Branching Deregulation and Merger Optimality*

Ana Lozano-Vivas
Universidad de Málaga

Miguel A. Meléndez-Jiménez
Universidad de Málaga

Antonio J. Morales
Universidad de Málaga and LINEEX

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Abstract

The U.S. banking industry has been characterized by intense merger activity in the absence of economies of scale and scope. We claim that the loosening of geographic constraints on U.S. banks is responsible for this consolidation process, irrespective of value-maximizing motives. We demonstrate this by putting forward a theoretical model of banking competition and studying banks' strategic responses to geographic deregulation. We show that even in the absence of economies of scale and scope, bank mergers represent an optimal response. Also, we show that the consolidation process is characterized by merger waves and that some equilibrium mergers are not profitable per se -they yield losses- but become profitable as the waves of mergers unfold.

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1 Introduction

For most of the twentieth century, the structure of the U.S. banking system remained largely unchanged. However, in the last quarter of the century it underwent a major transformation. Much work has been devoted to understanding the causes of this change and its effects. According to Berger et al. (1995), these changes are linked to two major innovations: (i) regulatory changes, from deposit deregulation in the early 1980s to deregulation of branching and banking activities a decade later, and (ii) innovations in technology, information processing, derivatives, and loan securitization, among others.

In this paper, we focus on regulatory changes. Historically, U.S. banks were constrained from crossing state lines by the MacFadden Act of 1927, and from establishing branches across county lines by state laws. By 1975, no state allowed out-of-state bank holding companies (BHCs) to buy in-state banks. By 1990, all states but Hawaii allowed out-of-state BHCs to buy in-state banks, and all but three states allowed state-wide branching. These developments marked a deregulatory process that was completed with the passage by Congress of the Riegle-Neal Interstate Banking and Branching Efficiency Act (IBBEA) in 1994. Implemented in June 1997, IBBEA removed all remaining federal restrictions on interstate banking and encouraged states to permit interstate branching. (See Kroszner and Strahan, 1999, for a detailed chronology of deregulatory changes at the state level).

This rapid structural change in the U.S. banking industry has been characterized by consolidation in commercial banking (according to data from FDIC, the number of commercial banks decreased by one-third between 1985 and 1997) and by concentration of assets among the largest banking organizations (the concentration ratio of the top five banks raised from 0,14 in 1985 to 0,19 in 1997).

There are, however, some puzzles about the consolidation process. According to the literature, there are two main motives behind banking consolidation: (i) value maximization, such as economies of scale, economies of scope, and risk diversification, and (ii) other motives, such as self-serving interests of managers. Extensive research on the effects of consolidation has found evidence of profit efficiency (Berger, 1998), but no evidence of economies of scale (Berger et al., 1999) or economies of scope (Stiroh, 2004). As a consequence, there are no improvements in cost efficiency. However, these findings conflict with the opinions expressed by bank managers, who cite gains in cost efficiency as the main motive for consolidation. As Jones and Critchfield (2005) state, the lack of economies of scope and scale in the consolidation of the U.S. banking system represents a fairly substantial puzzle.

This paper sheds light on this puzzle by putting forward a theoretical model of banking competition and studying banks' strategic responses to geographic deregulation. To our knowledge, Economides et al. (1996) is the only paper that attempts a theoretical approach to deregulation; it does so from the point of view of attempting to uncover regulators' motivations for protecting small banks from entry by large corporations.

We borrow from Economides et al. (1996) the monopolistic competition model à la Salop (1979). We model the deregulatory process as an incumbent–entrant game with several markets. The novelty of this approach is that all banks are incumbents in some markets and entrants in others. We keep our version of the model particularly simple, especially with respect to the cost function. Given that there is no empirical evidence of economies of scope or scale in the U.S. consolidation process, we do not include them in our cost function.[1]

We consider the following stylized model of the U.S. banking deregulation process. Initially, banks can only run a single branch in their local market. After that, there is a first stage in which local banks can operate in other territories only via mergers and acquisitions. Finally, full branching deregulation across territories takes place.[2] Note that local banks face two kinds of decisions sequentially: first, they decide in which markets to be incumbents (or equivalently, with whom to merge, if so) and then, once the distribution of incumbents has arisen endogenously, they must decide whether to accommode entry and how to expand their branch networks across territories (this is the incumbent-entrant part of the model).

We show that for every distribution of incumbents and market density across territories, incumbents find it optimal to expand their branch networks, although not enough to deter entry. As a result, each territory experiences an increase in the number of branches, with the former incumbent controlling half of the territory, whereas the other half is shared by the entrants. This implies the following features of the concentration ratios at the local market level: the r-bank concentration ratio is slightly above one half and the Herfindahl-Hirschman index is slightly above one quarter.

Given the advantage in terms of market share to incumbents in local markets, it

[1] Some claim that scale economies exist but that the methodology used to find them is not adequate. See for example, Hughes et al. (2001), which finds scale economies based on the role of the bank as an intermediary.

[2] We abstract from mergers and acquisitions after deregulation. The main motive is that most of the merging activity in the U.S. banking industry took place before 1997, the year of full liberalization. In the period 1975-1997, there were 8281 unassisted mergers while in the period 1998-2008 there were 3689. The analysis of mergers in the post regulated era is left for future research.

becomes intuitive that banks are eager to become incumbents. Considering that the only way is through mergers and acquisitions, and that in addition an acquisition reduces by one the number of future entrants, it is therefore not surprising that mergers are optimal responses in the model. What it is somehow remarkable is that for a general class of merger protocols, whenever a merger occurs in equilibrium, a complete merger wave is predicted.

Quite interestingly, we prove that there are merger waves initiated for strategic reasons; i.e., they are not profitable per se for the merging institutions but later become profitable because they trigger further mergers. Moreover, we show that the gains to be realized through mergers and acquisitions, are more easily realized if large banks are protected and have a priority in merger activities.

These findings are more notable when compared to other theoretical papers that deal with mergers in different industries. First, economies of scale are generally assumed -and hence mergers are more likely to ocurr- and exogenous initial conditions are considered. In our paper, there is no choice as to the initial industry configuration and the industry technology: they are determined by the regulatory body and the banking technology.

Second, we offer general results whereas other papers, given the complexity of the strategy space, usually confine the main analysis to a small number of firms. Qiu and Zhou (2007) develops a model of sequential endogenous mergers in the context of Cournot competition in which, following a merger, the less efficient constituent firm disappears. Gorton et al (2009) considers a model of sequential exogenous mergers with managerial incentives in which firms can only acquire a smaller firm. Both predict the existence of merger waves and mergers with negative profits for some initial conditions following a industry-level regime shift.

We do also compute the explicit (merger) equilibrium strategies for a small number of banks (cf. Section 5) and show that some of the assumptions imposed in Gorton et al (2009) protocol (the "large" bank is safe from being acquired) arise endogenously. Additionally, we show that the optimal behaviour in our case displays a race for size, similar to the behaviour found by Gorton et al (2009), but of a different nature: Our model lacks of agency problems, whereas the defensive mergers in Gorton et al (2009) result from the disaligment of incentives between the shareholders and the manager.

Our conclusions endorse the ideas of Berger (1998), which observes that "many of the merger participants in the 1980s focused on expanding their geographic bases to gain strategic long-run advantage by getting footholds in new locations, rather than on reducing costs or raising profits in the short run. Merger participants in the 1990s appear to be more focused on cutting costs..." We however, believe that the strategic explanation can be extended to most of the 1990s, as full deregulation was implemented in June 1997 by the IBBEA.

The remainder of this paper proceeds as follows. Section 2 sets forth the basic model, which is solved in Section 3. Mergers and acquisitions are considered in Section 4. Section 5 shows the detailed equilibrium (merger) dynamics for a small number of banks. In Section 6 we discuss our findings and finally Section 7 concludes. All proofs are contained in the Appendix.

2 The Branching Deregulation Game

We assume that there are K territories and K banking institutions, each initially operating as a monopolist in its own territory. The regulatory agency announces that at a given date, cross border activity will be allowed. Following the history of the U.S. deregulation process we assume that, prior to full deregulation, banks can operate in other territories via mergers and acquisitions. We model the Branching Deregulation Game as an incumbent–entrant model. The timing of the game is set forth in the following definition.

Definition 1 *The timing of the Branching Deregulation Game is as follows:*[3]

Stage 1. *As a first deregulatory step, banks are allowed to expand to other territories via Mergers and Acquisitions (M&A stage).*

Stage 2. *Prior to full deregulation, incumbents decide simultaneously the number of branches to open in their own territories (Incumbent stage).*

Stage 3. *Upon observing incumbents' decisions in Stage 2, entrants decide simultaneously how many branches to open in new territories (Entry stage).*

Stage 4. *Price competition takes place in each territory (Price competition stage).*

These stages can be grouped into two: The *M&A stage* (Stage 1) and the *incumbent-entrant (sub)game* (Stages 2, 3 and 4). The M&A stage determines which banks remain in the market and how they are distributed as the incumbents of the different territories. Once the M&A process finishes, banking institutions play the incumbent-entrant game. They use two strategic variables: branch networks and interest rates.

[3] The Branching Deregulation Game is a stylized version of U.S. branching deregulation. One missing feature is the opt-out option that the IBBEA permitted regarding the opening of de novo branches by out-of-state banks. In our game, banks are free to open as many de novo branches as they wish. We later discuss the role of this assumption.

In this paper, we consider a dynamic model in which the long-term variable is the branch network, whereas the short-term strategic variable is price (interest rates).

Moreover, note that, after the M&A stage, a given banking institution faces a complex problem as it acts as incumbent in some territories (in Stage 2) and as entrant in the remaining ones (in Stage 3), so it needs to take K decisions. In order to simplify the problem, we assume that:

(i) the territories are isolated from each other -we do not allow customers to fulfill their banking needs outside their territory; and

(ii) the cost function depends in a linear fashion on the total number of branches open throughout all territories. We purposely neglect other cost variables.

These two assumptions cause decisions in one territory to be independent of decisions in other territories. Hence, we focus on competition in one arbitrary territory.

Summarizing, in our model, before banking institutions are allowed to open *de novo* branches in other territories, they can expand via M&A (Stage 1). Once the M&A stage finishes, each of the remaining active banks acts as an incumbent in all the territories of its constituent banks. In Stage 2, prior to full deregulation, the incumbents decides how many branches to open in their territories. Then, in Stage 3, each bank is allowed to open (de novo) branches outside its territories. Finally, in Stage 4, price competition occurs and payoffs are realized.

In the next sections, we solve for the subgame perfect equilibrium of the Branching Deregulation Game. The Branching Deregulation Game has four types of subgame: (i) price competition subgames, (ii) entry subgames, (iii) incumbent subgames, and (iv) M&A subgames. Inductive arguments apply. First, given a distribution of incumbents across territories, we analyze the incumbent-entrant game, i.e., subgames of type (i), (ii) and (iii). Then, we step back and analyze the M&A stage.

3 The incumbent-entrant game

Let n be the number of active banks resulting from the M&A stage, i.e. those banks that still remain in the market after Stage 1. Since we assume from the outset that full monopolization of the banking sector is not permitted, $n \geq 2$. Let $\mathcal{K} = \{1, ..., K\}$ be the set of territories and $\mathcal{K}_i \subset \mathcal{K}$ be the set of territories in which bank i acts as incumbent (this is a partition of the set of territories among the n active banks).[4]

[4]Note that all the results of Sections 4 and 5 also hold for a situation in which no mergers take place in the M&A stage. Such a situation corresponds to the specific case $n = K$.

3.1 Price competition stage

Following Economides et al. (1996), in Stage 4 we model competition using Salop's spatial model (1979). Given that we are interested in analyzing geographic deregulation, we simplify the model by assuming that banks compete on deposits and that transportation costs are at infinity. Under this condition, customers optimally take their deposits to the closest branch, independent of interest rates. The Nash equilibrium strategy in this subgame is that all banks will set their deposit interest rates equal to zero. With the additional assumption of symmetric location of branches, profits of banking institution i in territory k amount to

$$\Pi_{i,k} = b_{i,k}\left(\delta_k r \frac{1}{b_k} - \Phi\right)$$

where δ_k represents market k density, r is market interest rate,[5] $b_{i,k}$ is the number of branches opened by banking institution i in market k, b_k is the total number of branches (of all banking institutions) in market k, and $\Phi > 0$ represents cost per branch opened. Let

$$B_k = \frac{\delta_k r}{\Phi}$$

Since $B_k \Phi = \delta_k r$, B_k corresponds to the (aggregate) number of branches in market k such that there are zero-profits (i.e., it is the competitive branch network in market k). Thus, as B_k is the maximum number of branches that can be sustained in market k, it shall be a useful reference in the subsequent analysis.

3.2 Entry stage

We step back now and address entry subgames (Stage 3). In this class of subgame, entrants decide on the number of branches to open in territory k, after observing the number of branches opened by the incumbent. Let I_k be the number of branches opened by the incumbent in territory k in the incumbent stage. Since there are n active banks in the market, the number of entrants in each territory is

$$\bar{n} = n - 1$$

Lemma 1 *In the entry subgame, the optimal number of branches opened by each entrant in territory k is*

$$e_k^*(I_k) = \begin{cases} \frac{1}{2\bar{n}^2}\left(\bar{n}\left(1 - 2\frac{I_k}{B_k}\right) - 1 + \sqrt{(\bar{n}-1)^2 + 4\bar{n}\frac{I_k}{B_k}}\right) B_k & \text{if } I_k < B_k \\ 0 & \text{if } I_k \geq B_k \end{cases}$$

[5] We assume that banks, regardless of their territory, can access the same market interest rate.

From the above proposition, the following corollary follows:

Corollary 1 *(i)* $e_k^*(0) > 0$, *(ii)* $\frac{\partial e_k^*(I_k)}{\partial I_k} < 0$ *and (iii)* $\frac{\partial e_k^*(I_k)}{\partial \bar{n}} < 0$.

Note that entry is not blocked, and it can only prevented by the incumbent upon opening B_k branches. Also, there is an inverse relationship between branches opened by incumbents and branches opened by entrants (strategic substitutes) and between the number of branches opened by each entrant and the number of entrants.

3.3 Incumbent stage

We now analyze the incumbent subgames (Stage 2). Recall that there is only one incumbent per territory.[6] In this class of subgame, incumbents decide on their own optimal opening of branches anticipating the optimal behavior of entrants described in Lemma 1.

Lemma 2 *The optimal number of branches opened by the incumbent of territory* k *is* $I_k^* = \frac{2\bar{n}-1}{4\bar{n}} B_k$.

Some properties of the optimal number of branches opened by the incumbent are contained in the following corollary:

Corollary 2 *(ii)* $\frac{\partial I_k^*}{\partial \bar{n}} > 0$ *(ii)* $\lim_{\bar{n}\to\infty} I_k^* = \frac{1}{2} B_k$, *(iii)* $\frac{\partial I_k^*}{\partial \Phi} < 0$ *and (iv)* $\frac{\partial I_k^*}{\partial \delta_k} > 0$.

In equilibrium, entry is not prevented but accommodated by the incumbent, as the expansion of the branch network in its own territory is smaller than the competitive amount. This expansion is inversely proportional to the cost per branch and increasing in the number of entrants or, equivalently, in the number of active banks (since $n = \bar{n} + 1$).

We can now set forth the equilibrium outcome of the incumbent-entrant game and the resulting concentration ratios at the market level.

[6] Either the bank initially original from the territory or the bank that has acquired it in the M&A stage.

Proposition 1 *In equilibrium, for territory k,*[7]

(i) *The incumbent opens* $\frac{2\bar{n}-1}{4\bar{n}}B_k$ *branches, with profit* $\frac{1}{4\bar{n}}\delta_k r$.

(ii) *Each entrant opens* $\frac{2\bar{n}-1}{4\bar{n}^2}B_k$ *branches, with profit* $\frac{1}{4\bar{n}^2}\delta_k r$.

(iii) *For each* $r \in \mathbb{N}$, *the r-bank concentration ratio (CRr) is* $\frac{1}{2} + \frac{r-1}{2\bar{n}}$.

(iv) *The Herfindahl-Hirschman index (HHI) is* $\frac{1}{4} + \frac{1}{4\bar{n}}$.

It is important to note that the number of branches opened by the different banking institutions in each market depends on the number of active banks (determined in the M&A stage) and on the parameters of the model, i.e., market densities and cost parameters. For example, the total number of branches opened in territory k is $\frac{2\bar{n}-1}{2\bar{n}}B_k$, i.e., a fraction of the competitive branch behaviour.

However, the market structure is parameter-free in equilibrium: Each incumbent gets a market share of 50% in its own territories (measured either in terms of branches or in terms of profits), whereas the remaining 50% is shared equally among the entrants. This implies that one-half is a lower bound for the r-bank concentration ratio. Moreover, $HHI \in (1/4, 1/2)$.

To conclude the equilibrium analysis of the Branching Deregulation Game, we focus on the equilibrium behavior of a given banking institution at the industry level. Recall that an active bank i acts as incumbent in $|\mathcal{K}_i|$ territories and as entrant in $|\mathcal{K}\backslash\mathcal{K}_i|$ territories; we next compute the branch network size of banking institution i:

$$N_i = \sum_{k\in\mathcal{K}_i} \frac{2\bar{n}-1}{4\bar{n}}B_k + \sum_{k\in\mathcal{K}\backslash\mathcal{K}_i} \frac{2\bar{n}-1}{4\bar{n}^2}B_k = \frac{2\bar{n}-1}{4\bar{n}^2}\left((\bar{n}-1)\sum_{k\in\mathcal{K}_i} B_k + \sum_{k\in\mathcal{K}} B_k\right)$$

It is important to stress that the size of the branch network of the various banking institutions are not symmetric and that they mimic the market density distribution of territories, as the following proposition shows.

Proposition 2 *For two active banks i and j, if* $\sum_{k\in\mathcal{K}_i}\delta_k > \sum_{k\in\mathcal{K}_j}\delta_k$, *then the total number of branches opened by bank i (in all territories including its own ones) exceeds that of bank j.*

If we now identify higher deposit-market densities with wealthier territories, then Proposition 2 has an economically relevant interpretation: banking institutions from richer territories enjoy larger branch networks once deregulation is completed. This is actually caused by the advantage that incumbents have in terms of market share in their local markets (the incumbent effect).

[7] The CRr is the sum of the market shares of the r largest banks, and the HHI is computed as the sum of the squares of the market shares of all banks.

4 Mergers and acquisitions stage

Finally, we step back and analyze the M&A subgames (Stage 1). We will work with a large set of merger protocols which comply with two properties: (i) sequential process and (ii) mutual agreement. In our model, mergers take place sequentially in multiple rounds, one round for each merger. When a merger proposal is accepted the merger game proceeds to the next round, in which the number of active banks is reduced by one but the merged bank is larger and acts as incumbent in all the territories of its constituent banks.

Let i be an active bank and α_i denote the percentage of total market density in which bank i is incumbent, i.e.,

$$\alpha_i = \frac{\sum_{k\in\mathcal{K}_i} \delta_k}{\sum_{k\in\mathcal{K}} \delta_k}$$

We can compute the *reservation values* in any round of the M&A stage with configuration $\{n, (\alpha_i)_{i=1}^n\}$ as the banks' profits in the associated incumbent-entrant game. To simplify further calculations we normalize by dividing by $r\sum_{k\in\mathcal{K}} \delta_k$. This yields the so-called normalized profits which, given Proposition 1, are:[8]

$$\pi(n,\alpha_i) = \frac{1}{r\sum_{k\in\mathcal{K}} \delta_k}\left(\sum_{k\in\mathcal{K}_i} \frac{\delta_k r}{4(n-1)} + \sum_{k\in\mathcal{K}\setminus\mathcal{K}_i} \frac{\delta_k r}{4(n-1)^2}\right) = \frac{1+(n-2)\alpha_i}{4(n-1)^2}$$

We now review some of the properties of the reservation values.

Remark 1 *We note that*

(i) $\pi(n,\alpha_i)$ *is increasing in* α_i *and decreasing in* n*,*

(ii) the duopoly profits are $\pi^d = \pi(2,\alpha_i) = 1/4$*, independent of* α_i*,*

(iii) $\sum_i \pi(n,\alpha_i) = \frac{1}{n-1}2\pi^d$*,*

(iv) for each $n > 2$*,* $\frac{1}{(n-1)^2} < \frac{\pi(n,\alpha_i)}{\pi^d} < \frac{1}{n-1}$ *and*

(v) $\pi(n,\alpha_i) - \pi(n,\alpha_j) = \frac{n-2}{(n-1)^2}(\alpha_i - \alpha_j)\pi^d$*.*

Point (i) states that the reservation value to a given bank is increasing in the percentage of total market density in which the bank is incumbent (this is the incumbent effect) and it is decreasing in the number of active banks n. The latter is actually the free-rider effect associated with mergers, first noted by Salant, Switzer and Reynolds

[8] Recall that $n = \bar{n} + 1$.

(1983) in Cournot competition. Also, the reservation value does not depend on the number of territories in which it is incumbent, but on the size of the fraction of the pie in which the bank acts as incumbent, α_i.

The reservation values are dependent of $(\alpha_i)_{i=1}^n$ except for the duopoly. In this case, each duopolist shares half of each territory and gets $\pi^d = 1/4$.[9] In fact, the duopoly profits appears as a natural bound for the reservation values in every merger round. The industry profits in any given round amounts to a fraction $1/(n-1)$ of the duopoly industry profit. Hence, as a whole, the industry benefits from reducing the number of banking institutions. Also, the dispersion of bank profits is of the order of $1/n$ at any given round.

With the help of the reservation values, a merger can be classified as either *naive* or *strategic*. A merger is *naive* if the reservation value of the merged bank $\pi(n-1, \alpha_i + \alpha_j)$ exceeds the sum of the reservation values of the two merging banks $\pi(n, \alpha_i) + \pi(n, \alpha_j)$. A naive merger is profitable per se, that is, it generates a positive surplus. A non-naive merger is termed *strategic* because if it happens in equilibrium, the merging institutions expect other mergers to happen in the future that render profitable their own merger.

The next lemma characterizes naive mergers.

Lemma 3 *The merger $i + j$ is naive if and only if $\alpha_i + \alpha_j \geq c(n)$, where $c(n) = \frac{n^2-6n+7}{n^2-5n+5}$.*

The analysis of the threshold $c(n)$ forms the content of next corollary:

Corollary 3 *(i) $c(n) < 0$ if and only if $n < 5$, (ii) $c(n)$ is increasing in n, and (iii) for every n, $c(n) < 1$.*

The first point in this corollary implies that for a small number of banks $(n < 5)$ all possible mergers are naive. And although $c(n)$ is increasing in the number of banks, it never reaches unity what implies that for any number of active banks, there are configurations for which there exist naive mergers.

Given the sequential nature of the merger stage, the strategy space even for a small number of banks and territory distribution $\{n, (\alpha_i)_{i=1}^n\}$ is a complex object. Also, the equilibrium concept to be used is that of subgame perfection, which renders more complicated the analysis. It is this complexity what have led other authors

[9] Recall that the incumbent gets a half of the market in which she acts as incumbent and that the other half is shared by the entrants. Given that in a duopoly there is only one entrant, the statement follows.

to confine the analysis to a small number of players so as to explicitly compute the subgame perfect equilibrium strategies.

In contrast, we will be able to prove general results concerning the equilibrium outcomes of the Branching Deregulation Game for sequential merger protocols with mutual agreement. As a first step, we prove that the existence of naive mergers triggers a complete merger wave.

Proposition 3 *If there exists at least one naive merger in initial configuration* $\{n, (\alpha_i)_{i=1}^n\}$*, then there is a complete merger wave in equilibrium.*

The above proposition does not say that in equilibrium all mergers will be naive. It simply states that the presence of a naive merger is a sufficient condition for a merger wave. We next prove an even stronger statement regarding the equilibrium outcomes of the M&A Stage.

Proposition 4 *In equilibrium, for every initial configuration* $\{n, (\alpha_i)_{i=1}^n\}$*, either there is no merger at all or a complete merger wave occurs.*

Let us summarize our findings thus far. Under the unique assumption of a sequential merger protocol with mutual agreement, we have shown that (i) in the branching deregulation game there are only two possible equilibrium outcomes: either no mergers at all or a complete merger wave, (ii) the existence of naive mergers is a sufficient condition for merger waves to occur, and (iii) for any n, there are initial configurations for which naive mergers exist.

Note that these results hinge on the existence of naive mergers, which by definition never yield losses. However, a recurrent issue in the literature is the presence of unprofitable mergers. The discussion of optimal unprofitable mergers would require a full description of the merger decision protocol.[10] A number of merger protocols have been proposed in the literature. They range from cooperative to non-cooperative analyses and from exogenous to endogenous merging proposals order. Qiu and Zhou (2007) and Gorton et al (2009) are two notable examples based on the non-cooperative approach to the merger issue which belong to the class of merger protocols we deal with in this paper.

In the merger protocol by Qiu and Zhou (hereafter QZ protocol), the proposal order is endogenous. At the beginning of each round, a bank is randomly selected as proposer. The proposer can either pass or offer a merger. If the proposer passes, a new bank is selected to act as merger proposer. If all active banks pass, the M&A

[10] The order in which firms decide on whether to merge; the allowed number of mergers; etc.

stage ends. If the proposer offers a price to acquire another bank (respondent), the latter can either accept or reject. If the respondent accepts, then the merger takes place (and the price is paid) and the merger game proceeds to the next round. If the respondent rejects, the M&A stage ends.

The merger protocol proposed by Gorton et al (hereafter GKR protocol) is more restrictive. Besides a fixed merger proposal order starting from the largest firm and ending with the smallest one they further assume that firms cannot acquire larger ones.[11]

We next analyse the existence of strategic mergers in equilibrium in our branching deregulation model under the QZ and GKR merger protocols. Note that we need to focus on the case $n \geq 5$ because Corollary 3 says that there are no strategic mergers if $n < 5$. We start considering in first place the QZ protocol. We prove an existence result of strategic mergers in equilibrium for an arbitrarily large number of banks.

Proposition 5 *Under the QZ merger protocol, there exists a configuration* $\{\widetilde{n}, (\alpha_i)_{i=1}^{\tilde{n}}\}$ *with* $\widetilde{n} \in [n, 2n+1]$ *whose merger wave is triggered by a strategic merger in equilibrium for each* $n \geq 5$*.*

The proof of the existence of strategic merger under the QZ protocol for arbitrarily large number of banks uses a particularly asymmetric distribution of bank sizes; that of a dominant bank and a fringe of small ones (see Appendix). The dominant bank is not large enough for a naive merger to exist, but as soon as one merger takes place, naive mergers (involving the dominant bank) are available. The proof proceeds by showing that two small banks will prefer to merge to each other if all other banks remain passive.

More symmetric configurations are harder to analyse under the QZ protocol. The reason is that the more symmetric configuration, the longer the chain of strategic mergers till a naive one is available. Since there is no preference for larger banks, then starting a race to increase size through mergers is highly risky as it might well be that the large bank is acquired before recovering its "investment".

An extreme position in protecting large banks is the one considered in the GKR protocol. There, large banks move first in the merger proposal and they cannot be acquired by small ones. We write down these properties.

[11] GKR also assume that there is at most one acquisition per firm. This assumption is restrictive. Thus, we shall exclude it from our analysis when studying the effects of incorporating some features of GKR protocol to our model (cf. Proposition 6).

Definition 2 ***Size protection.*** *A merger protocol protects large banks if a bank cannot acquire larger banks*

Definition 3 ***Size priority.*** *A merger protocol prioritizes large banks if the order of merger proposals in a given round goes from the largest to the smallest bank.*

Next proposition shows that in the QZ protocol with priority and protection of large banks, in equilibirum a complete merger wave occurs starting from any initial configuration.

Proposition 6 *In the QZ merger protocol with size protection and size priority, for any initial banking configuration* $\{n, (\alpha_i)_{i=1}^n\}$*, in equilibrium there is a complete merger wave.*

This proposition highlights one of the main points of the paper: starting from any bank distribution of sizes, there are gains to be realized through mergers and acquisitions. They can be easily realized if large banks are protected and have a priority in the merger proposal. Note that this is true for all bank configurations, regardless of the existence of naive mergers.

Remark 2 *The strategy proposed to prove Proposition 6 is not necessarily an equilibrium (see Appendix). Note that the large bank would have incentives to reach the duopoly by spending as less as possible in acquiring banks. In this vein, given the size protection and size priority, its optimal strategy would be to let others merge unless such merges might let it loose its dominant position. As we showed, in case no one merges, the largest bank has incentives to acquire banks by itself.*

For a precise description of equilibrium strategies for arbitrary bank distribution of sizes, we join the standard practice in the literature of confining the analysis to a small number of banks. This is the purpose of the next Section. We will use the more general QZ protocol for $n = 4$,[12] and we will show that some of the restrictive assumptions (size protection) and results (defensive mergers: to increase firm size to avoid being eaten) of Gorton et al (2009) arise as equilibrium outcomes applying the QZ protocol to our model.

[12] We expose the results of $n = 4$ because it is the simplest case that allows for a merger wave (in this case, a sequence of two mergers). More complex merger dynamics for the cases $n = 5$ and $n = 6$ (assuming initial symmetry, i.e., $\alpha_i = 1/n$ for each $i \in \{1, ..., n\}$) are available from the authors upon request.

5 Merger Dynamics

The paper so far has discussed a general case with n banks. In the following we analyze in detail the case of 4 banks ($n = 4$) to study the precise equilibrium dynamics under the QZ protocol.

Given the backwards induction arguments needed in the analysis of $n = 4$, we start analysing the case $n = 3$. Assume, without loss of generality, $\alpha_1 > \alpha_2 > \alpha_3$. Recall that the profits that a bank gets if a duopoly forms is not dependent on α_i (cf. Remark 1). Hence, if a duopoly arises both banks obtain the same profits, independent of the number (and density) of territories in which each of them act as incumbent.

This has an interesting implication: To reach to a duopoly is very attractive for banks. However, to this aim, they may need to acquire other banks, which is costly (the more costly the "larger" the acquired bank). This cost incurred to grow may be worthy in order to reach the duopoly, but the size the bank has acquired (the higher the more costly) does not have an effect in its final earnings. Thus a bank would prefer to reach a duopoly at the least cost (either because other banks are the ones who merge or because it acquires "small" banks).

Having said that, we now solve for the equilibrium strategy.

Proposition 7 *Assume the QZ protocol and let $n = 3$ and $\alpha_1 > \alpha_2 > \alpha_3$. In equilibrium,*

(i) *bank 1 passes, unless it is the last proposer, in which case it proposes bank 3,*

(ii) *bank 2 proposes bank 3, unless bank 3 has already passed and bank 1 has not yet proposed, in which case it passes, and*

(iii) *bank 3 proposes bank 2.*

From the above proposition, the following corollary follows:

Corollary 4 *In equilibrium, bank 1 stands alone and banks 2 and 3 merge to each other. The equilibrium profits to bank 1 are π^d and for each $i \in \{2,3\}$ and $j \in \{2,3\}\backslash\{i\}$ the expected profits to i are $\frac{1}{2}\pi^d + \frac{1}{2}\left(\pi\left(3,\alpha_i\right) - \pi\left(3,\alpha_j\right)\right)$.*

Interestingly, in equilibrium, the largest bank 1 is always too large to be eaten, showing that it is relative size rather than absolute size what matters for being safe. Thus, in our case, the "large" bank is safe from being acquired, not because it is exogenously imposed (as Gorton et al, 2009), but because the other banks have no incentives to acquire it. Thus, in the less restrictive QZ merger protocol, the two smaller banks are eager to acquire each other in order to avoid being acquired, and the big one safely reaches to the duopoly.

Step back and consider $n = 4$, which allows for 2 rounds of merger proposals. Assume, without loss of generality, $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$. We formalize the equilibrium strategy in the first round of proposals (the subsequent equilibrium behavior corresponds to the case $n = 3$ and it is described in the previous proposition)

Proposition 8 *Assume the QZ protocol and let $n = 4$ and $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$. In equilibrium, in the first round of proposals, for each $i \in \{1, 2, 3, 4\}$, bank i proposes bank j, with $j = \arg\min_{j' \in \{1,\ldots,4\} \setminus \{i\}} \alpha_{j'}$ such that $\alpha_i + \alpha_{j'} > \max\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$.*

Recall that we learnt from the case $n = 3$ that the largest bank goes to the duopoly for free (getting π^d without needing to acquire any bank in the second round of proposals). Thus, becoming the largest bank after the first round of proposals is attractive. Given that it is relative rather than absolute size what matters, then it follows that eating the smallest bank that allows becoming the largest one is the optimal behaviour.

Hence, the optimal behaviour in the first round of mergers displays a race for size, similar to the behaviour found by Gorton et al (2009), but of a different nature. The defensive mergers in Gorton et al (2009) comes from an agency problem (the disaligment of incentives between the shareholders and the manager), whereas in our model there are no such agency problems.

6 Discussion

This paper develops a stylized model that captures the geographic deregulation process of the U.S. banking industry. In this section we review their main assumptions and results and test the fit to the evidence from the U.S banking industry consolidation process. The data we use come from FDIC Summary of Deposits for the period 1985-2008; banks are defined as FDIC-insured commercial banks.

Figure 1 displays the evolution of the number of U.S. bank mergers.

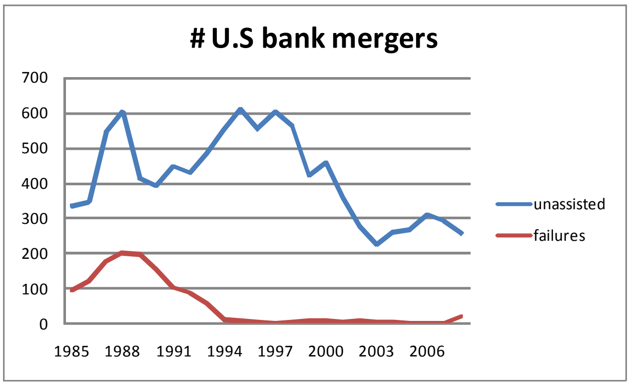

Figure 1. Evolution of U.S bank mergers

Several features are worth discussing as regards the two properties of the merging protocols considered in the model. First, merging activities span over a large period of time, although with clear peaks (1987-88) and 1994-1997. Hence, the sequential nature of the M&A protocol seems well taken (the average number of mergers is 488 per year). Also, the vast majority of mergers in the period are unassisted, that is, they did not required help from the government, and therefore they are not caused by variables outside the model (the exception happens in the second half of the 1980s, in which bank failures resulting in a merger due to economic crisis) but by mutual agreement of the parties.

Having verified the appropriateness of the class of the merging protocols, we next discuss our main conclusion: the existence of a complete merger wave. To this end, Figure 2 below displays the evolution of the number of banking institutions and number of branches for the period 1985-2008 (in thousands).

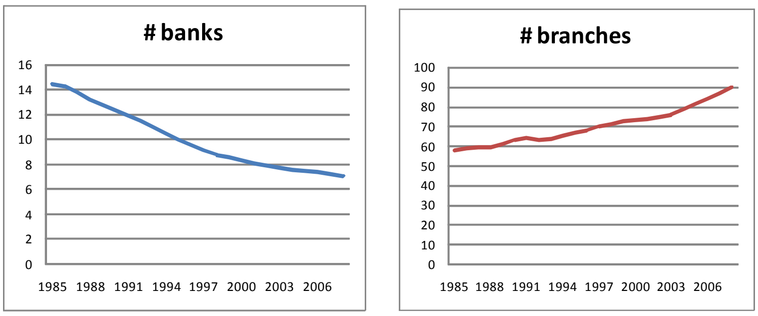

Figure 2. Evolution of number of U.S. banks and branches

The number of banks was 14.427 back in 1985 and 7.097 by the end of the period. Hence, a reduction of one half is obtained. Certainly, this is far from the complete merger wave predicted by the model. However, there is a missing feature in the model that can account for this discrepancy. Our model does not allow for banks entries. And in fact, the decrease of 7.330 banks was accomplished with 11.238 exits (mergers) and 3.908 entries. Hence, the merger wave was 77,89%, much more closer to the predicted 100% merger wave.

Even though the subgame perfect equilibrium outcome of the model is not observed in practice, the model yields precise predictions for all subgames, not only for those subgames belonging to the equilibrium path. We now review these predictions.

First, Propositions 4, 5 and 6 imply a reduction in the number of banking institutions (due to the presence of mergers waves) together with an increase in the number of branches (Proposition 1).[13] This is actually one of the main features of the evolution of the number of banks and branches in the U.S. as displayed in Figure 2.

Second, Proposition 2 captures the so-called incumbent effect: incumbents have an advantage in terms of branch share in their own markets. Given that the absolute number of branches depends on market densities, it follows that those banking organizations from richer markets are to be leading the ranking of top branch network. (see Jeon and Miller, 2003).

In third place, we deal with concentration ratios. The literature usually works with two measures: the *r-bank concentration ratio*, CRr, calculated as the deposit market share of the r largest banks in the market and the *Herfindahl-Hirschman index*, HHI, computed as the sum of the squares of the deposit market shares of all participantes in the market.

Table 1 displays the concentration ratios in 1997, computed at the market level, where a market is either a Metropolitan Statistical Area (MSA) or not (Non-MSA), together with the predicted value in parenthesis. The variable n refers to the average number of banks over markets.[14]

	CR1	CR3	HHI	n
MSA	0,26 (0,5)	0,66 (0,54)	0,15 (0,26)	26,8
NON-MSA	0,46 (0,5)	0,88 (0,75)	0,35 (0,31)	5,4

Table 1. Concentration ratios, year 1997.

As we see, these predictions are quite close to the observed concentration measures, specially for non-metropolitan markets. We believe that the discrepancy is mainly

[13] We can actually quantify the final number of branches in the industry $\frac{1}{2}\sum_k B_k$.

[14] There are 358 metropolitan markets and 1907 non-metropolitan markets. 75% of branches are in metropolitan markets.

related to the entry of new banks, a phenomenon which is more important in urban, metropolitan markets, which are more attractive for entry than rural (smaller) ones. (See Amel and Liang, 1997).

As regards the concentration at the industry level, we can compute the CRr in the regulated and post-regulation periods.[15] Let $\Delta = \sum_{k=1}^{k=K} \delta_k$. In the regulated period each bank acted as a local monopolist in its own local market, then the concentration ratio at the industry level is

$$CRr^R = \sum_{k=1}^{k=r} \frac{\delta_k}{\Delta}$$

where it is assumed that $\delta_1 \geq \delta_2 \geq ... \geq \delta_K$.

After deregulation, let d_i be the fraction of the sizes (market densities) of all the targets acquired by bank i with respect to the size of the acquiree. Then, the concentration ratio post-deregulation is as follows.[16]

$$CRr^P = \left(\frac{1}{2} + \frac{1}{2}\frac{r-1}{\bar{n}}\right) \frac{\sum_{k=1}^{k=r} (1+d_k)\,\delta_k^P}{\Delta} + \frac{r}{\bar{n}}\frac{1}{2}\left(1 - \frac{\sum_{k=1}^{k=r} (1+d_k)\,\delta_k^P}{\Delta}\right)$$

Note that there are two effects: The first summand reflects the loss of market share enjoyed by the r-largest banks when acting as incumbents (they lose half of their own markets). The second and third summands reflect the market shares gained by the r-largest banks when acting as entrants in other territories.

When the number of banks in the industry is large (large $\bar{n}$), the second effect vanishes and, therefore, the concentration ratio in the post-regulated period can be approximated as follows:

$$CRr^P \simeq \frac{1}{2}\frac{\sum_{k=1}^{k=r} (1+d_k)\,\delta_k^P}{\Delta}$$

We finally need to say something about the distribution of δ_k and δ_k^P, that is, the market shares of the $r-$largest banks before and after deregulation. As this regards, note that if merging activities do not involve the presence of large banks (banks which act as incumbents in rich markets), then it follows that

$$\frac{\sum_{k=1}^{k=r} (1+d_k)\,\delta_k^P}{\Delta} \simeq CRr^R$$

and

$$CRr^P \simeq \frac{1}{2} CRr^R$$

[15] The HHI is harder to work with since it involves the sum of the squares of the shares of all banks.

[16] Again assuming $\delta_1^P \geq \delta_2^P \geq ... \geq \delta_n^P$. Note that $\sum_{k=1}^{k=n} \delta_k^P = \sum_{k=1}^{k=K} \delta_k = \Delta$.

Notice that the concentration ratio would halve with respect to the regulated period.

On the contrary, if the $r-$largest banks are actively involved in the M&A process, then the concentration index would go up (through an increase in d_k) and even overcoming the concentration levels in the regulated period. Jeon and Miller (2003) actually reports the U-shaped pattern of all concentration ratios for $r = 5, 10, 20, 50$ and 100, falling in the 1980s and raising through the 1990s (the reported decrease in the $CR5$ and $CR10$ is 40%, quite close to the predicted value of 50%). This U-shaped patter is consistent with our model assuming the merger pattern in the U.S. banking industry, where large-banks were involved in mergers in the 1990s.

7 Conclusion

Although one of the main motives behind banking consolidation is the value maximization of the consolidated banks (such as economies of scale, economies of scope, and risk diversification), many empirical studies find a lack of economies of scale and scope in the U.S. banking industry. This paper attempts to shed light on this puzzle by developing a stylized theoretical model based on the main features of the geographic deregulation that the U.S. banking industry has experienced in the last decades. The deregulation process is modelled as an incumbent-entrant game (all banks are incumbents in some markets and entrants in others) with several markets, preceded by a "merger and acquisition" stage that (endogenously) determines which banks are incumbents in each territory.

Within this framework, the paper analyzes the banks' strategic responses to geographic deregulation. The main outcome of the model is that mergers are optimal responses and that in the absence of frictions associated to merger activities (financial constraints, legal constraints), the gains associated to mergers are so large that a complete merger wave is obtained in equilibrium. We prove that there are merger waves initiated for strategic reasons and not for value-maximization goals. This result may explain the merger waves observed in the sector despite the lack of evidence of scale and scope economies found in the empirical literature. Moreover, we show that the gains to be realized through consolidation are easily achieved if large banks are protected and have a priority in the merger protocol.

The main assumptions and results of the model are compared to real data from the U.S. banking industry in terms of number of bank mergers, number of banks, number of branches, and concentration ratios of the industry. The comparisons suggest that the proposed model offers a good approximation to the U.S. geographic deregulation process.

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Appendix

Proof of Lemma 1. Let the entrant i open e branches and the remaining $n-2$ entrants open m branches on aggregate. Then, entrant $i's$ profits in market k are

$$\Pi_{i,k} = \left(\frac{1}{I_k + e + m} - \frac{1}{B_k}\right)\delta_k re$$

The first order condition is

$$\frac{\partial \Pi_{i,k}}{\partial e} = \frac{m + I_k - \frac{1}{B_k}(m + e + I_k)^2}{(m + e + I_k)^2}\delta_k r = 0$$

By imposing symmetry, i.e., $m = e(\bar{n} - 1)$ and solving for e^* we get

$$e_k^*(I_k) = \frac{1}{2\bar{n}^2}\left(\bar{n}\left(1 - 2\frac{I_k}{B_k}\right) - 1 + \sqrt{(\bar{n}-1)^2 + 4\bar{n}\frac{I_k}{B_k}}\right)B_k$$

It is easy to show that the optimal number of branches opened by each entrant is null when the incumbent opens the competitive amount of branches B_k. $\square$

Proof of Lemma 2. The profits to incumbent i in territory k from opening I_k branches are

$$\Pi_{i,k} = \left(\frac{1}{I_k + \bar{n}e^*(I_k)} - \frac{1}{B_k}\right)I_k\delta_k r$$

The first order condition yields

$$\frac{\partial \Pi_{i,k}}{\partial I_k} = -4\bar{n}^2 \left(\frac{I_k}{B_k}\right)^2 \left(-2\bar{n} + 4\bar{n}\frac{I_k}{B_k} + 1\right) \delta_k r = 0$$

And solving for I_k^* we get

$$I_k^* = \frac{2\bar{n} - 1}{4\bar{n}} B_k. \ \square$$

Proof of Proposition 1. The proof follows from Lemmas 1 and 2. $\square$

Proof of Proposition 2. Banks i and j open the same number of branches in territories in $k \in \mathcal{K} \backslash (\mathcal{K}_i \cup \mathcal{K}_j)$. Hence, any difference in branches must stem from behavior in territories in $\mathcal{K}_i \cup \mathcal{K}_j$. In fact, this difference favors the incumbent of the territories with the higher aggregate density, as Proposition 1 states. $\square$

Proof of Lemma 3. The proof makes use of the definition of naive mergers.

$$\begin{aligned} \pi(n-1, \alpha_i + \alpha_j) &\geq \pi(n, \alpha_i) + \pi(n, \alpha_j) \\ \frac{1 + (n-3)(\alpha_i + \alpha_j)}{4(n-2)^2} &\geq \frac{2 + (n-2)(\alpha_i + \alpha_j)}{4(n-1)^2} \\ \alpha_i + \alpha_j &\geq \frac{n^2 - 6n + 7}{n^2 - 5n + 5}. \ \square \end{aligned}$$

Proof of Proposition 3. Let α^M be the size of the naive merger. Lemma 3 and Corollary 3 imply that there exists at least one naive merger in later rounds. This is so because by definition (i) the number of active banks grows smaller as mergers are consummated, and (ii) there are at least two banks for which their merger size is at least α^M. Once the presence of a naive merger is assured in any round of the M&A stage, it becomes clear that in equilibrium in any round there must be a merger, because at least two banks have an incentive to merge; these are the banks that can engage in a naive merger. $\square$

Proof of Proposition 4. Assume for the sake of contradiction that there exists a shorter merger wave. We then focus on the last merger. This merger, by Proposition 3 cannot be naive, otherwise it would trigger a merger wave and therefore could never be the last merger. But if the last merger is not naive, it cannot be part of any equilibrium, because the merged banks would be better off not engaging in the merger. $\square$

Proof of Proposition 5. Let $n \geq 5$ and $\widehat{n} = 2n$. Consider $\widehat{n}$ banks and banking configuration $\widehat{C} = \left\{\widehat{n}, \left(\widehat{\alpha}, \frac{1-\widehat{\alpha}}{\widehat{n}-1}, ..., \frac{1-\widehat{\alpha}}{\widehat{n}-1}\right)\right\}$. This configuration considers one bank with size $\widehat{\alpha}$ and each of the remaining $\widehat{n}-1$ banks with size $\frac{1-\widehat{\alpha}}{\widehat{n}-1}$. Let us also assume that $\widehat{\alpha}$ is such that $\widehat{\alpha}+\frac{1-\widehat{\alpha}}{\widehat{n}-1}=c(\widehat{n})$. Given that $c(\widehat{n})$ is larger than 0.79 for $\widehat{n}\geq 10$, this banking configuration assumes that there exists one large bank (with size $\widehat{\alpha}$) and a fringe set of small banks in the banking system (with aggregate size $1-\widehat{\alpha}$) and that naive mergers necessarily require the participation of the large bank.

Assume that there are no strategic mergers for any $\tilde{n}\in[n,\widehat{n}]$. Otherwise, the proposition would be trivially true. Our aim is to show a configuration with $\widehat{n}+1$ banks for which a merger wave is triggered by a strategic merger.

We focus on symmetric equilibria. There are three possible symmetric equilibrium strategies: (i) no bank passes, (ii) the largest bank passes but not the smallest ones, and (iii) the smallest banks pass but not the largest one. Let us not forget that only naive mergers occur in equilibrium, and therefore the target bank in any merger is the largest one for the smallest banks, and any small bank (we assume randomly picked) for the largest one. It is trivial to show that regardless of the equilibrium strategy, the probability that a small bank passes from one round to the next without being involved in any merger is $\frac{\widehat{n}-2}{\widehat{n}-1}$.

Given that any naive merger gives rise to another highly symmetric configuration of the same sort, i.e., one larger bank and the small ones, again with size $\frac{1-\widehat{\alpha}}{\widehat{n}-1}$, we easily compute the probability that a small bank survives t rounds without being involved in mergers: $\frac{\widehat{n}-1-t}{\widehat{n}-1}$.

Let us now consider the equilibrium expected payoff for a small bank in configuration $\widehat{C}$. There are three possible events in a given round: (i) the bank is acquired by the large bank, (ii) the bank acquires the large bank, and (iii) the bank passes to the next round with no involvement in a merger. Any history up to any given round will consist of a succession of these three event types, one for each round.

Of all possible histories up to a given round, the following plays a prominent role in our proof: a small bank surviving $\widehat{n}/2$ periods with no involvement in a merger. Note that this history has probability $\frac{\widehat{n}-1-\frac{\widehat{n}}{2}}{\widehat{n}-1}=\frac{1}{2}\frac{\widehat{n}-2}{\widehat{n}-1}$.

There are of course other possible histories, but they all have positive expected payoffs. We now show why this is so. Some histories involve being acquired in the very first round—the payoff is therefore positive because the bank is paid the reservation value. Other histories entail acquiring the large bank in the first round; the expected payoff attached to such an acquisition is positive because this merger is naive. The remaining histories begin with passing to the second round with no involvement in a merger. Again, this subset is composed of a history in which the bank is acquired in the second round (with a positive payoff), another history in which the bank acquires the large bank (again with a positive payoff), and finally the remaining histories in which the bank survives with no involvement in a merger.

This argument shows that the equilibrium expected payoff of a small bank in the original configuration is bounded by the expected payoff attached to the event "surviving $\widehat{n}/2$ periods." And we consider the worst-case scenario, being acquired and therefore receiving the reservation payoff, $\pi_{i+j}\left(\widehat{n}-\frac{\widehat{n}}{2},\frac{1-\widehat{\alpha}}{\widehat{n}-1}\right)$.

Consider now initial configuration $\widetilde{C}=\left\{\widehat{n}+1,\left(\widehat{\alpha},\frac{1-\widehat{\alpha}}{\widehat{n}-1},...,\frac{1-\widehat{\alpha}}{\widehat{n}-1},\alpha_a,\alpha_b\right)\right\}$ in which one of the small banks in $\widehat{C}$ is split into two smaller banks with sizes α_a and α_b such that $\alpha_a+\alpha_b=\frac{1-\widehat{\alpha}}{\widehat{n}-1}$. Notice that given that $c(n)$ is increasing in n, it happens that no merger is naive in this configuration.[17]

We claim that banks with sizes α_a and α_b find it profitable to merge in configuration $\widetilde{C}$. Note that the expected payoff in equilibrium of the merger in $\widetilde{C}$ is the equilibrium expected payoff of any small bank in configuration $\widehat{C}$, which we have shown to be bounded below by

$$\Gamma=\frac{\widehat{n}-2}{2\left(\widehat{n}-1\right)}\pi\left(\widehat{n}-\frac{\widehat{n}}{2},\frac{1-\widehat{\alpha}}{\widehat{n}-1}\right)=\frac{1}{4}\frac{2\widehat{n}^2-9\widehat{n}+6}{\left(\widehat{n}-2\right)\left(\widehat{n}-1\right)\left(\widehat{n}^2-5\widehat{n}+5\right)}$$

We need to show that this payoff is larger than the opportunity cost of the merger, which is

$$\Omega=\pi\left(\widehat{n}+1,\alpha_a\right)+\pi\left(\widehat{n}+1,\alpha_b\right)=\frac{1}{4}\frac{\left(\widehat{n}-3\right)\left(2\widehat{n}-3\right)}{\widehat{n}^2\left(\widehat{n}^2-5\widehat{n}+5\right)}$$

This is actually the case, as follows

$$\Gamma-\Omega=\frac{1}{4}\frac{\left(45\widehat{n}-18\right)+2\widehat{n}^2\left(3\widehat{n}-17\right)}{\widehat{n}^2\left(\widehat{n}-2\right)\left(\widehat{n}-1\right)\left(\widehat{n}^2-5\widehat{n}+5\right)}>0 \text{ for } \widehat{n}\geq 10$$

We therefore show that in configuration $\widetilde{C}$ "no merge" is not an equilibrium outcome. It then follows from Proposition 4 that a merger wave necessarily occurs, and given that there are no naive mergers, the claim follows. □

Proof of Proposition 6 Consider initial configuration $\{n,(\alpha_i)_{i=1}^n\}$. We will prove that no merger is not equilibrium. In case $n<5$, $c(n)<0$ and, by Lemma 3, all possible mergers are naive. Thus, by Proposition 3, a complete merger wave occurs.

Hence, assume $n\geq 5$ and assume, without loss of generality, that $\alpha_1\geq\alpha_2\geq...\geq\alpha_n$. Clearly, since $\sum_{i=1}^{i=n}\alpha_i=1$, for each $x\leq n$, $\alpha_x\leq\frac{1}{x}$. Consider the following strategy for bank 1: In the first round of merger proposals acquire bank 3 and, then in each subsequent round of proposals in which there are $x\leq n-1$ active banks, acquire bank x. Stop acquiring banks when there are 4 banks left (including bank 1). Note that, since there is size protection and size priority, this strategy is available to

[17]Note that the size of the largest merger is $\widehat{\alpha}+\frac{1-\widehat{\alpha}}{\widehat{n}-1}=c(\widehat{n})<c(\widehat{n}+1)$.

bank 1. This strategy allows bank 1 to reach a situation with 4 active banks: banks 1, 2, 4 and n. Since when there are 4 active banks all possible mergers are naive, size priority and size protection allows bank 1 to reach the duopoly without acquiring any further bank: Since the size of bank 1 is at least $\alpha_1+\alpha_3+\alpha_5$, which exceeds $\alpha_2+\alpha_4$ bank 1 can just pass in the last two rounds of merger proposals (size priority), no one will acquire bank 1 (size protection) and the other banks will merge until the duopoly (naive mergers).

To prove the proposition it suffices to show that the proposed strategy provides bank 1 with a higher payoff than if there is no merger at all, in which case it earns $\pi(n,\alpha_1)$.[18] The cost (of acquiring banks) for bank 1 if it follows the proposed strategy is:

$$\pi(n,\alpha_3)+\sum_{x=5}^{n-1}\pi(x,\alpha_x)$$

Since $\alpha_x \leq \frac{1}{x}$ and $\pi(x,\alpha_x)$ is increasing in α_x,

$$\sum_{x=5}^{n-1}\pi(x,\alpha_x) < \sum_{x=5}^{n-1}\pi\left(x,\frac{1}{x}\right) = \sum_{x=5}^{n-1}\frac{2}{(x-1)\,x}\pi^d$$

Hence, since $\pi(n,\alpha_1)+\pi(n,\alpha_3)=\frac{2+(n-2)(\alpha_1+\alpha_3)}{(n-1)^2}\pi^d$, it suffices to show that

$$\frac{2+(n-2)(\alpha_1+\alpha_3)}{(n-1)^2}\pi^d+\sum_{x=5}^{n-1}\frac{2}{(x-1)\,x}\pi^d<\pi^d$$

Rearranging terms,

$$\sum_{x=5}^{n-1}\frac{2}{(x-1)\,x}<1-\frac{2+(n-2)(\alpha_1+\alpha_3)}{(n-1)^2}$$

Since the RHS of the inequality is increasing in n (recall that $n\geq 5$) and decreasing in $\alpha_1+\alpha_3$, it suffices to show that

$$\sum_{x=5}^{n-1}\frac{2}{(x-1)\,x}<1-\frac{2+(5-2)}{(5-1)^2}=\frac{11}{16},\text{ i.e. }\sum_{x=5}^{n-1}\frac{1}{(x-1)\,x}<\frac{11}{32}$$

We show that this is the case

$$\sum_{x=5}^{n-1}\frac{1}{(x-1)\,x}<\sum_{x=5}^{n-1}\frac{1}{(x-1)^2}=\sum_{x=4}^{n-2}\frac{1}{x^2}<\sum_{x=4}^{\infty}\frac{1}{x^2}=\frac{\pi^2}{6}-\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}\right)<\frac{11}{32}$$

[18] Note that it would not imply that the proposed strategy constitutes an equilibrium, but that there must necessarily be a merger wave, since otherwise the largest bank would have incentives to acquire other banks.

where we have used Euler's sum of the reciprocals of the squares of the natural numbers $\sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{\pi^2}{6}$. $\square$

Proof of Proposition 7. We follow a backwards induction argument. Note that there is only one round of merger proposals. Consider first that two banks have passed in the sequence of mergers proposal. Then the optimal action of the remaining bank, say i, is to acquire the smallest rival bank, say j (note that $j = 3$ if $i \in \{1, 2\}$ and $j = 2$ if $i = 3$), at price $\pi(3, \alpha_j)$; and j accepts. To see it note that, for each $\alpha, \alpha' \in (0, 1)$, $\alpha > \alpha'$, $\pi(3, \alpha) > \pi(3, \alpha')$ and $\pi^d > 2\pi(3, \alpha)$ (cf. Remark 1).

Thus, let us go to the previous proposal opportunity. It is clear that if the proposer is bank 1, it should pass (since this let her reach the duopoly without incurring in the cost to acquiring any bank). If the proposer is bank 3, it should acquire 2 (since, if it passes, it will be acquired in the last step by the remaining proposer). If the proposer is bank 2, her optimal choice depends on whether 3 has already received the proposal opportunity: If 3 has already passed, then 2 should pass ("forcing" 1 to acquire 3 in the last step and, therefore, reaching the duopoly without incurring in any cost); and if 3 has not acted yet, then 2 should acquire 3 (since if it passes, it will be acquired by 3 in the last step).

Finally, we analyze the first proposal opportunity. As in the previous case it is clear that, if the proposer is bank 1, it passes (and reachs the duopoly). If the proposer is $i \in \{2, 3\}$, it optimally acquires $j \in \{2, 3\} \backslash \{i\}$ at price $\pi(3, \alpha_j)$ and j accepts (since, otherwise, i will be acquired by j as soon as it receives the proposal opportunity). $\square$

Proof of Proposition 8. We follow a backwards induction argument. If there is no merger, each bank gets $\pi(4, \alpha_i)$. In case there is a merger in the first round of merger proposals, we denote by $\alpha_a > \alpha_b > \alpha_c$ the resulting post-merger configuration (with $\{a, b, c\} \subseteq \{1, 2, 3, 4\}$). In such a case the equilibrium outcome (including the optimal behavior in the second round of proposals) is described in Proposition 7. Hence, we focus on the first round of merger proposals.

Consider first that three banks have passed in the sequence of mergers proposal. Let us study the optimal action of the remaining bank, say i: if i passes its (normalized) profits are $\pi(4, \alpha_i) < \frac{1}{3}\pi^d$ (cf. Remark 1). If i acquires a bank j (paying its reservation value $\pi(4, \alpha_j)$) its profit depends on whether i becomes a, b or c for the next round of merger proposals (by using Lemma 7 and Remark 1):

- If i becomes a, its payoff is $\pi^d - \pi(4, \alpha_j) > \pi^d - \frac{1}{3}\pi^d = \frac{2}{3}\pi^d$.

- If i becomes b, its payoff is $\frac{1}{2}\pi^d + \frac{1}{2}(\pi(3, \alpha_b) - \pi(3, \alpha_c)) - \pi(4, \alpha_j) < \frac{1}{2}\pi^d +$

$\frac{1}{16}\pi^d < \frac{2}{3}\pi^d$.[19]

- If i becomes c, its payoff is $\frac{1}{2}\pi^d + \frac{1}{2}\left(\pi\left(3, \alpha_c\right) - \pi\left(3, \alpha_b\right)\right) - \pi\left(4, \alpha_j\right) < \frac{1}{2}\pi^d$ (since $\pi\left(4, \alpha_j\right) > 0$ and $\pi\left(3, \alpha_c\right) \leq \pi\left(3, \alpha_b\right)$).

Hence, i acquires the smallest bank that allows her to become bank a in the next round, i.e. $j = \arg\min_{j' \in \{1,\ldots,4\} \setminus \{i\}} \alpha_{j'}$ such that $\alpha_i + \alpha_{j'} > \max\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$. In this way i becomes the largest bank at the smallest cost.

Consider now that we are in a previous proposal opportunity (within the first round), and let i' be the proposer. If i' passes, then given the previous reasoning, either it will be acquired (earning its reservation value $\pi\left(4, \alpha_i\right) < \frac{1}{3}\pi^d$) or it will become b or c in the next round, earning at most $\frac{1}{2}\pi^d + \frac{1}{2}\left(\pi\left(3, \alpha_b\right) - \pi\left(3, \alpha_c\right)\right) < \frac{2}{3}\pi^d$. Hence it is preferable to acquire a bank and become a. In this case, i' also chooses the smallest bank that allows her to become a in the next round. Hence, in equilibrium, the first bank that receives a proposal opportunity, immediately acquires $\hat{\jmath} = \arg\min_{\hat{\jmath}' \in \{1,\ldots,4\} \setminus \{i\}} \alpha_{\hat{\jmath}'}$ such that $\alpha_{i'} + \alpha_{\hat{\jmath}'} > \max\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ paying the reservation value $\pi\left(4, \alpha_{\hat{\jmath}}\right)$. $\square$

[19] Note that $\pi\left(4, \alpha_j\right) > 0$ and, since $\alpha_b - \alpha_c < \frac{1}{2}$, $\pi\left(3, \alpha_b\right) - \pi\left(3, \alpha_c\right) = \frac{1}{4}(\alpha_b - \alpha_c)\pi^d < \frac{1}{8}\pi^d$ (cf. Remark 1).