

Linear prices vs. contracts for a dominant firm^{*}

Manel Antelo^a and Lluís Bru^b

March 2010, preliminary version.

Abstract

The paper compares linear prices and bilateral contracts as selling procedures for a dominant firm in the presence of a fringe. When a buyer is served by the dominant firm, a contract establishes an efficient relationship between them; the number of buyers that the dominant firm chooses to serve, however, is inefficiently low. Consumers are always worse off with contracts than with linear prices, whereas the effect on total welfare is ambiguous and depends on the specific values of the parameters that characterize the industry. Finally, it is shown the dominant firm has incentives to consolidate the industry, exacerbating the effects of contracts on welfare.

Keywords: Dominant firm, linear prices, bilateral contracts

JEL Classification: L11, L14

^{*} We wish to thank Miguel González-Maestre and Ramon Caminal for valuable comments and suggestions that improved more preliminary versions of this article. Both authors gratefully acknowledge financial support from the Galician Autonomous Government (Xunta de Galicia) through Grant PGIDIT06PXIB201002PR and Bru also from the Spanish Ministry of Education and Science (Grant SEC2007-67895-C04-03/ECON).

^a Departamento de Fundamentos da Análise Económica, Universidade de Santiago de Compostela, Campus Norte, 15782 Santiago de Compostela, Spain (e-mail: manel.antelo@usc.es).

^b Departament d'Economia de l'Empresa, Universitat de les Illes Balears, Campus Cra. de Valldemossa, km. 7, 07122 Palma de Mallorca, Spain (e-mail: lluis.bru@uib.es).

1. Introduction

Two pricing strategies often used in real-life industries are linear-posted prices and bilateral contracts (or non-linear prices), and much effort in the literature has been devoted to the analysis of their performance. Posted prices mean that buyers purchase the good at the same unit price in an organized and anonymous market. Bilateral contracts, on the other hand, imply that a given amount of good is sold to a certain customer in exchanging of a fixed payment.

The literature on price discrimination mainly analyzed a monopoly moving from linear prices to first-degree price discrimination or bilateral contracts (Tirole, 1988). The new foreclosure doctrine in vertical structures also explores why buyers accept prejudicial contracts under the feature that the outside option has endogenous value, but the alternative is less valuable, the larger the number of buyers signed into a contract. Foreclosure works to limit the number of available potential customers for firms that evaluate the possibility of entering into the market.

Our analysis can be compared to Biais et al. (1998). Our linear prices structure is similar to their limit order market, whereas our contract structure is similar to their dealership market. They consider an oligopoly of risk averse liquidity suppliers, whereas we consider a dominant firm plus a fringe of competitive liquidity suppliers.¹

The issue of the possible inefficiencies of contracts (or perfect discrimination) has been recently analyzed in Leeson and Sobel (2008) and Bhaskar and To (2004). Leeson and Sobel assume that setting a contract is costly; they show that a monopoly firm has excessive incentives to set contracts. Bhaskar and To analyze monopolistic competition, and show that contracts lead to excessive entry. There has been also some analysis of discrimination in oligopoly; Armstrong and Vickers (2001) show that competition between suppliers protects consumers against surplus extraction.

¹ Other modelling: Duffie et al. (2005, 2007) consider over-the-counter market as markets with bilateral bargaining and search.

In this paper, we explore the choice of linear prices and contracts as selling procedures in a more general set-up than a monopoly; namely, in an industry in which there is a large firm jointly with a fringe of firms. Second, and contrariwise to the literature on foreclosure, we do not analyze the role of contracts in foreclosing entry, since the fringe is already there. Third, in our case, the buyers' alternative is more valuable the more buyers have accepted a contract; rival firms are already in the market, no more entry is expected, and less contracts reduces the production capacity available per capita.

In our model, we find a rationale for bilateral contracting between the dominant firm and buyers to take place. Compared with the standard case of a monopoly, the presence of a fringe of firms creates an explicit alternative for buyers. And this alternative has an endogenous value in such a way that whenever the fringe's supply is inelastic (i.e., whenever it cannot adapt immediately to a change in prices driven by a change in demand), the less the number of buyers the dominant seller serves, the worse are the remaining buyers served by the fringe.

When buyers are homogeneous, bilateral contracts between the dominant firm and some buyers are globally efficient (i.e., they maximize joint surplus), and thus the dominant firm commits to them if such contracts are publicly observed. This conclusion parallels the one obtained in a monopoly with no fringe. Nevertheless, our results deviate from the pure monopoly case in that what is inefficient in our setting is the number of contracts offered: the large firm strategically reduces the number of contracts it offers to reduce the reservation value for buyers. This allows the dominant seller to increase its share of the rent created by the contractual relationship with buyers, and, as a consequence, bilateral contracts reduce consumer surplus compared to linear prices. In terms of total surplus, however, results are not unambiguous because contracts lead to an efficient relationship between the dominant firm and each buyer served in this way, but contracts also lead to a reduction in the overall efficiency of the allocation of total production (that of the dominant seller and of the fringe). Which effect dominates depends crucially on the size of the fringe with respect to the dominant firm. Hence, if we compare our analysis with the standard case of a monopoly that moves from linear prices to two-part tariffs (see Tirole, 1988), we find that both the increase in the efficiency of

contracting and the shift of rents from the buyer to the seller survive in our setting, but not necessarily the overall increase in welfare.

Another feature we analyze is the incentives of the dominant firm to consolidate the industry. If the dominant firm is limited to the use of linear prices, any increase in profits from further consolidation is fully passed away to fringe firms in the form of higher payments for capacity; hence the dominant firm has no incentive to acquire capacity from the fringe. If the dominant firm can set contracts with consumers,

The rest of the paper is organized as follows. In Section 2 the general model is outlined. Sections 3 to 5 explore the incentive of a dominant firm to set contracts instead of posted prices and the effect on welfare. Section 6 discusses capacity acquisition by the dominant firm. Section 7 concludes and discusses directions of future research. An Appendix contains the proofs of the results.

[2. The argument of the paper through a simple example]

Although contracts increase the efficiency of trades between the dominant firm and the customers it serves in this way, the number of buyers served by this firm is, however, inefficiently low.

Extreme examples show that total welfare may increase or decrease under contracts. When there is no fringe, $k=0$, i.e., instead of a dominant firm we have a monopoly. In this case, it is a well known fact (see, for instance, Tirole, 1988) that perfect discrimination (contracts) achieves the first-best outcome.

Consider instead the case where the dominant has fixed capacity k^d , denote by k^f the capacity of the fringe, and assume that under linear prices the dominant firm sells at full capacity, $k^d < Q(p^*)$, where $p^* = \arg \max_p RD(p)$. Under contracts, the dominant firm will also use all its productive capacity. But then, with linear prices all buyers are served the same quantity of product, $k^d + k^f$, whereas a straightforward adaptation of Proposition 1 shows that

the dominant firm serves less buyers under contracts, $a^c < \frac{k^d}{k^d + k^f} = a^{lp}$. Buyers served by the dominant firms receive more product than those served by the fringe, $\frac{k^d}{a^c} > \frac{k^f}{1-a^c}$; as a consequence, total surplus decreases with contracts. That is,

$$a^c U\left(\frac{k^d}{a^c}\right) + (1-a^c) U\left(\frac{k^f}{1-a^c}\right) < U(k^d + k^f).$$

Below, we show that the dominant firm always reduces the number of buyers sellers when contracts are available. As a consequence, the inefficient distribution of buyers between firms is always present with contracts, and buyers to the fringe receive an excessively low level of product, $\frac{k^f}{a^c} < q^{lp} < q^c$.]

2. The model

Consider an industry comprised by consumers and producers of a homogeneous good. There is a continuum of symmetric and homogeneous buyers of size one that purchase the good. Each buyer has the same quasi-linear utility function $u(q,m) = U(q) + m$, where $U(q)$ is utility derived from the consumption of the good and m represents the numéraire. As usual, utility of the consumption good satisfies $U(0) = 0$, $U'(q) > 0$, $U''(q) < 0$ and $U'''(q) + 2U''(q) < 0$.²

The production side of the industry is formed by a dominant firm and a fringe of competitive sellers. Let Q be the level of production of the dominant firm, an amount that is supplied according to an increasing and convex cost function $C(Q)$, $C'(Q) > 0$ and $C''(Q) \geq 0$. We will assume for simplicity a fixed (or perfectly inelastic) supply k of the fringe, although all we actually need for our results below is that the fringe's supply is not perfectly elastic.

² $U'''(q) + 2U''(q) < 0$ amounts to guarantee the concavity of the monopolist's problem selling to price-taking consumers with demand function $p = P(q) = U'(q)$.

The optimal production and distribution of product between sellers: $q^d=q^f$, $U'(q)=C'(a \cdot q)$.

We consider two market procedures concerning transactions between buyers and sellers. They may take place either through linear prices in an anonymous and organized market, or through bilateral contracting.

3. The impact of linear prices and contracts

Under a linear-uniform price, demand and supply of the good are formed, and the price adjusts to clear the market. If so, the (individual) demand $q = D(p)$ of each price-taking buyer is defined by

$$q = \arg \max_q U(q) + m - pq, \quad (1)$$

whose first-order condition, $U'(q) - p = 0$, leads to $D(p)$ such that $D'(p) = \frac{1}{U''(D(p))} < 0$.

Thus the (market-clearing) equilibrium price is the one for which $D(p) = Q + k$ holds.

The dominant firm, in turn, faces a residual demand $RD(p) = D(p) - k$ and posts the price that maximizes its profits,

$$\Pi(p) = RD(p)p - C(RD(p)), \quad (2)$$

The resolution of the first order condition

$$0 = \frac{\partial \Pi(p)}{\partial p} = D'(p)p + RD(p) - C'(RD(p))D'(p) \quad (3)$$

yields the optimal price p^{lp} . Finally, the dominant firm's production in a linear-price regime is the one given by $Q^{lp} = RD(p^{lp})$.

On the other hand, when the dominant firm resorts to bilateral contracts with some customers, it offers contracts $\{q_i^c, T_i\}$ to an amount a of buyers, $0 \leq a \leq 1$, where q_i^c is the quantity sold to each buyer i in exchange of payment T_i .³ Equivalently, the dominant firm offers a two-part tariff $T(q_i^c) = t + wq_i^c$, where w denotes marginal price and t is the fixed payment. Buyers simultaneously decide whether to accept or reject the offer. All remaining buyers ($1-a$ buyers if the contracts are chosen judiciously so that all the buyers approached by the dominant firm accept the proposal) not served by contract can purchase in the anonymous market to the fringe at the corresponding linear-clearing price, i.e. at price p satisfying $(1-a)D(p) = k$. Since this price is decreasing in a , the number of buyers served by the dominant firm, $\frac{\partial p}{\partial a} = \frac{D(p)}{(1-a)D'(p)} < 0$, the consumer surplus of buyers in the anonymous market, i.e., the

reservation value for buyers when evaluating any offer of the dominant firm,

$$CS(a) = U(D(p(a))) - U'(D(p(a)))D(p(a)), \quad (4)$$

is increasing in a ; namely, $CS'(a) \equiv \frac{\partial CS(a)}{\partial a} = -U''(\cdot)D(\cdot)D'(\cdot)\frac{\partial p}{\partial a} > 0$.

If the objective were to maximize total surplus,

$$TS = \int_0^a U(q_i)di + (1-a)U\left(\frac{k}{1-a}\right) - C\left(\int_0^a q_i di\right), \quad (5)$$

then the optimal quantity of product q_i shipped to each customer would be the one that satisfies

³ Superscript c stands for contracting.

$$U'(q_i) = C' \left(\int_0^a q_i di \right). \quad (6)$$

That is, in the first-best each customer of the dominant firm would be offered the quantity $q(a)$ that satisfies $U'(q(a)) = C'(aq(a))$,⁴ and the optimal number of buyers served by the dominant firm a^{fb} would be the one that solves

$$\max_a aU(q(a)) + (1-a)U\left(\frac{k}{1-a}\right) - C(aq(a)), \quad (7)$$

whose first order condition is

$$\begin{aligned} 0 = \frac{\partial TS(a)}{\partial a} &= U(q) - \left[U\left(\frac{k}{1-a}\right) - U'\left(\frac{k}{1-a}\right) \frac{k}{1-a} \right] - C'\left(\frac{k}{1-a}\right) q \\ &= [U(q) - U'(q)q] - \left[U\left(\frac{k^f}{1-a}\right) - U'\left(\frac{k^f}{1-a}\right) \frac{k^f}{1-a} \right], \end{aligned} \quad (8)$$

where we make use of condition (6). Solving (8), the optimal number of contracts a^{fb} is the one that satisfies $q(a^{fb}) = \frac{k}{1-a^{fb}}$,⁵ so all buyers are served equally. In sum, the first-best may be achieved if the dominant firm behaved competitively (were price-taker).

But we are in a set-up in which the dominant firm behaves strategically. In this case, it is immediate that it prefers to use contracts rather than linear prices. The intuition of this conclusion relies on the fact that under contracts, it can be at least as well off as under linear prices, since the market outcome under linear prices can be replicated offering to exactly the

⁴ Of course, the quantity served to each buyers would be decreasing in a , $\frac{\partial q}{\partial a} = \frac{qC''}{U'' - aC''} < 0$.

⁵ The problem is concave, since $\frac{\partial^2 TS(a)}{\partial a^2} = -U''(q) \cdot q \cdot \frac{\partial q}{\partial a} + U''\left(\frac{k}{1-a}\right) \cdot \frac{k^2}{(1-a)^3} < 0$.

same amount of buyers served with linear prices, $a^{lp} = \frac{RD(p^{lp})}{D(p^{lp})}$, a contract as such

$$\{q_i^c, T_i\} = \{D(p^{lp}), p^{lp} D(p^{lp})\}.$$

Anyway, the dominant firm does not choose such a contract. Given a customers, contracts offered by the dominant firm will be those that solve the problem

$$\max_a \Pi^D = \int_0^a t_i di - C\left(\int_0^a q_i di\right), \text{ s.t: } U(q_i) - t_i \geq CS(a). \quad (9)$$

The right-hand side of the buyers' participation constraint represents the reservation utility of each buyer, i.e. the consumer surplus he would obtain by rejecting the offer and buying to the fringe at a linear price, provided that everyone else accepts the contract the dominant firm offers.

Expressing transfers from the binding constraints and substituting them in the dominant firm's objective function, the problem to be solved by this firm becomes

$$\max_{q_i, a} \int_0^a [U(q_i) - CS(a)] di - C\left(\int_0^a q_i di\right). \quad (10)$$

The resolution of this problem yields the quantity traded q_i given the number of buyers a the dominant firm serves. Such a quantity is the one that satisfies

$$U'(q_i) = C'\left(\int_0^a q_i di\right), \quad (11)$$

which coincides with (6). Given a customers, the quantity $q(a)$ sold to each customer is the efficient one, i.e., that which maximizes the joint surplus in the relationship between these buyers and the dominant firm.

What about the number of buyers the dominant firm serves through contracts, a^c ? The number of buyers served by means of contracts is the one that solves the problem

$$\max_a \Pi(a, q) = a[U(q^c(a)) - CS(a)] - C(a, q^c(a)). \quad (12)$$

The first-order condition of problem (12),

$$\begin{aligned} 0 &= \frac{\partial \Pi(a, q)}{\partial a} = U(q) - CS(a) - a CS'(a) - C'(\cdot) q \\ &= U(q) - U'(q) q - CS(a) - a CS'(a), \end{aligned} \quad (13)$$

where we make use of the participation constraint stated in problem (9), leads the dominant firm to choose a such that

$$U(q) - U'(q) q > CS(a) = U\left(\frac{k}{1-a}\right) - U'\left(\frac{k}{1-a}\right) \frac{k}{1-a}, \quad (14)$$

since $CS'(a) > 0$.

If $CS(a)$ were exogenous (did not depend on a), the dominant firm would choose a such that $U(q) - U'(q) q = CS(a)$, that is, $q^c = q^f$. The first best would be achieved. Condition (14) implies, first, that $q^c(a^c) > \frac{k}{1-a}$, and, second, that an inefficiently low number of buyers is served by the dominant firm, $a^c < a^{fb}$.

Stated as a Proposition? We have inefficiency with contract, $a^c < a^{fb}$.

So, in contrast with the standard result under monopoly, contracts are inefficient. Then the issue is if contracts can still be more efficient than linear prices. When a^c is compared with the

number of buyers served by the dominant firm through linear price, $a^{lp} = \frac{RD(p^{lp})}{D(p^{lp})}$, we can observe that (14) reflects two opposite effects. On the one hand, increased efficiency achieved through contracts implies that higher rents can be obtained in this way, which potentially leads the dominant firm to serve more buyers through contracts than through linear prices.⁶ On the other hand, the fact that $CS'(a) > 0$ provides the dominant firm with an incentive to reduce the number of buyers it serves in order to reduce their reservation utilities.

In the Appendix we prove the following result, where superscripts c and lp stand for contracts and linear prices, respectively.

Proposition 1. *Contracts lead the dominant firm to restrict the number of buyers served with respect to those served through linear pricing; namely, $a^c < a^{lp}$.*

That is, whenever a dominant firm can offer the optimal two-part tariff or contract, it serves less buyers than this firm would serve if it could only offer linear prices. Contracts imply a more efficient bilateral relationship with buyers, but the number of buyers attended is more inefficient than under linear prices.

An immediate consequence of Proposition 1 is that consumer surplus decreases whenever the dominant firm uses contracts.

Corollary. $q^c > q^{lp}$ but $CS^c < CS^{lp}$.

Proof. See the Appendix.

In words, although less customers are served through contracts than through linear prices, customers served by means of contracts obtain a quantity higher than that these customers

⁶ Evaluated at a^l the first part of $\frac{\partial \Pi(a, q)}{\partial a}$ is positive, since $U(q(a^l)) - qU'(q(a^l)) - CS(a^l) > 0$.

would obtain if they were served through linear prices. However, as the reservation value for buyers is increasing in a , buyers are worse off with contracts.

Some remarks are worthwhile in this respect.

Remark 1. Although the model is stated as if firms in the fringe could only serve through linear prices, this is not necessary for our results; the only thing we need is that fringe firms can not affect market outcomes. To see this, consider the bargaining between the dominant firm and a buyer, if the dominant firm can not affect the alternative of the buyer; condition (7) becomes $U(q_i) - t_i \geq CS$, where now CS does not depend on a . The dominant firm faces the problem

$$\max_{q_i, a} \int_0^a [U(q_i) - CS] di - C \left(\int_0^a q_i di \right). \quad (15)$$

and the optimal quantity satisfies (7) as before, but now the optimal number of attended buyers satisfy

$$\begin{aligned} 0 &= \frac{\partial \Pi(a, q)}{\partial a} = U(q) - CS - C'(\cdot) q \\ &= U(q) - U'(q) q - CS \end{aligned} \quad (16)$$

It is then optimal for the seller to serve additional buyers until the condition $U(q) - U'(q) q = CS$ holds: The contract offers to buy at market prices p the quantity q that satisfies $U'(q) = p$. Since $U''(q) < 0$, it is better for the dominant firm to serve an additional buyer (that values at p the marginal unit of product) that serve more output to a given customer (that will value less any additional unit if $U'' < 0$, $U'(D(p)+1) < p$).

Remark 2. *Buyers served by the dominant firm through contracts exert a positive externality on those buyers served by the fringe.*

This is because there is more quantity of the good that is left for buyers served by fringe firms. The consequence is that the dominant firm has an incentive to decrease the number of contracts in order to reduce the value of the buyers' outside option, namely being served by the fringe.

Hence, and contrary to the literature on foreclosure, rivals are better off with a dominant firm than sets contracts (there are more buyers left for them); the logic on the role of contracts on buyers, instead, is similar: the dominant firm profits from the lack of coordination by buyers.

4. The role of the fringe

Our results deviate from those of the standard monopolist case when the buyers' reservation value is exogenously given and the participation constraint becomes $U(q) - U'(q)q - t \geq CS$. Formally,

Remark 3. *A monopolist would serve the same number of buyers with linear posted prices than with two-part tariffs. The quantity served under each regime, however, may differ.*

To see why, assume that a buyer has an exogenous reservation value CS (that can be zero as a special case). For low levels of the reservation value CS , a monopolist would serve all buyers both with bilateral contracts and linear prices, $a^c = a^p = a^* = n$, whenever $U(q) - U'(q)q - CS > 0$. The quantity served, however, differs. Quantity q^c served with contracts is that which satisfies $U'(q^c) = C'(nq^c)$, and quantity q^l served with linear prices would be a lower one, because it satisfies $U'(q^l) + U''(q^l)q^l = C'(nq^l)$. For intermediate values of reservation value CS , q^l would satisfy $CS(q^l) \equiv U(q^l) - U'(q^l)q^l = CS$, with $q^l < q^c$, and the dominant firm would still serve all buyers. Finally, for higher levels of CS , the quantity offered and the number of buyers served

would be the same under contracts and linear posted prices, and they would both satisfy $U(q) - U'(q)q = CS$ and $U'(q) = C'(a^*q)$, with $a^c = a^l = a^* < n$.

To summarize, we obtain: (i) $a^* = n$, if $CS(f(n)) - CS > 0$. This is the case in particular under the usual assumption that the reservation value CS adopts the value zero, and (ii) $a^* < n$, if $CS(f(n)) - CS < 0$, and then a^* is the number of contracts that satisfy the condition $CS(f(a^*)) = CS$.

Remark 4. *At the privately optimal number of contracts a^c , the dominant firm has an incentive to serve the anonymous market. Hence it must commit to the number of contracts a , and also not to serve the market (i.e. to have just one selling procedure).*

Proof. See the Appendix.

In words, for the dominant firm it is not sufficient to set publicly the amount of contracts a^* ; it must be able to commit, in addition, to the selling procedure. The only selling procedure will be contracting (which we may interpret as an indirect commitment to $Q=0$). And buyers must be able to check that the dominant firm is not producing for the market.

5. Contracts, linear prices, and welfare

Interestingly, as we discuss below, the effect of contracts on total welfare is ambiguous. Although they increase the efficiency of trades between the dominant firm and the customers it serves in this way, the number of buyers served by this firm is, however, inefficiently low.

Nevertheless, there are some polar cases that give unambiguous results. One is that in which there is no fringe, $k=0$, i.e, instead of a dominant firm we have a monopoly. In this case, it is a well known fact (see, for instance, Tirole, 1988) that perfect discrimination (contracts) achieves the first-best outcome.

Consider instead the case where the dominant has fixed capacity k^d , denote by k^f the capacity of the fringe, and assume that under linear prices the dominant firm sells at full capacity, $k^d < Q(p^*)$, where $p^* = \arg \max RD(p) p$. Under contracts, the dominant firm will also use all its productive capacity. But then, with linear prices all buyers are served the same quantity of product, $k^d + k^f$, whereas a straightforward adaptation of Proposition 1 shows that the dominant firm serves less buyers under contracts, $a^c < \frac{k^d}{k^d + k^f} = a^{lp}$. Buyers served by the dominant firms receive more product than those served by the fringe, $\frac{k^d}{a^c} > \frac{k^f}{1-a^c}$; as a consequence, total surplus decreases with contracts. That is,

$$a^c U\left(\frac{k^d}{a^c}\right) + (1-a^c) U\left(\frac{k^f}{1-a^c}\right) < U(k^d + k^f) \quad (17)$$

5.1. A numerical example

For illustrative purposes, consider the following example. Assume that each buyer has the utility function given by $U(q) = (1 - \frac{s}{2}) q$ and that the dominant firm has production costs $C(Q) = \frac{s}{2} Q^2$, where $s > 0$ represents the convexity degree of the cost function. Numerical simulations allows us to obtain the following result

Proposition 2. *In comparing linear prices and contracts the following holds.*

(i) *The quantity the dominant seller serves to each buyer is $q^{lp} = \frac{1-k}{2+s}$ under linear pricing and $q^c(a,s) = \frac{1}{1+s a}$ under contracting; that is $q^{lp} > q^c$.*

(ii) *Total surplus achieved in each regime is as follows:*

(ii.1) *If $s \in [0,1]$, then $TS^c > TS^{lp}$ whenever $k \in [0,1)$, and $TS^c = TS^{lp}$ if $k = 1$.*

(ii.2) *If $s > 1$, $TS^c > TS^{lp}$, and $TS^c = TS^{lp}$ if $k = 1$.*

(ii.3) A threshold value of k exists, $\underline{k} \in (0,1)$, such that $TS^c > TS^{lp}$ if $k \in (0, \underline{k})$, $TS^c = TS^{lp}$ if $k \in \{\underline{k}, 1\}$, and $TS^c < TS^{lp}$ if $k \in (\underline{k}, 1)$.

(ii.4) Moreover \underline{k} is decreasing in s , and $\lim_{s \rightarrow 1} \underline{k} = 1$.

Proof. For the functional forms proposed above, we can prove analytically the result for $s=0$ and $s=1$. For $s=0$, with linear prices the dominant seller chooses $q^{lp} = \frac{1-k}{2}$ and total surplus is equal to

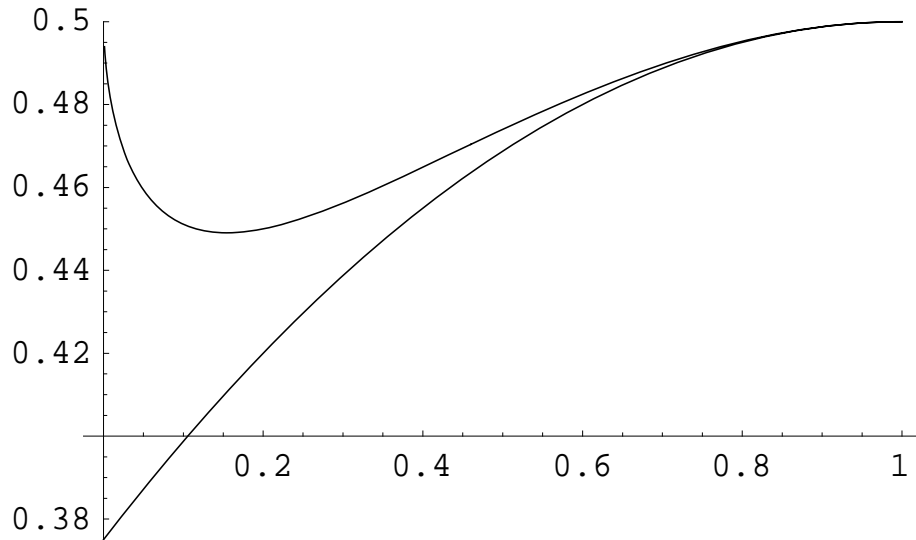
$$TS^{lp} = \frac{(3-k) \cdot (1+k)}{8}. \quad ()$$

With contracts, the dominant seller chooses $q=1$ and serves to

$$a^c(k) = 1 - \frac{(k)^2}{3^{1/3} [\sqrt{3} \sqrt{(k)^4 [27 + (k)^2]} - 9(k)^2]^{1/3}} + \frac{[\sqrt{3} \sqrt{(k)^4 [27 + (k)^2]} - 9(k)^2]^{1/3}}{3^{2/3}}$$

buyers. This is decreasing in k .

If we evaluate TS^c at $a^c(k)$, we see that it is larger than TS^{lp} unless $k=1$, in which case all buyers are served $q=1$ by the fringe. This can be illustrated in the Figure below for $s=0$, when k evolves from 0 to 1.



For $s=1$, with linear prices the dominant seller chooses $q^{lp} = \frac{1-k}{3}$ and total surplus is equal to

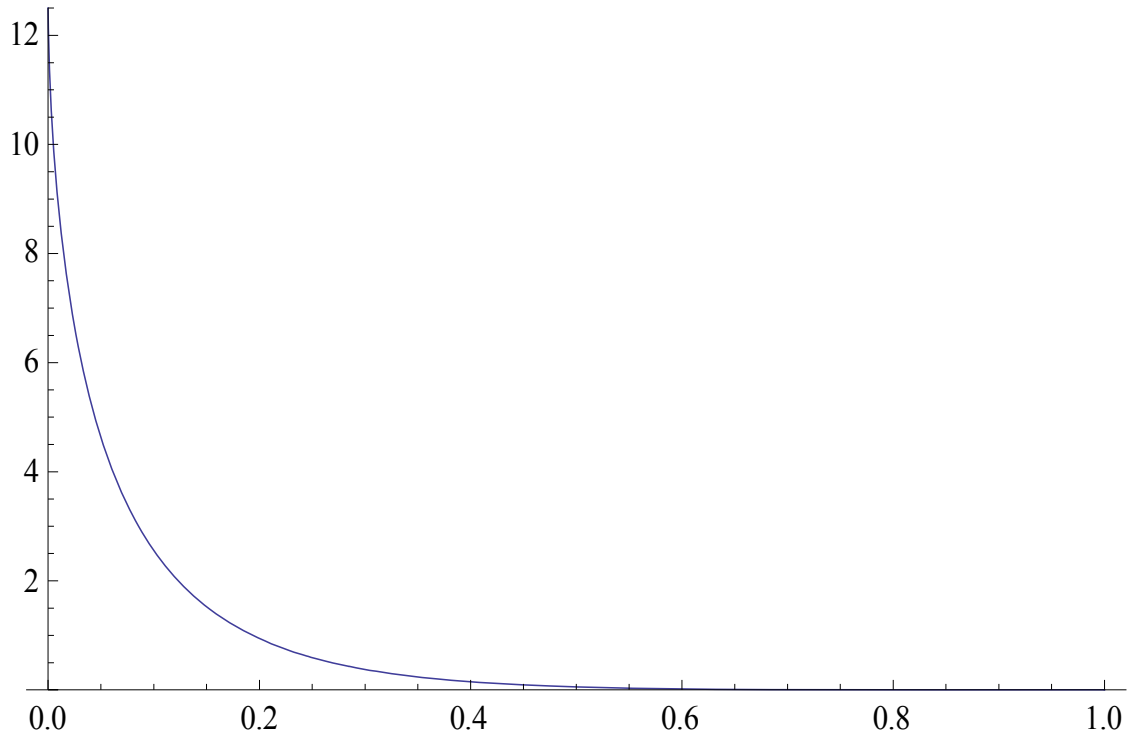
$$TS^{lp} = \frac{4 + 5 \cdot k \cdot (2 - k)}{18}. \quad (9)$$

With contracts, the dominant seller serves $q^c(a, s) = \frac{1}{1+a}$ to

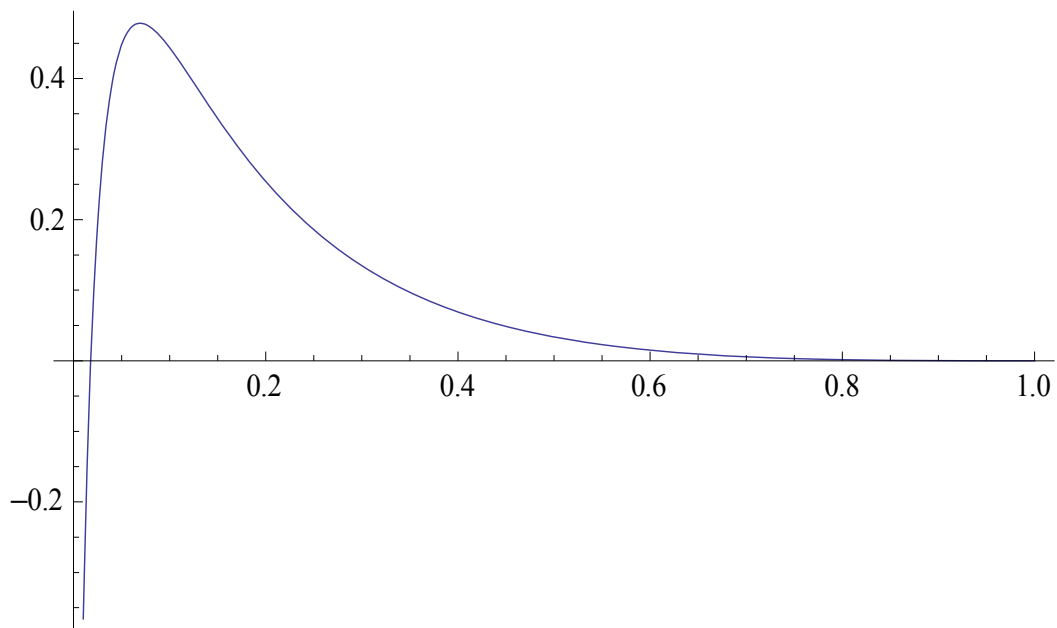
$$a^c(k) = \frac{1 - (k)^2 - 2(k)^{2/3} + 2(k)^{4/3}}{1 + (k)^2} \text{ buyers. This is decreasing in } k.$$

Finally, if we evaluate TS^c at $a^c(k)$, we see that it is larger than TS^{lp} unless $k=1$, in which case all buyers are served, $q=1$, by the fringe.

The plot of the percent increase on TS , if we move from linear prices to contracts for $s=1$, when k evolves from 0 to 1.



If we move to $s > 1$, then $TS^{dp} > TS^c$ whenever the fringe is sufficiently large (and the larger s , the smaller can be the fringe for this result). For instance, in the particular case in which $s=5$, the percentage of increase on total surplus, if we move from linear prices to contracts for, when k evolves from 0 to 1, can be plotted as follow.



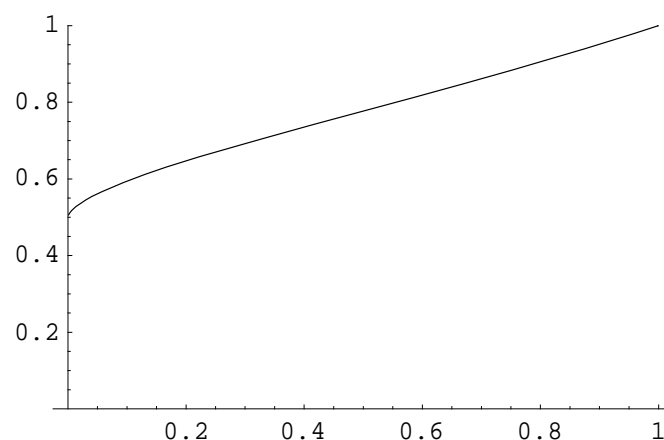
Admittedly the increase in welfare if contracts are banned is very small. But we have two points to make: first, so easily we can have a constellation of parameters for which there is not an increase in welfare if we move from linear prices to contracts; but consumer surplus decreases markedly if we allow contracts.

We may give more weight to consumer surplus than to the profit of firms in our measure of welfare,

$$W = CS + \mu \Pi, \text{ with } \mu \in [0,1] \quad ()$$

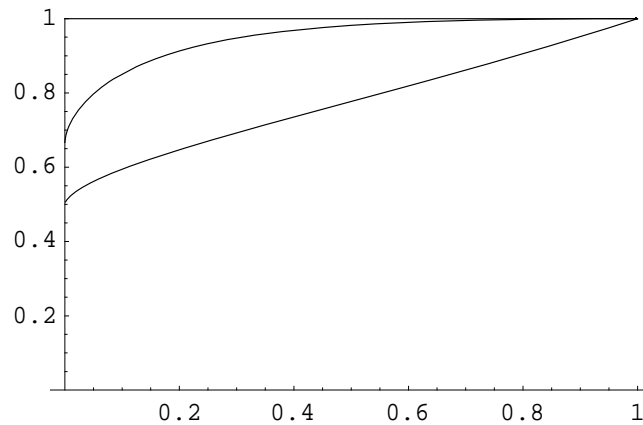
Although sometimes it is argued that μ should be set equal to zero, some authors consider that the measure of welfare must give the same weight to consumer surplus and profits; for instance, Motta (2004, pp. 20-22) argues against $\mu=0$, and we are inclined to read him as inclined to setting $\mu=1$; but then the literature on regulation and public finance (Laffont and Tirole (1993), Baron and Myerson (1981), Atkinson and Stiglitz (1980)) favors setting $0 < \mu < 1$. In any case, we are not aware of any author that argues in favor of setting μ above one!

In this latter case, the range of parameters for which linear prices are preferred over contracts from a social point of view is increased substantially.

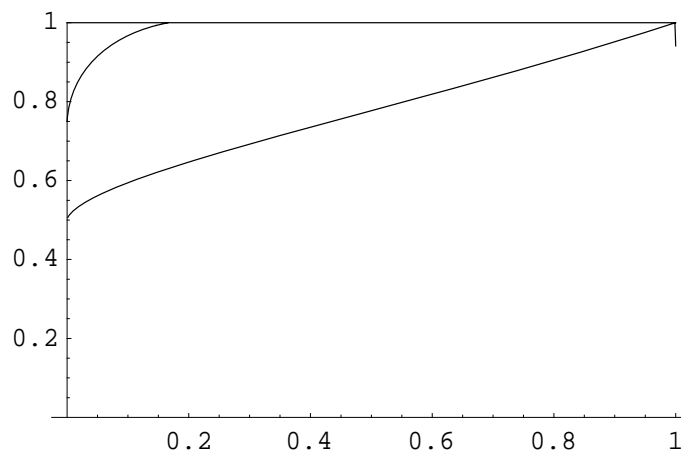


In this plot, we assume that $s=0$, that is, the dominant firm has no production costs. In the horizontal axis there is again the capacity of production of the fringe and takes values on $[0,1]$. In the vertical axis we have the parameter μ that represent the weight of industry profits in the measure of welfare. We have $W^c > W^{lp}$ above the curve and $W^c < W^{lp}$ below the curve, and it is immediate to conclude that the region of optimality of contracts decreases substantially as k^f , the size of the fringe, increases.

The region of optimality of contracts his region also decreases substantially when $s>0$ (strictly positive and convex costs of production). This can be illustrated by plotting the outcome when $s=1$:



and when $s=2$:



6 Outsourcing by the dominant firm

Assume that, previous to the opening of the final market, there is a forward market for the product. We evaluate if the DF wants to buy product from the fringe. If the fringe and the DF have the same costs of production, the results readily extend to the evaluation of DF's incentives to acquire capacity of production from the fringe.

We denote f the number of forwards. The dominant firm decides sequentially (i) The quantity of product acquired from the fringe (the amount of forward contracts) and later on (ii) the number of buyers to attend and the contract $\{t, q^c\}$ offered to them. We calculate the rational expectations equilibrium where the price of a forward equals the expected final price of the product in the anonymous market where buyers and fringe firms interact (i.e. in equilibrium fringe firms obtain the same profit selling forwards or in the final market). We assume further that the number of forwards is observed; hence the signing of more forward contracts by the DF affects the price paid for forwards, since the final price depends on the number of forwards.

6.1. Forwards and linear prices

The dominant firm faces a residual demand $RD(p) = D(p) - k^f + f$ and posts the price that maximizes its profits,

$$\Pi(p, f) = RD(p)p - C(RD(p) - f) - p^e f, \quad (.)$$

The optimal price satisfies the first order condition

$$0 = \frac{\partial \Pi(p, f)}{\partial p} = D'(p)p + RD(p) - C'(RD(p) - f)D'(p)$$

In the forward market, the dominant firm maximizes $\Pi(p(f), f)$. If the number of forward contracts is observed and the expected price reacts to changes in the number of forwards contracted by the dominant firm,

$$\frac{\partial \Pi(p(f), f)}{\partial f} = p - p^e - \frac{\partial p^e}{\partial f} f = -\frac{\partial p^e}{\partial f} f$$

Since $p^e = p$ and $\frac{\partial p^e}{\partial f} > 0$,⁷ the optimal number of forwards is $f^* = 0$.

6.2. Forwards and contracts

The problem (12) that the dominant firm solves when choosing the number of contracts with buyers changes into $\max_a \Pi(a, q^c(a), f) = a[U(q^c(a)) - CS(a)] - C(a, q^c(a) - f) - p^e f$

when a quantity f of product has been bought forward from fringe producers.

When choosing the number of forwards, we take into account that

$D(p^e) = q^f = \frac{k^f - f}{1 - a^c(f)}$, and the dominant firm chooses the number of contracts a^c optimally

given f (we denote anyway a^c instead of $a^c(f)$ the number of contracts with buyers). We have

$$\begin{aligned} \frac{\partial \Pi(a, q^c(a), f)}{\partial f} &= -a \frac{\partial CS(a)}{\partial f} + C'(a, q^c(a) - f) - p - \left(\frac{\partial p}{\partial f} + \frac{\partial p}{\partial a} \frac{\partial a}{\partial f} \right) f = \\ &= -a \frac{\partial CS(a)}{\partial f} - (U'(q^f) - U'(q^c)) - \left(\frac{\partial p}{\partial f} + \frac{\partial p}{\partial a} \frac{\partial a}{\partial f} \right) f. \end{aligned}$$

⁷ $\frac{\partial p^e}{\partial f} = -\frac{\frac{\partial^2 \Pi}{\partial p \partial f}}{\frac{\partial^2 \Pi}{\partial f^2}} = -\frac{1}{(-)} > 0$.

Where we rewrite $p = U'(q^f)$ and take into account that the optimal contract offers a quantity q^c that satisfies $C'(a q^c - f) = U'(q^c)$.

$$\text{Evaluated at } f=0, \left. \frac{\partial \Pi(a, q^c(a), f)}{\partial f} \right|_{f=0} = -a \frac{\partial CS(a)}{\partial f} - (U'(q^f) - U'(q^c)).$$

The first term is positive (a reduction in the size of the fringe reduces the value of the alternative for buyers), the second negative (the unit of capacity f is better used by the fringe, that sells a lower amount of product to each individual buyers and hence obtains a larger marginal payment for this unit of capacity).

Proposition 3. *Under contracts, $f^* > 0$.*

Proof. In the second stage, when solving for the number of contracts, and writing q^c for the optimal quantity per contract given a and f : $\max_a \Pi(a, q^c, f) = a[U(q^c) - CS(a)] - C(a q^c - f) - p^e f$, the FOC is

$$\begin{aligned} \frac{\partial \Pi(a, q^c, f)}{\partial a} &= U(q^c) - CS(a) - a \frac{\partial CS(a)}{\partial a} - C'(a q^c - f) q^c = \\ &= U(q^c) - CS(a) - a \frac{\partial CS(a)}{\partial a} - U'(q^c) q^c = 0, \end{aligned} \quad (*)$$

using $C'(a q^c - f) = U'(q^c)$. From $D(p^e) = q^f = \frac{k^f - f}{1 - a^c(f)}$, and given that

$$\frac{\partial CS(a)}{\partial a} = -U''(q^f) \cdot q^f \cdot \frac{\partial q^f}{\partial a} \quad \text{and} \quad \frac{\partial CS(a)}{\partial f} = -U''(q^f) \cdot q^f \cdot \frac{\partial q^f}{\partial f}, \quad \text{we obtain}$$

$$\frac{\partial CS(a)}{\partial a} = -q^f \frac{\partial CS(a)}{\partial f}. \quad \text{Then we may use (*) to write}$$

$$\begin{aligned} \left. \frac{\partial \Pi(a, q^c(a), f)}{\partial f} \right|_{f=0} &= \frac{1}{q^f} \cdot \{U(q^c) - CS(a) - U'(q^c)q^c\} - (U'(q^f) - U'(q^c)) = \\ &= \frac{1}{q^f} \cdot \{U(q^c) - U(q^f) - U'(q^c)[q^c - q^f]\} = \frac{1}{q^f} \cdot \int_{q^f}^{q^c} \{U'(s) - U'(q^c)\} ds > 0 \text{ whenever } U'' < 0 \text{ and} \\ & q^c > q^f. \quad \blacksquare \end{aligned}$$

So we obtain that, with contracts, the dominant firm has incentive to use forward contracts. Notice that forwards can be interpreted as subcontracting production; if the fringe and the dominant firm have the same production costs (for instance the same unit costs of production $c_{df} = c_f$), the result can immediately be extended to the presence of incentives to acquire capacity from the fringe. (For capacity acquisition, there is an additional incentive to acquire capacity, if the dominant firm is more efficient than the fringe, $C'_{df}(\cdot) < c_f$).

Which is the effect of forward contracts on welfare? Since $f^* > 0$ implies a larger concentration of capacity of production in the dominant firm, and hence a smaller alternative source of the output for buyers, consumer surplus is reduced under contracts. Total welfare increases if k^f is small (and the effect is more important, the more cost-efficient is the dominant firm), since the allocation of product among buyers is improved (the dominant seller serves more customers, and most of them must be served by the dominant firm); and total welfare decreases if k^f is large. Compared with total welfare under linear prices, the range of values for which welfare under linear prices is larger is increased (but for small fringes the dominance of contracts is increased).

For the specific functional functions $U(q) = \left(1 - \frac{q}{2}\right)q$ and $C(Q) = \frac{s}{2}Q^2$, we further obtain:

1. $\frac{\partial a^c}{\partial f} > 0$: the dominant seller serves more buyers when the fringe's sales to final buyers is reduced through forwards.

2. $\frac{dq^f}{df} = \frac{\partial q^f}{\partial f} + \frac{\partial q^f}{\partial a^c} \cdot \frac{\partial a^c}{\partial f} < 0$: CS decreases if forward contracts are allowed.

3. From point 2., since $p = 1 - q^f$, it is immediate that $\frac{\partial p}{\partial f} + \frac{\partial p}{\partial a} \frac{\partial a}{\partial f} > 0$ and the third term in the

derivative, $-\left(\frac{\partial p}{\partial f} + \frac{\partial p}{\partial a} \frac{\partial a}{\partial f}\right)f < 0$, so that acquiring forward becomes increasingly costly,

because fringe profits (per unit of capacity) are increasing in the amount of forwards that the dominant seller acquires.

Some simulations

parameters $\{s, k\}$ take values on $[0, 2]$ and $[.1, .9]$ respectively
 each column is for a value of k ($k=.1, k=.2$ and so on)
 each file is for a value of s ($s=0, s=.25$ and so on)

Compare total surplus under contracts ($f=0$) and linear contracts

Table[100 (wlp[s, k]-wc[s, k])/wlp[s, k], {s,0,2,.25}, {k,.1,.9,.1}];
 TableForm[%]

-13.13	-7.14	-3.98	-2.17	-1.11	-0.51	-0.20	-0.05	-0.00	
-8.44	-4.26	-2.25	-1.17	-0.58	-0.26	-0.09	-0.02	-0.00	
-5.60775	-2.60607	-1.2865	-0.634801	-0.300254	-0.129713	-0.0473672	-0.0124107	-0.0013975	
-3.78176	-1.58876	-0.720203	-0.328962	-0.144693	-0.0582809	-0.0198698	-0.00486288	-0.000511391	
-2.55824	-0.943077	-0.376082	-0.149622	-0.0561478	-0.0186321	-0.00493066	-0.000836338	-0.0000459141	
-1.71331	-0.522408	-0.162177	-0.0424429	-0.00499134	0.00360443	0.00322938	0.00131169	0.000197213	
-1.11652	-0.243435	-0.0275289	0.0220283	0.0245436	0.0159641	0.00760581	0.00242535	0.000319268	
-0.687823	-0.0564425	0.0574566	0.0605083	0.0412388	0.0225798	0.0098212	0.00295739	0.000374159	
-0.376008	0.0693932	0.110621	0.0828414	0.0501664	0.0258008	0.0107857	0.00315901	0.000391528	

Compare total surplus under contracts when forward contracts are allowed ($f>0$ evaluated at f optimal for the dominant seller) and linear contracts

Table[100 (wlp[s, k]-wcf[s, k])/wlp[s, k], {s,0,2,.25}, {k,.1,.9,.1}];
 TableForm[%]

-25.0917	-17.7942	-10.8795	-3.26837	1.99605	0.753594	0.256812	0.0643791	0.00931466	
-17.6843	-11.6848	-5.76483	2.30233	1.51927	0.626371	0.222557	0.0575218	0.0110495	
-13.0373	-7.88036	-2.3894	2.56882	1.22174	0.519883	0.18884	0.0498317	0.00974285	
-9.9145	-5.32149	0.146012	2.10418	1.00602	0.434908	0.159072	0.0421369	0.00863562	
-2.54376	-3.49216	2.00479	1.75125	0.841184	0.365446	0.134554	0.0372776	0.00757105	
-6.09122	-2.11367	2.48367	1.47947	0.711629	0.309688	0.114685	0.0308515	0.00663966	
-4.86756	-1.02382	2.32235	1.26505	0.607965	0.264427	0.098658	0.0274116	0.00584633	
-3.9184	-0.12435	2.07703	1.09207	0.523733	0.227493	0.0833481	0.0222139	0.00343625	
-3.16613	0.639199	1.84402	0.950359	0.453164	0.197174	0.0720405	0.0193925	0.00303087	

Compare consumer surplus under contracts ($f=0$) and linear contracts

Table[100 (cslp[s, k]-csc[s, k])/cslp[s, k], {s,0,2,.25}, {k,.1,.9,.1}];
TableForm[%]

50.7843	30.5556	18.6322	11.1992	6.48592	3.51362	1.69325	0.651281	0.142129
51.6136	31.1333	19.0186	11.4585	6.65711	3.62069	1.75304	0.677862	0.148795
51.5631	30.8694	18.7358	11.232	6.5019	3.52744	1.7052	0.658823	0.144586
51.047	30.1876	18.1394	10.7885	6.20627	3.35047	1.61334	0.621413	0.136045
50.2695	29.2953	17.4004	10.255	5.85657	3.14318	1.50628	0.577882	0.126098
49.342	28.3026	16.6067	9.69444	5.49466	2.93087	1.39743	0.533845	0.116069
48.3297	27.2712	15.8052	9.13915	5.14102	2.72551	1.29295	0.491828	0.106544
47.2725	26.2368	15.0212	8.6053	4.80534	2.53247	1.19548	0.452867	0.0977544
46.196	25.2201	14.2679	8.10054	4.49171	2.35375	1.1059	0.41727	0.0897635

Compare consumer surplus under contracts when forward contracts are allowed ($f>0$ evaluated at f optimal for the dominant seller) and linear contracts

Table[100 (cslp[s, k]-cscf[s, k])/cslp[s, k], {s,0,2,.25}, {k,.1,.9,.1}];
TableForm[%]

99.5738	98.2651	95.638	89.0134	32.6259	13.9452	5.98461	2.13825	0.493268
99.4635	97.7647	93.883	71.7025	25.4599	11.7744	5.16226	1.86273	0.501429
99.3398	97.1616	91.1508	47.5118	21.2361	10.1313	4.49569	1.63466	0.453252
99.2023	96.4269	86.1919	38.1645	18.2588	8.85212	3.94421	1.43483	0.414815
51.3471	95.5176	75.6585	32.4408	15.9918	7.80804	3.49117	1.2982	0.379171
98.8824	94.3671	62.3277	28.3392	14.1872	6.95093	3.11678	1.14013	0.347682
98.698	92.868	53.0962	25.1789	12.7094	6.23376	2.80564	1.04387	0.320194
98.4955	90.8404	46.6364	22.6312	11.4729	5.62723	2.5056	0.910055	0.226106
98.2733	87.9817	41.7402	20.5179	10.4039	5.10984	2.27002	0.828398	0.207697

Under contracts, total surplus increases when forward are allowed when m or s have small values

Table[100 (wc[s, k]-wcf[s, k])/wc[s, k], {s,0,2,.25}, {k,.1,.9,.1}];
TableForm[%]

-10.5694	-9.94121	-6.63336	-1.07447	3.07812	1.2652	0.45717	0.119944	0.0158735
-8.51689	-7.11122	-3.43614	3.43553	2.08897	0.886001	0.321452	0.084354	0.0141623
-7.03501	-5.14033	-1.0889	3.18341	1.51744	0.648755	0.236095	0.0622346	0.0111402
-5.90927	-3.67435	0.860021	2.42516	1.14905	0.492902	0.178907	0.0469975	0.00914696
0.014123	-2.52526	2.37195	1.89803	0.896829	0.384007	0.139477	0.0381136	0.00761696
-4.30417	-1.58299	2.64157	1.52127	0.716584	0.306095	0.11146	0.0295402	0.00644246
-3.70962	-0.778487	2.34923	1.2433	0.583565	0.248503	0.0910592	0.0249869	0.00552708
-3.20851	-0.0678691	2.02074	1.03219	0.482693	0.20496	0.0735342	0.0192571	0.0030621
-2.77967	0.570201	1.73532	0.868237	0.403199	0.171417	0.0612614	0.016234	0.00263935

Under contracts, buyers are always worse off when forward are allowed

Table[100 (csc[s, k]-cscf[s, k])/csc[s, k], {s,0,2,.25}, {k,.1,.9,.1}];
TableForm[%]

99.134	97.5018	94.6391	87.6278	27.953	10.8115	4.36528	1.49672	0.351638
98.8911	96.7542	92.4464	68.0405	20.1438	8.46	3.47005	1.19296	0.353159
98.637	95.8941	89.1106	40.8703	15.7588	6.84531	2.83889	0.982313	0.309113
98.3705	94.8818	83.1322	30.6866	12.8501	5.69237	2.36909	0.818501	0.27915
2.16688	93.6603	70.5308	24.721	10.7657	4.81624	2.01524	0.724504	0.253393
97.7939	92.1435	54.8258	20.6463	9.1979	4.14144	1.74372	0.609539	0.231882
97.4802	90.1938	44.2913	17.6531	7.97852	3.60654	1.53251	0.554766	0.213878
97.1466	87.5824	37.2037	15.3465	7.00408	3.17517	1.32597	0.459268	0.128477
96.7908	83.9284	32.0443	13.5119	6.19026	2.82252	1.17714	0.412851	0.118039

Conclusions (to be added)

References

- Atkinson and J. Stiglitz, 1980, Lectures on Public Economics, McGraw-Hill.
- Bhaskar, V., To, T., 2004. Is perfect price discrimination really efficient? An analysis of free entry. *Rand Journal of Economics* 35, 762–776.
- Biais, B., Foucault, T., Salanie, F., 1998. Floors, dealer markets and limit order markets. *Journal of Financial Markets* 1, 253–284.
- Darrell Duffie, Nicolae Garleanu, and Lasse Heje Pedersen, (2005). “Over-the-counter markets,” *Econometrica*, Vol. 73, No. 6 1815–1847.
- Darrell Duffie, Nicolae Garleanu, and Lasse Heje Pedersen (2007). "Valuation in Over-the-Counter Markets," *The Review of Financial Studies*, vol. 20, no. 5, pp 1865-1900.
- Iyer G. and M. Villas-Boas (2003), A bargaining theory of distribution channel, *Journal of Marketing Research* 40, 80-100.
- Leeson, P. T., R. S. Sobel (2008), Costly price discrimination, *Economics Letters* 99, 206–208.
- Makowsky, L. and Ostroy, J. (2001). "Perfect Competition and the Creativity of the Market", *Journal of Economic Literature*, Vol. 39, No. 2.
- Motta, M., (2004), *Competition Policy. Theory and Practice* Cambridge University Press.
- Milliou, C. and Petrakis, E., 2007. "Upstream horizontal mergers, vertical contracts, and bargaining," *International Journal of Industrial Organization*, Elsevier, vol. 25(5), pages 963-987, October.
- Rey, P. and J. Tirole "A Primer on Foreclosure", *Handbook of Industrial Organization*, Mark Armstrong and Rob Porter (eds.), vol. III, North Holland, p. 2145-2220.
- Tirole, J. (1988), *The Theory of Industrial Organization*, MIT Press.

Appendix A

Proof of Proposition 1.

To sort out which effect outweighs, assume that the dominant firm cannot choose the fixed part t in the two-part tariff (t is given). Linear prices are equivalent to being forced to set a two-part tariff with $t=0$. Given t , the dominant firm solves

$$\max_{q,a} \Pi(a, q; t) = a[t + U'(q)q] - C(aq), \text{ s.t.: } t \leq U(q) - U'(q)q - CS(a), \quad (\text{A1})$$

which yields the following result.

(i) From (A1), the dominant firm chooses (q, a) that satisfy $t = U(q) - U'(q)q - CS(a)$. We have

that $q = h(a, t)$, $\frac{\partial q}{\partial t} = -\frac{1}{U''(q)q} > 0$ and $\frac{\partial q}{\partial a} = -\frac{CS'(a)}{U''(q)q} > 0$, i.e. the dominant firm must offer a

larger quantity of product if t and/or a increases. Since q is a function of a and t , $q = h(a, t)$, the first-order condition is then

$$\begin{aligned} 0 &= \frac{\partial \Pi(a, q)}{\partial a} = \frac{\partial \Pi(a, h(a, t))}{\partial a} + \frac{\partial \Pi(a, h(a, t))}{\partial q} \frac{\partial h(a, t)}{\partial a} \\ &= t + q[(U'(q) - C'(aq))] - a[U''(q)q + U'(q) - C'(aq)] \frac{CS'(a)}{U''(q)q}. \end{aligned} \quad (\text{A2})$$

At $a^* = g(t)$, $U''(q)q + U'(q) - C'(aq) < 0$ given that $t \geq 0$ and $U'(q) - C''(aq) \geq 0$.

(ii) To determine how a^* evolves with t , we need to evaluate $\frac{\partial a}{\partial t} = -\frac{\frac{\partial}{\partial t} \frac{\partial \Pi(a, t)}{\partial a}}{\frac{\partial^2 \Pi(a, t)}{\partial a^2}}$. From here, it

follows that $\text{sign}\left(\frac{\partial a}{\partial t}\right) = \text{sign}\left(\frac{\partial}{\partial t} \frac{\partial \Pi(a, t)}{\partial a}\right)$. And that $\text{sign}\left(\frac{\partial}{\partial t} \frac{\partial \Pi(a, t)}{\partial a}\right) < 0$ is proved as

follows, where FOC stands for $\frac{\partial \Pi(a, q)}{\partial a}$.

$$\begin{aligned} \frac{\partial}{\partial t} \frac{\partial FOC}{\partial t} &= 1 + \frac{\partial FOC}{\partial q} \frac{\partial q}{\partial t} \\ &= - \left\{ [U'(q) - C'(aq) - aqC''(aq)] - aCS'(a) \frac{\partial}{\partial q} \left[\frac{U''(q)q + U'(q) - C'(aq)}{U''(q)q} \right] \right\} \frac{1}{U''(q)q} < 0 \quad (A2) \end{aligned}$$

if

$$U'(q) - C'(aq) < aqC''(aq) + aCS'(a) \frac{\partial}{\partial q} \left[\frac{U''(q)q + U'(q) - C'(aq)}{U''(q)q} \right] \quad (A3)$$

and (A3) holds because

$$\begin{aligned} aCS'(a) &= \frac{U''(q)q}{U''(q)q + U'(q) - C'(aq)} \{t + q[U'(q) - C'(aq)]\} \\ &> \frac{U''(q)q^2}{U''(q)q + U'(q) - C'(aq)} [U'(q) - C'(aq)] \quad (A4) \end{aligned}$$

since $t \geq 0$, and

$$\begin{aligned} \frac{\partial}{\partial q} \left\{ \frac{U''(q)q + U'(q) - C'(aq)}{U''(q)q} \right\} &= \frac{\partial}{\partial q} \left\{ \frac{U'(q) - C'(aq)}{U''(q)q} \right\} \\ &= \frac{1}{[U''(q)q]^2} \{ [U''(q) - aC''(aq)]U''(q)q - [U'(q) - C'(aq)][U'''(q)q + U''(q)] \} \\ &= \frac{[U''(q) - aC''(aq)][U''(q)q + U'(q) - C'(aq)] - [U'(q) - C'(aq)][U'''(q)q + 2U''(q) - aC''(aq)]}{[U''(q)q]^2} > 0 \quad (A5) \end{aligned}$$

In particular,

$$\frac{\partial}{\partial q} \left\{ \frac{U''(q)q + U'(q) - C'(aq)}{U''(q)q} \right\} > \frac{U''(q)q + U'(q) - C'(aq)}{U''(q)q^2} > 0. \quad (\text{A6})$$

Hence, $aCS'(a) \frac{\partial}{\partial q} \left\{ \frac{U''(q)q + U'(q) - C'(aq)}{U''(q)q} \right\} > U'(q) - C'(aq)$. ■

Proof of Corollary 1. From Proposition 1 it holds that $a^c < a^{lp}$. Moreover we know that q^c and q^{lp} must satisfy

$$U'(q^c) = C'(a^c q^c) \quad (\text{A7})$$

and

$$U'(q^{lp}) + U''(q^{lp}) \frac{a^{lp} q^{lp}}{n} = C'(a^{lp} q^{lp}) \quad (\text{A8})$$

respectively. Suppose otherwise to the statement in the corollary, $q^c < q^{lp}$. Then

$$U'(q^c) = C'(a^c q^c) < C'(a^{lp} q^{lp}) < U'(q^{lp}), \quad (\text{A9})$$

where the first inequality comes from $a^c < a^{lp}$, $q^c < q^{lp}$ and $C''(\cdot) > 0$, and the second inequality comes from (A8). But then, $U''(\cdot) < 0$ implies $q^c > q^{lp}$, a contradiction. ■

Proof of Remark 4.

Note that, from the first-order condition (6),

$$a^* [U(q^c) - q^c U'(q^c) - CS(a^*)] - a^* CS'(a^*) = 0, \quad (\text{A10})$$

implies that $U(q^c) - q^c U'(q^c) > CS(a^*)$, from which $q^c > D(p(a^*))$. When the dominant firm serves a^* buyers, the residual demand at the anonymous market would be

$RD(p) = (n - a^*)D(p) - Y$, and the price p^m that clears the market satisfies $RD(p^m) = 0$.

Without commitment, the dominant firm would solve

$$\max \Pi(p) = RD(p)p - C(a^*q^c + RD(p)). \quad (\text{A11})$$

The derivative

$$\frac{\partial \Pi(p)}{\partial p} = RD'(p)p + RD(p) - C'(a^*q^c + RD(p))RD'(p) \quad (\text{A12})$$

evaluated at p^m , the expected equilibrium price if the dominant firm does not serve the market, becomes

$$\frac{\partial \Pi(p)}{\partial p} = RD'(p^m)[p^m - C'(a^*q^c)], \quad (\text{A13})$$

which is strictly negative if $p^m > C'(\cdot)$. This is indeed the case, since at p^m we have, from (6), that $p^m = U'(q^m) > U'(q^c) = C'(\cdot)$. But this implies that the dominant firm has an incentive to set $p < p^m$ and enter the anonymous market. ■