if investment can be negative only one type of capital is used each period.** □

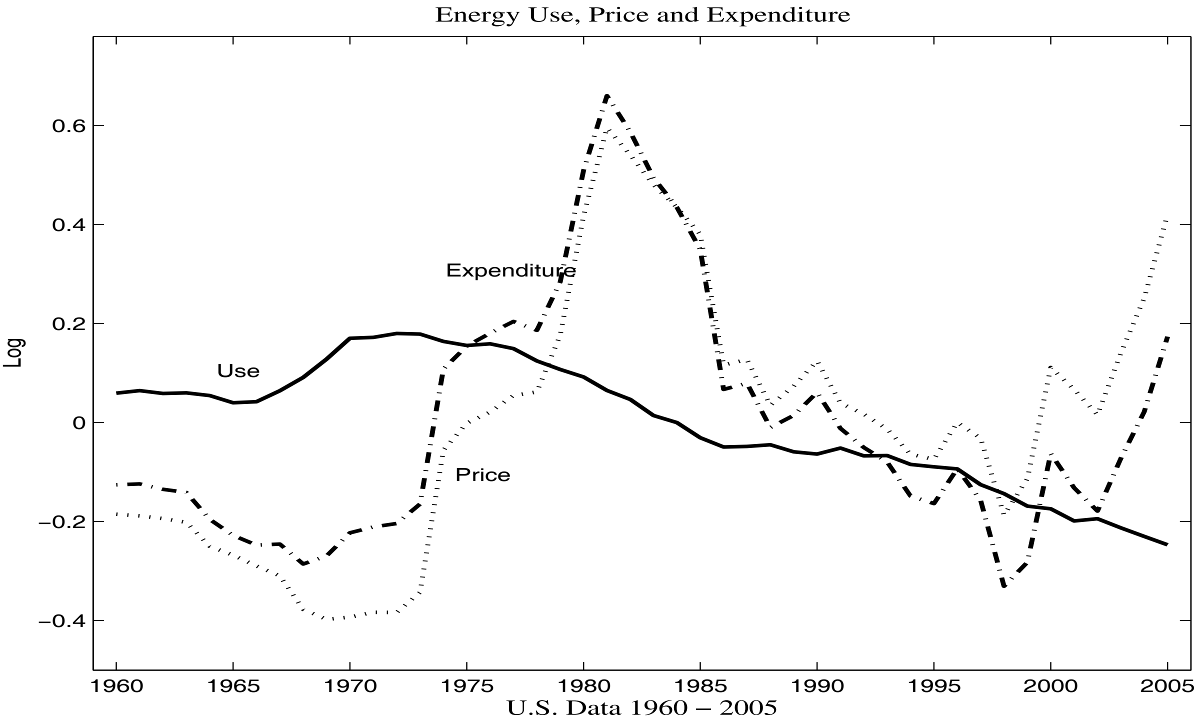
Energy Use, Price and Expenditure
Log
0.6
0.4
0.2
0
−0.2
−0.4
Expenditure
Use
Price
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
U.S. Data 1960 − 2005

Figure 1a

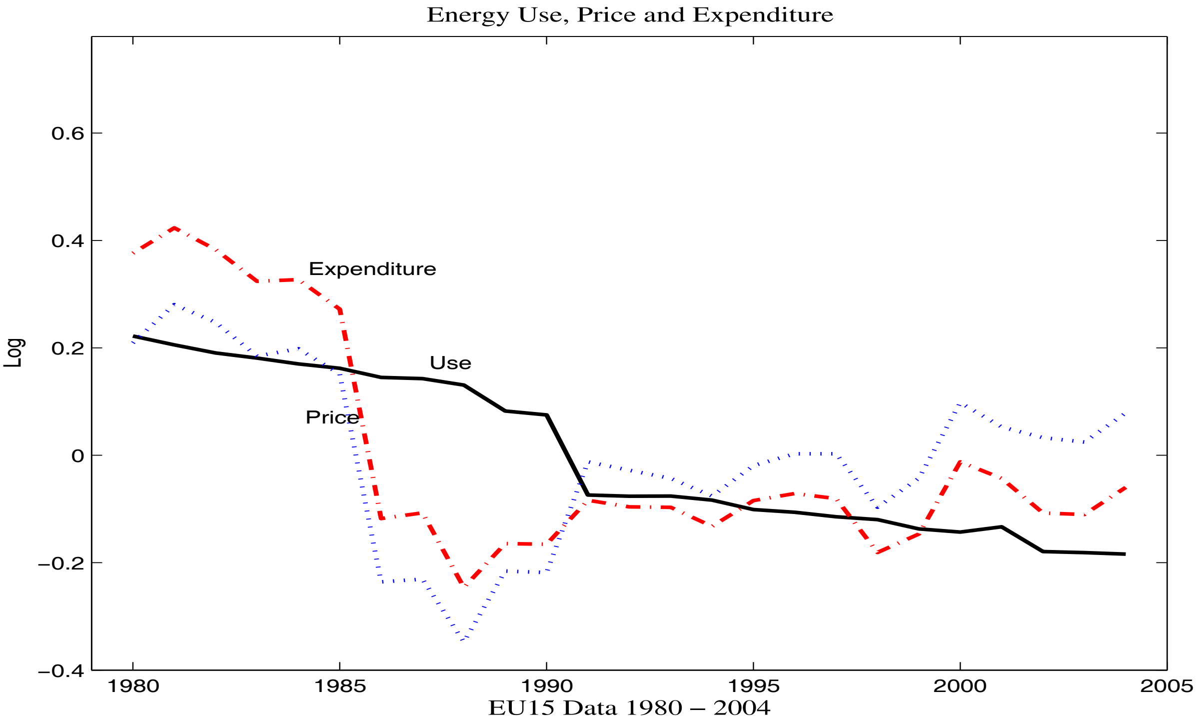
Energy Use, Price and Expenditure
Log
0.6
0.4
0.2
0
−0.2
−0.4
1980
1985
1990
1995
2000
2005
Expenditure
Use
Price
EU15 Data 1980 − 2004

Figure 1b

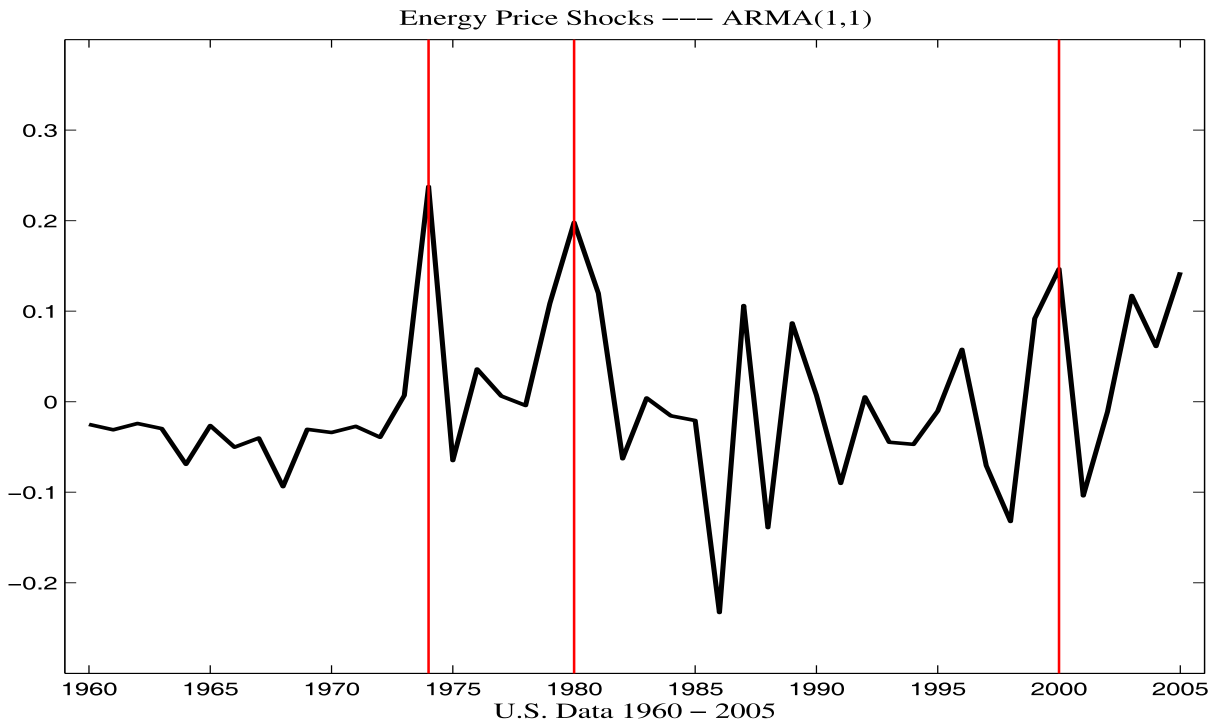

Figure 2

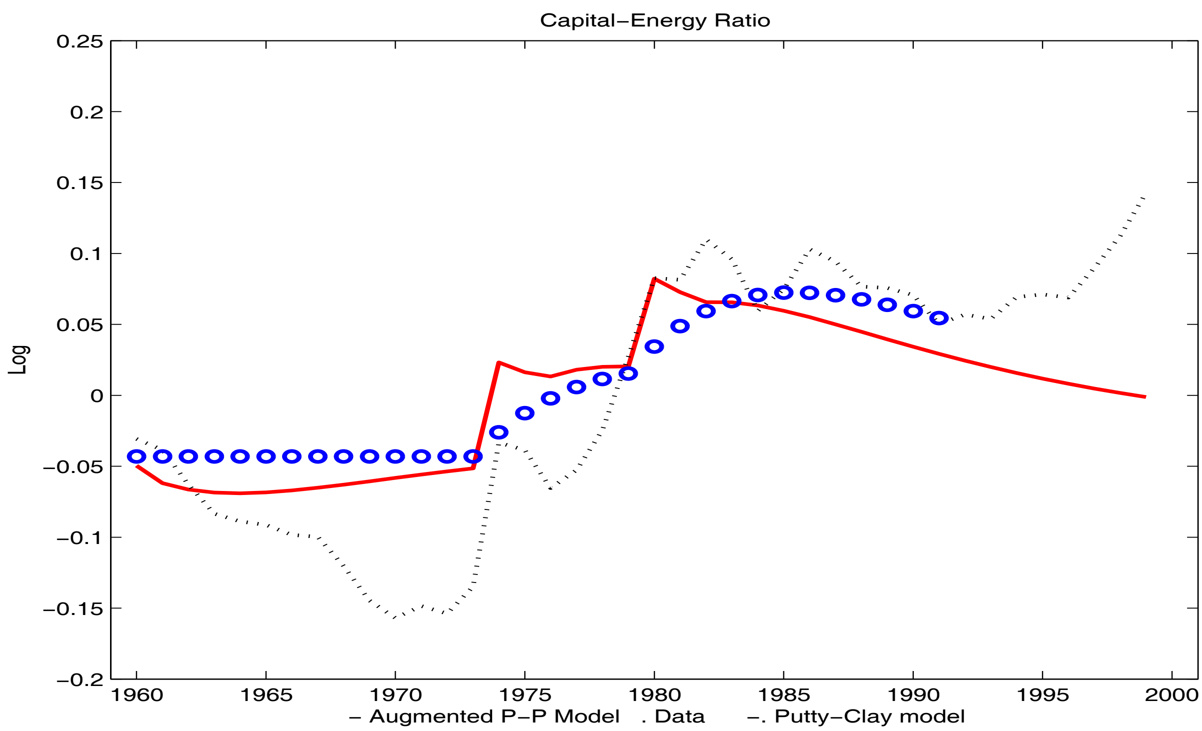

Figure 3

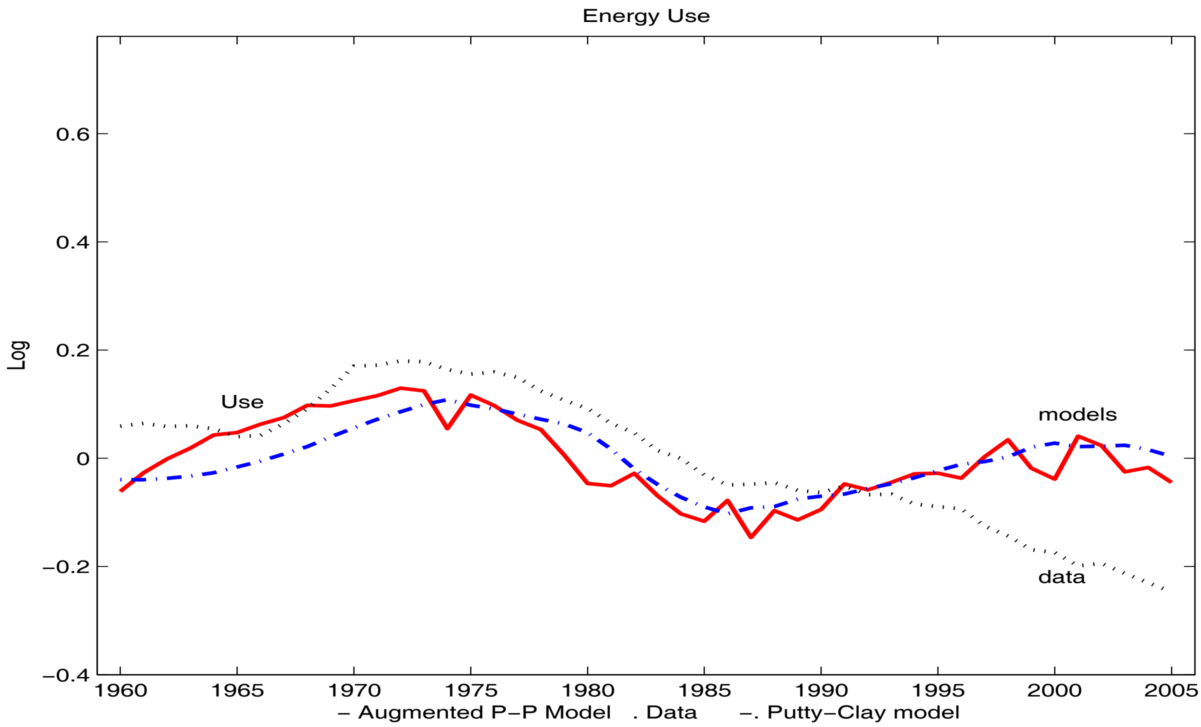

Figure 4

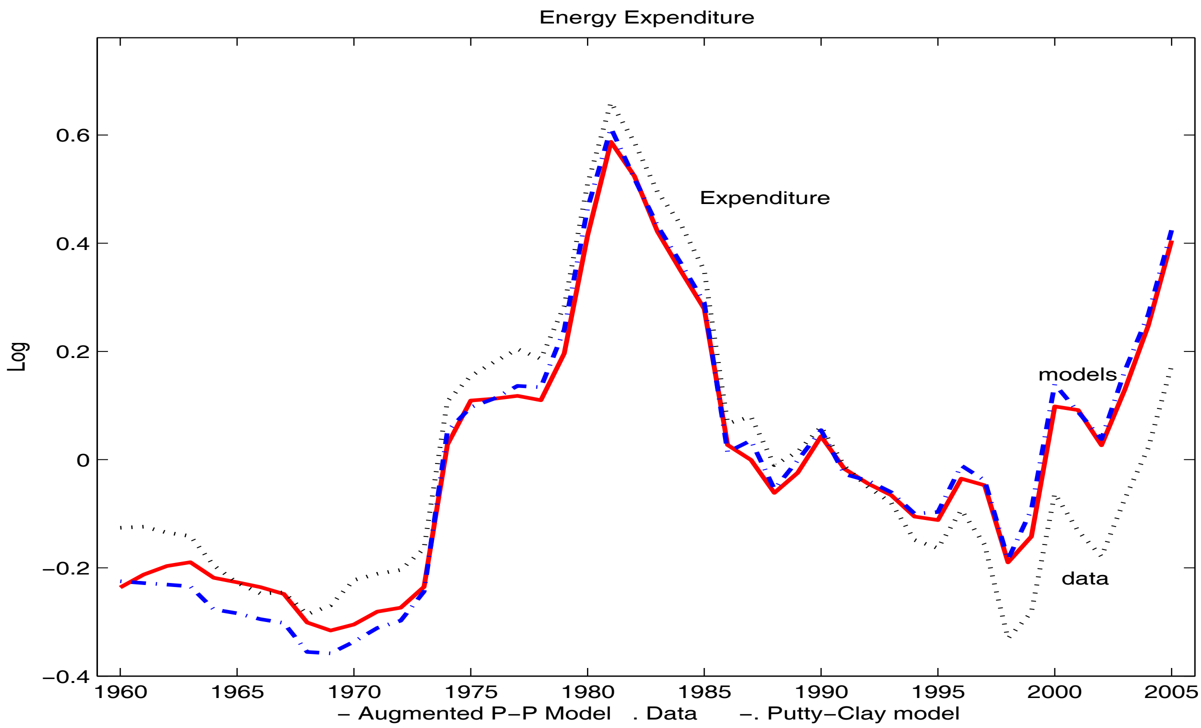

Figure 5

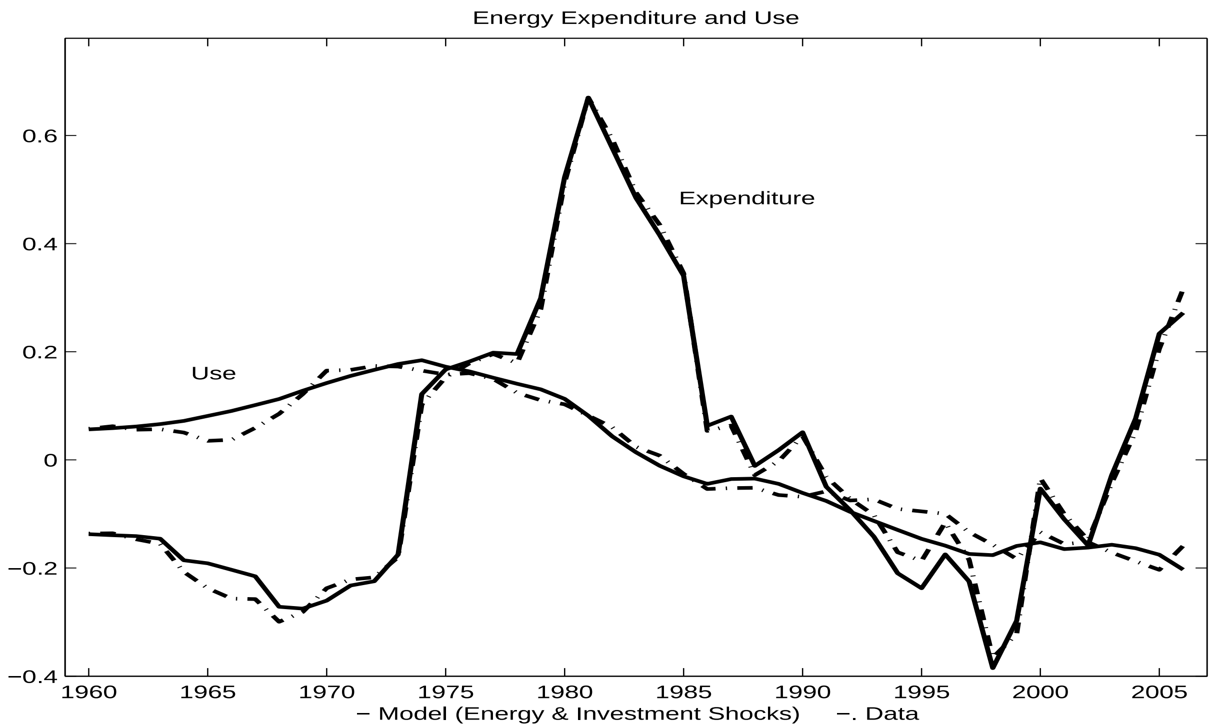
Energy Expenditure and Use
Expenditure
Use
0.6
0.4
0.2
0
−0.2
−0.4
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
− Model (Energy & Investment Shocks)   −. Data

Figure 6