

# A structural analysis of the inflation moderation\*

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## Abstract

U.S. inflation has experienced a great moderation in the last two decades. This paper examines the factors behind this and other stylized facts, such as the weaker correlation of inflation and the nominal interest rate (Gibson paradox). Our findings point at lower exogenous variability of supply-side shocks and, to a lower extent, structural changes in money demand, monetary policy, and firms' pricing behavior as the main driving forces of the changes observed in recent U.S. business cycles.

Keywords: DSGE monetary model, inflation moderation, structural changes.

JEL codes: E32, E47.

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# 1 Introduction

Since 1995, there has been a great moderation of U.S. inflation, characterized by a low average rate and mild fluctuations around it (see Figure 1). This is particularly striking when compared to high average inflation and strong volatility observed in the three decades before 1995. The inflation moderation might be connected to changes in other economic variables. Indeed, the Fed funds rate also displays low levels and low variability after 1995. The decline in volatility is also found in the rate of growth of some monetary aggregates.<sup>1</sup>

Apart from changes in volatilities, other statistics measuring cyclical correlation and persistence have also shifted in the post-1995 period. To illustrate these changes, Figure 2 shows dynamic correlation functions of inflation and the nominal interest rate computed from an empirical VAR using U.S. quarterly data.<sup>2</sup> A comparison across different samples helps us to highlight three additional important changes of inflation and the nominal interest rate dynamics across periods: (i) inflation persistence is much lower since 1995, (ii) the positive correlation between inflation and the nominal interest rate observed in the first period almost vanishes (i.e. Gibson paradox) in the most recent period,<sup>3</sup> (iii) the Fed funds rate seems to anticipate inflation movements by 10 quarters in the most recent period but not before 1995. The first two additional stylized facts were recently uncovered by Cogley, Sargent and Surico (2012), CSS (2012) from now on.

This paper considers an extended version of the canonical DSGE model (Smets and Wouters,

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<sup>1</sup>This stylized fact holds for some definitions of money such as Money with Zero Maturity (MZM) and Divisia recently used in the monetary economics literature (Altig, Christiano, Eichenbaum, Linde, 2011; McCallum and Nelson, 2010), but does not hold for some others (for example, M1).

<sup>2</sup>A comprehensive analysis of the actual vector autocorrelations of the four variables included in the VAR (inflation, nominal interest rate, nominal money growth and output growth rates) is provided below. See Hamilton (1994, pp. 264-266) for a derivation of the analytical expressions of the vector autocorrelation functions. Fuhrer and Moore (1995) and Ireland (2003) are two prominent papers in this literature using unconstrained VAR to summarize data features in this way. In contrast to these papers, we consider output growth instead of a measure of detrended output to characterize the dynamic comovement between economic activity and the three nominal variables included in the VAR.

<sup>3</sup>The term Gibson paradox was coined by Keynes (1930) in honor of A. H. Gibson who detected historical episodes during the Gold standard period where nominal interest rates were positively correlated with the aggregate price level, but a weak/null correlation with inflation, which contradicts conventional monetary theory. More recently, many papers (Friedman and Schwartz, 1982; Barsky, 1987; Barsky and Summers, 1988; Cogley, Sargent and Surico, 2012; among others) have revisited the Gibson paradox.

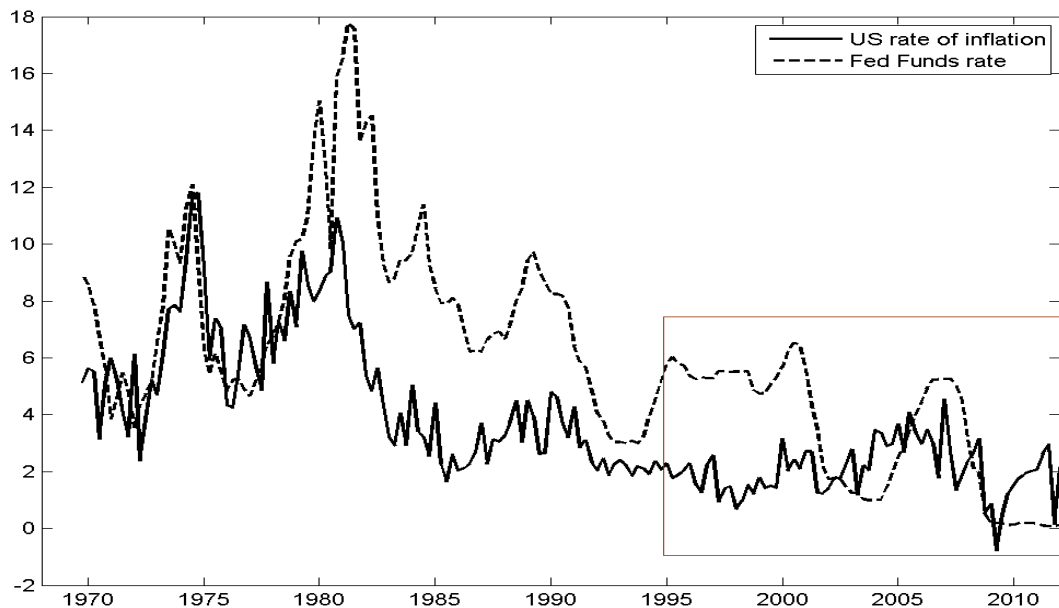


Figure 1: U.S. inflation and nominal interest rates, 1968-2011.

2007) to understand the underlying forces explaining these changes in U.S. business cycles.<sup>4</sup> The extended version of the model incorporates a transactions-facilitating role of money, and a variant of the Taylor rule, which allows for the partial reaction of the nominal interest rate to changes in nominal money growth.

Following Ireland (2003), Canova and Ferroni (2012) and CSS (2012), we estimate the DSGE model with money in two different sub-sample periods, which allows to identify the sources of the changing patterns.<sup>5</sup> In particular, our attention is focused on explaining the business cycle dynamics of output growth, inflation, nominal interest rate and nominal money growth. CSS (2012) do it with a highly stylized four-equation model. However, both our model and those of Ireland (2003) and Canova and Ferroni (2012) estimate medium-scale DSGE models, which in particular incorporate price/wage indexation schemes, consumption habits, variable capital utilization and adjustment

<sup>4</sup>As discussed below, this extension is mainly motivated because the cash-less model of Smets and Wouters (2007) fails to reproduce some observed dynamic shifts such as the Gibson paradox.

<sup>5</sup>Both Ireland (2003) and Canova and Ferroni (2012) focus their attention on the changing patterns observed around 1980. Meanwhile, CSS (2012) and this paper focus on the changing patterns observed since 1995.

costs for changes in investment. These sluggishness mechanisms improve the fitting of sticky-price models to regularities found in actual business cycle fluctuations (see Kimball, 1995; King and Watson, 1996; Casares and McCallum, 2000; among others). Moreover, Ireland (2003) has shown that using data on both consumption and investment in addition to output, as opposed to only using output in CSS (2012), helps to estimate adjustment cost parameters for both capital and prices. In addition, CSS (2012) take the great inflation period (1968-1983) as the first subsample, whereas ours covers the period 1968-1994. The reason is that by estimating a longer subsample that also includes the disinflation period (1984-1994) the comparison between the pre-1995 and the post-1995 periods is not affected. We also extend our second subsample in order to cover the most recent period after the subprime mortgage crisis, which is ignored in CSS (2012).

The estimation results show that the structural parameter estimates are fairly stable across the two sample periods studied. However, many parameters describing the shock processes have changed significantly across periods. We assess model's performance by reporting key second-moment statistics obtained from both actual US data and synthetic data. The model does a good job in capturing the volatility reduction of inflation, the nominal interest rate and the growth of nominal money since 1995. The model also reproduces the mild increase of the inertia in both nominal interest rates and the nominal money growth and the lower persistence of inflation in this period. Moreover, the model performs well in replicating the comovement of inflation with the nominal interest rate, nominal money growth and output growth rates. In particular, the model explains the re-emergence of the Gibson Paradox in the post-1995 period. The simulations carried out in the paper show that a lower volatility in price mark-up shocks and less persistent in wage mark-up shocks are the two main factors that explain the inflation moderation observed after 1995. In addition, price stickiness rises, the sensitivity of money demand to the nominal interest rate increases and the Fed exerts less aggressive and slower responses to changes in inflation, the output gap and nominal money growth.

The rest of the paper is organized as follows. Section 2 describes a DSGE-style model that incorporates both money demand and monetary policy behavior. Section 3 discusses the estimation strategy and provides the empirical results. Section 4 analyzes model's performance by comparing actual and simulated second-moment statistics. Based on the estimates of model parameters obtained from the two sub-samples, Section 5 illustrates the changes in the structural equations

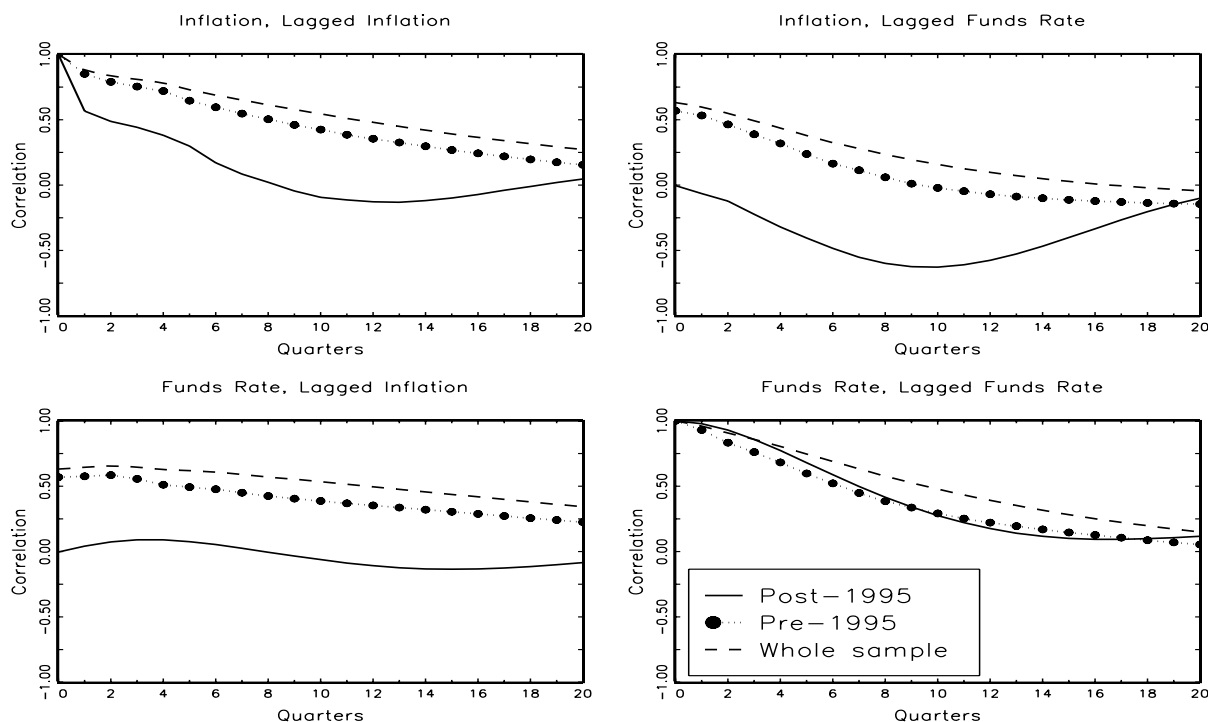


Figure 2: Vector autocorrelation and cross-correlation functions from US inflation and nominal interest rates.

from 1995 onwards. In addition, this section runs a large set of counterfactual exercises to assess the relative importance of key model elements to explain the swing of business cycle patterns. Finally, Section 6 concludes.

## 2 A DSGE model with money

This paper considers a modified version of the Smets and Wouters (2007) model to introduce money. The role of money is defined by its specific function: being the medium-of-exchange to carry out transactions. Thus, a transactions technology is presented where the stock of real money can be used to save transaction costs.<sup>6</sup> Any increase in real money holdings has a negative impact on transactions costs, with decreasing marginal returns. Meanwhile, money is supplied by the central

<sup>6</sup>The introduction of money through a transaction cost technology instead of a money-in-the-utility function specification used by Ireland (2003) and Canova and Ferroni (2012) is empirically motivated. The former approach is somewhat more flexible than the latter to accommodate the observed dynamic shifts.

bank to support the implementation of a Taylor (1993)-style stabilizing monetary policy rule.

Households maximize intertemporal utility that is non-separable between consumption and labor as in Smets and Wouters (2007). There is also an external consumption habit component and a consumption preference shock. Meanwhile, the budget constraint incorporates (real) spending on transaction costs, and the possibility of using savings for a net increase in real money balances. Based on the monetary model of Casares (2007), let us consider the following transactions technology for a  $j$  representative household

$$H_t(j) = a_0 + a_1 C_t(j) \left( \frac{C_t(j)}{\exp(\varepsilon_t^X) \left( \frac{M_t(j)}{P_t} - \lambda_m \frac{M_{t-1}}{P_{t-1}} \right)} \right)^{\frac{a_2}{1-a_2}}, \quad (1)$$

where  $H_t(j)$  is the amount of real income required to cover the transaction costs of a household that consumes  $C_t(j)$  and holds the amount of real money  $\frac{M_t(j)}{P_t}$ . The parameters of the transactions technology function satisfy  $a_0, a_1 > 0$ ,  $0 < a_2 < 1$ ,  $0 < \lambda_m < 1$ , while  $\varepsilon_t^X$  is a money-augmenting AR(1) shock.<sup>7</sup> As a distinctive characteristic from Casares (2007), there is (external) monetary habits that measure endogenous inertia on the demand for real money. The first-order conditions bring about the money demand equation

$$-H_{\frac{M_t(j)}{P_t}} = \frac{R_t}{1 + R_t}, \quad (2)$$

that equates the marginal return of monetary services,  $-H_{\frac{M_t(j)}{P_t}}$ , to the discounted value of the nominal interest rate,  $\frac{R_t}{1+R_t}$ , as the marginal (opportunity) cost of money holdings. The fluctuations of the marginal service of transactions-facilitating money can be taken from the partial derivative of (1) and substituted in the optimality condition (2) to yield the semi-log real money demand equation

$$m_t = (\lambda_m/\gamma) m_{t-1} + (1 - \lambda_m/\gamma) c_t - \frac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}} R_t - a_2 (1 - \lambda_m/\gamma) \varepsilon_t^X, \quad (3)$$

where  $m_t$  and  $c_t$  denote respectively log fluctuations of real money and consumption with respect to their steady-state levels,  $\gamma$  is the steady state output growth and  $R^{ss}$  is the steady-state nominal interest rate. The algebra involved is shown in the appendix.

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<sup>7</sup>As shown in the Appendix, the partial derivatives of the transaction costs function imply the desirable properties:

$$H_{C_t(j)} > 0, H_{C_t(j)C_t(j)} > 0, H_{\frac{M_t(j)}{P_t}} < 0, H_{\frac{M_t(j)}{P_t} \frac{M_t(j)}{P_t}} > 0 \text{ and } H_{C_t(j) \frac{M_t(j)}{P_t}} < 0.$$

Transaction costs and money also affect the decision of how much to consume. On the one hand, consumption is costly in terms of the transaction cost required to be able to carry out the purchases of goods (transaction costs rise with the level of consumption in the transactions technology function). On the other hand, real money facilitates consumption as it can be used to save some transaction costs through the entry in the denominator of the transactions technology. Taking into account both effects results in the following IS curve with real-money balance effects (see the appendix for details)

$$(1 + c_4) c_t = c_1 c_{t-1} + c_2 E_t c_{t+1} + c_3 (l_t - E_t l_{t+1}) + c_4 \left( \frac{1}{1 - \lambda_m / \gamma} m_t - \frac{\lambda_m / \gamma}{1 - \lambda_m / \gamma} m_{t-1} \right) - c_4 \left( \frac{1}{1 - \lambda_m / \gamma} E_t m_{t+1} - \frac{\lambda_m / \gamma}{1 - \lambda_m / \gamma} m_t \right) - c_5 (R_t - E_t \pi_{t+1}) + c_6 \varepsilon_t^x + c_7 \varepsilon_t^b, \quad (4)$$

where  $l_t$  is log fluctuations of hours,  $R_t - E_t \pi_{t+1}$  is the real interest rate,  $\varepsilon_t^b$  is an AR(1) consumption preference shock, and the  $c_i$  coefficients depend upon the structural parameters. The consumption equation of Smets and Wouters (2007) is the particular case  $c_4 = 0$  that comes in when dropping the marginal transaction cost of consumption.

The labor supply decision is also affected by the introduction of transaction costs and money. The marginal rate of substitution between hours and consumption receives an effect from the amount of real money holdings. Thus, a higher level of real money increases the shadow value of consumption because shopping is less costly. In turn, real money reduces the marginal rate of substitution between hours and consumption. In a log-linear approximation, we have

$$mrs_t = \sigma_l l_t + \left( \frac{1}{1 - \lambda / \gamma} + \frac{H_C a_2}{1 - a_2} \right) c_t - \frac{\lambda / \gamma}{1 - \lambda / \gamma} c_{t-1} - \frac{H_C a_2}{1 - a_2} \left( \frac{1}{1 - \lambda_m / \gamma} m_t - \frac{\lambda_m / \gamma}{1 - \lambda_m / \gamma} m_{t-1} \right) - \frac{H_C a_2}{1 - a_2} \varepsilon_t^x, \quad (5)$$

where  $mrs_t$  is the log deviation of the marginal rate of substitution with respect to the steady-state level, and  $H_C$  is the steady-state marginal transaction cost of consumption. In Smets and Wouters (2007), the case  $H_C = 0$  determines  $mrs_t$  in (5).

Finally, log fluctuations of transaction costs,  $h_t$ , appear in the log-linearized aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + h_y h_t + g_y \varepsilon_t^g, \quad (6)$$

where  $c_y = \frac{C}{Y} = 1 - g_y - i_y$ ,  $i_y = \frac{I}{Y} = (\gamma - 1 + \delta) \frac{K}{Y}$ ,  $z_y = r^k \frac{K}{Y}$ ,  $h_y = \frac{H}{Y}$ , and  $g_y = \frac{G}{Y}$  are steady-state ratios that provide the weights of spending on consumption, investment, variable capital utilization,

transaction costs, and exogenous spending. As in Smets and Wouters (2007), the component of exogenous spending  $\varepsilon_t^g$  is determined by an AR(1) process correlated with productivity innovations to collect possible fiscal or net exports shocks. The value of  $h_t$  is provided by the loglinearized transactions technology function

$$h_t = \frac{1-(a_0/H)}{1-a_2} \left( c_t - a_2 \left( \frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - a_2 \varepsilon_t^X \right). \quad (7)$$

Regarding the central-bank behavior, systematic monetary policy actions are governed by a Taylor (1983)-type rule extended with a stabilizing response to changes in the growth of nominal money,  $\mu_t = \log M_t - \log M_{t-1}$ . In addition, there is a smoothing component,  $0 < \rho < 1$ , that brings a partial adjustment between the previous nominal interest rate and the Taylor-style targeting as follows:<sup>8</sup>

$$R_t = \rho R_{t-1} + (1 - \rho)[r_\pi \pi_t + r_y (y_t - y_t^p) + r_\mu \mu_t] + \varepsilon_t^R, \quad (8)$$

where  $y_t$  and  $y_t^p$  denote output and potential output, respectively; and  $\varepsilon_t^R$  is a random disturbance following an AR(1) process with persistence parameter denoted by  $\rho_R$  and the standard deviation of the innovations of the AR(1) process denoted by  $\sigma_R$ .

The complete set of loglinearized dynamic equations of the DSGE monetary model and the set of structural parameters are available in the appendix.

### 3 Estimation results

The DSGE model with money has been estimated for two sub-samples of quarterly U.S. data, 1968:1 to 1994:4 and 1995:1 to 2011:3, taking the first quarter of 1995 as the starting period of the great moderation of inflation with low average rates and little variability. The choice of the sample split can be found by both simple inspection of the U.S. inflation time series plot (see Figure 1) and the changes in persistence and correlation patterns of inflation and the nominal interest rate

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<sup>8</sup>We have experimented with additional formulations for the policy rule, included the *hybrid* policy rule considered by Ireland (2003), where the central bank monitors monetary policy by adjusting a linear combination of the nominal interest rate and the money growth rate in response to output gap and inflation movements from their steady-state values. The qualitative results of the paper are robust to all policy specifications considered. The rationale for this empirical finding is simple: equation (8) can be viewed as an observational equivalent reduced-form of a wide range of alternative monetary rules where either the nominal interest rate or the nominal money growth is the instrument monitored by the central bank. Of course, the interpretation of the monetary policy shock changes accordingly.



shown in Figure 2. Interestingly, a more sophisticated method suggested in CSS (2012), based on a VAR with drifting coefficients and stochastic volatility, points out 1995 as the year when the Gibson paradox re-emerged and inflation persistence fell.

As a monetary model, one of the observable variables is the log difference of "Money with Zero Maturity" (MZM) calculated by the Federal Reserve Bank of St. Louis.<sup>9</sup> The rest of the observables corresponds to the list used in Smets and Wouters (2007): the inflation rate, the Federal funds rate, the log of hours worked and the log differences of the real GDP, real consumption, real investment and the real wage.

The estimation follows a two-step Bayesian procedure. In the first step, the log posterior function is maximized in a way that combines the prior information of the parameters with the empirical likelihood of the data. In a second step, we perform the Metropolis-Hastings algorithm to compute the posterior distribution of the parameter set.<sup>10</sup>

In terms of the priors, we select the same set of distributions as in Smets and Wouters (2007), which are shown in the first three columns of Tables 1A and 1B and where we have also borrowed their notation for the structural parameters. The introduction of transactions-facilitating money brings about a few additional parameters. The prior distribution of  $\lambda_m$ , the parameter that measures monetary habits, is identical to the one associated with consumption habits,  $\lambda$ , with the exception of the standard deviation, which is twice larger for  $\lambda_m$  reflecting our rather diffuse prior knowledge of this parameter. The prior distribution of the elasticity parameter in the transaction costs technology,  $a_2$ , is described by a beta distribution with mean 0.5 and standard deviation 0.2. The prior distribution of the policy parameter  $r_\mu$  is a normal distribution with mean 0.5 and standard deviation 0.2. Finally, the prior

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<sup>9</sup>As discussed in McCallum and Nelson (2010) and Altig, Christiano, Eichenbaum and Linde (2011), there are hybrid definitions of money such as MZM or Divisia more adequate for providing a representation of the medium-of-exchange role of money than more conventional aggregates such as the Monetary Base, M1 or M2. Moreover, the choice of MZM is empirically motivated from the analysis of the overall performance of alternative specifications of the medium-scale model in replicating the stylized facts described in the Introduction. These alternative specifications include money-in-the-utility function, alternative specifications of the policy rule and alternative definitions of money.

<sup>10</sup>All estimation exercises are performed with DYNARE free routine software, which can be downloaded from <http://www.dynare.org>. A sample of 250,000 draws was used (ignoring the first 20% of draws). A step size of 0.30 resulted in an average acceptance rate of roughly 31% (27%) in the estimation procedure of the first (second) sub-sample period.

distributions of the two parameters describing the money demand shock,  $\rho_\chi$  and  $\sigma_\chi$  are identical to the corresponding parameters describing the other shocks of the model. In addition to

Table 1A. Priors and estimated posteriors of the structural parameters

	Priors			Posteriors					
	Distr	Mean	Std D.	Pre-1995 model			Post-1995 model		
				Log density = -954.56	5%	95%	Log density = -481.65	5%	95%
$\varphi$	Normal	4.00	1.50	6.54	4.73	8.37	7.38	5.50	9.34
$\lambda$	Beta	0.70	0.10	0.73*	0.66	0.81	0.82*	0.74	0.90
$\lambda_m$	Beta	0.70	0.20	0.52	0.39	0.64	0.46	0.28	0.64
$\sigma_c$	Normal	1.50	0.37	1.75*	1.41	2.09	1.30*	0.98	1.61
$\sigma_l$	Normal	2.00	0.75	1.78	0.66	2.89	0.86	-0.15	1.90
$a_2$	Beta	0.50	0.20	0.77	0.70	0.85	0.75	0.68	0.82
$\xi_p$	Beta	0.50	0.10	0.62*	0.51	0.72	0.75*	0.66	0.84
$\xi_w$	Beta	0.50	0.10	0.73	0.62	0.84	0.74	0.64	0.83
$\iota_p$	Beta	0.50	0.15	0.34	0.14	0.53	0.28	0.11	0.45
$\iota_w$	Beta	0.50	0.15	0.61	0.43	0.82	0.44	0.19	0.66
$\psi$	Beta	0.50	0.15	0.37*	0.19	0.56	0.69*	0.52	0.86
$\Phi$	Normal	1.25	0.12	1.57*	1.44	1.70	1.41*	1.27	1.55
$\rho$	Beta	0.75	0.10	0.66*	0.57	0.75	0.79*	0.69	0.89
$r_\pi$	Normal	1.50	0.25	1.87	1.58	2.16	1.57*	1.19	1.95
$r_y$	Normal	0.12	0.05	0.16	0.09	0.22	0.09*	0.01	0.17
$r_\mu$	Normal	0.5	0.20	0.41	0.28	0.54	0.41	0.16	0.66
$\pi^{ss}$	Gamma	0.62	0.50	1.10**	0.81	1.40	0.53**	0.30	0.76
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	0.18	0.08	0.29	0.15	0.06	0.24
$\alpha$	Normal	0.30	0.05	0.20*	0.17	0.24	0.15*	0.11	0.18

Notes to Table 1: Table A in the appendix shows the definition for each estimated parameter. A double asterisk, \*\*, means that the two confidence intervals associated with a particular parameter do not overlap across sub-samples. Meanwhile, a single asterisk, \*, means that the estimated parameter does not lie inside the confidence interval obtained from the other sub-sample.

Table 1B. Priors and estimated posteriors of the shock processes

		Priors			Posteriors					
					Pre-1995 model			Post-1995 model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%	
$\sigma_a$	Invgamma	0.10	2.00	0.46	0.41	0.51	0.48	0.41	0.56	
$\sigma_b$	Invgamma	0.10	2.00	3.93	3.07	4.81	5.22*	3.35	7.06	
$\sigma_i$	Invgamma	0.10	2.00	0.59**	0.46	0.70	0.32**	0.24	0.41	
$\sigma_g$	Invgamma	0.10	2.00	0.54**	0.47	0.60	0.38**	0.32	0.43	
$\sigma_p$	Invgamma	0.10	2.00	0.15*	0.12	0.19	0.11*	0.08	0.14	
$\sigma_w$	Invgamma	0.10	2.00	0.18**	0.13	0.22	0.40**	0.33	0.48	
$\sigma_R$	Invgamma	0.10	2.00	0.43**	0.34	0.52	0.18**	0.10	0.26	
$\sigma_\chi$	Invgamma	0.10	2.00	5.48*	3.60	7.20	2.70*	1.74	3.75	
$\rho_a$	Beta	0.50	0.20	0.92	0.88	0.96	0.94	0.89	0.99	
$\rho_b$	Beta	0.50	0.20	0.14**	0.03	0.24	0.70**	0.55	0.85	
$\rho_i$	Beta	0.50	0.20	0.64**	0.54	0.76	0.88**	0.81	0.96	
$\rho_g$	Beta	0.50	0.20	0.90	0.81	0.98	0.87	0.78	0.96	
$\rho_p$	Beta	0.50	0.20	0.88	0.79	0.98	0.81	0.64	0.98	
$\rho_w$	Beta	0.50	0.20	0.87**	0.78	0.97	0.44**	0.18	0.71	
$\rho_R$	Beta	0.50	0.20	0.27**	0.14	0.40	0.60**	0.49	0.73	
$\rho_\chi$	Beta	0.50	0.20	0.93	0.87	0.99	0.96	0.92	0.99	
$\mu_p$	Beta	0.50	0.20	0.68	0.48	0.88	0.61	0.36	0.86	
$\mu_w$	Beta	0.50	0.20	0.59	0.37	0.82	0.47	0.22	0.72	
$\rho_{ga}$	Beta	0.50	0.20	0.58*	0.40	0.75	0.41	0.25	0.57	

the parameters fixed in Smets and Wouters (2007), the steady-state growth parameter,  $\gamma$ , was also fixed to the estimated value reported by them. The reason is that our empirical strategy splits up the period into two subsamples, which makes it harder to identify parameters characterizing long-run dynamics. Moreover, the scale parameter of transaction-cost technology,  $a_1$ , is calibrated to match the steady-state money velocity with the average ratio of nominal GDP/MZM over the whole sample period (1968-2011). The fixed transaction cost,  $a_0$ , is also predetermined at the value

that implies that the ratio of total transaction costs over consumption is equal to 0.01 in steady state.

Table 1 shows the estimation results by reporting the posterior mean values together with the 5% and 95% quantiles of the posterior distribution for the two sub-samples studied. Many of the estimates look rather stable across sub-samples. In particular, indexation parameters are rather similar in the two sub-samples whereas policy parameters  $\rho$  and  $r_\pi$  have only changed marginally. However, there are several noticeable differences. Thus, the consumption habit formation parameter,  $\lambda$ , and the price rigidity probability parameter,  $\xi_p$ , are higher in the after-1995 subsample. Meanwhile, the steady-state rate of inflation,  $\pi^{ss}$ , is much lower. Moreover, many of the estimates of shock processes change significantly across sub-samples. Thus, the persistence of preference, investment, and monetary policy shocks ( $\rho_b$ ,  $\rho_i$ , and  $\rho_R$ ) has increased in the recent period, whereas the opposite occurs for the persistence of wage indexation shocks ( $\rho_w$ ). Furthermore, the standard deviation of the innovations associated with most shocks have changed across sub-samples. Those corresponding to money demand, monetary policy, investment, government spending and price indexation shocks are higher in the first sub-sample than in the second whereas the opposite is true for consumption preference and wage indexation shocks.

## 4 Model performance

As described in the Introduction, business cycle dynamics have shifted in many ways during the great moderation of inflation. What of these changes can be captured by the estimated DSGE model extended with money? Table 2 reports changes in the standard deviations of key aggregate variables obtained from both actual U.S. data and synthetic data. The model does a great job in matching the standard deviations of inflation, nominal interest rate and nominal money growth in each period, which implies that the model explains very well the fall of volatility observed in the three nominal variables. Moreover, the model captures the lower volatility of output growth since 1995, though it generates greater output growth volatility than what it has been observed in actual data.

By using the vector autocorrelation function method, Figures 3 and 4 show a comprehensive analysis of model's performance based on dynamic auto-correlations and cross-correlations obtained

from actual and synthetic data across subsamples. A comparison of these two figures shows that in general the model fits well the two subsamples. More precisely, the model does a very good job in matching the shape of the serial correlation of output and nominal money growth. However, it falls short to characterize the persistence of inflation and the nominal interest rate in the pre-1995 period. The model also performs well in capturing the comovement of inflation with the nominal interest rate, nominal money growth and output growth. In particular, the model replicates the fall of the contemporaneous correlation between inflation and the nominal interest rate, which results in a weak correlation between the two variables (i.e. Gibson paradox) during the great moderation of inflation. Moreover, the model does fairly well when describing the low correlations of nominal money growth with the other three variables in the pre-1995 period. However, the model fails to capture the positive, although weak, correlation of the nominal interest rate with both output and money growth rates since 1995.

Table 2. The moderation of macroeconomic volatility.

	Pre-1995		Post-1995	
	US data	Model	US data	Model
Standard deviation, %				
$\sigma(\pi_t)$	0.61	0.65	0.23	0.38
		(0.51, 0.76)		(0.26, 0.47)
$\sigma(R_t)$	0.82	0.74	0.55	0.40
		(0.62, 0.84)		(0.31, 0.46)
$\sigma(\mu_t)$	2.17	2.30	1.41	1.55
		(1.97, 2.56)		(1.32, 1.80)
$\sigma(\Delta y_t)$	0.95	1.14	0.72	0.96
		(1.02, 1.23)		(0.84, 1.06)

Note: the posterior 5%-95% confidence interval for each estimated second-moment statistic is reported on parenthesis.

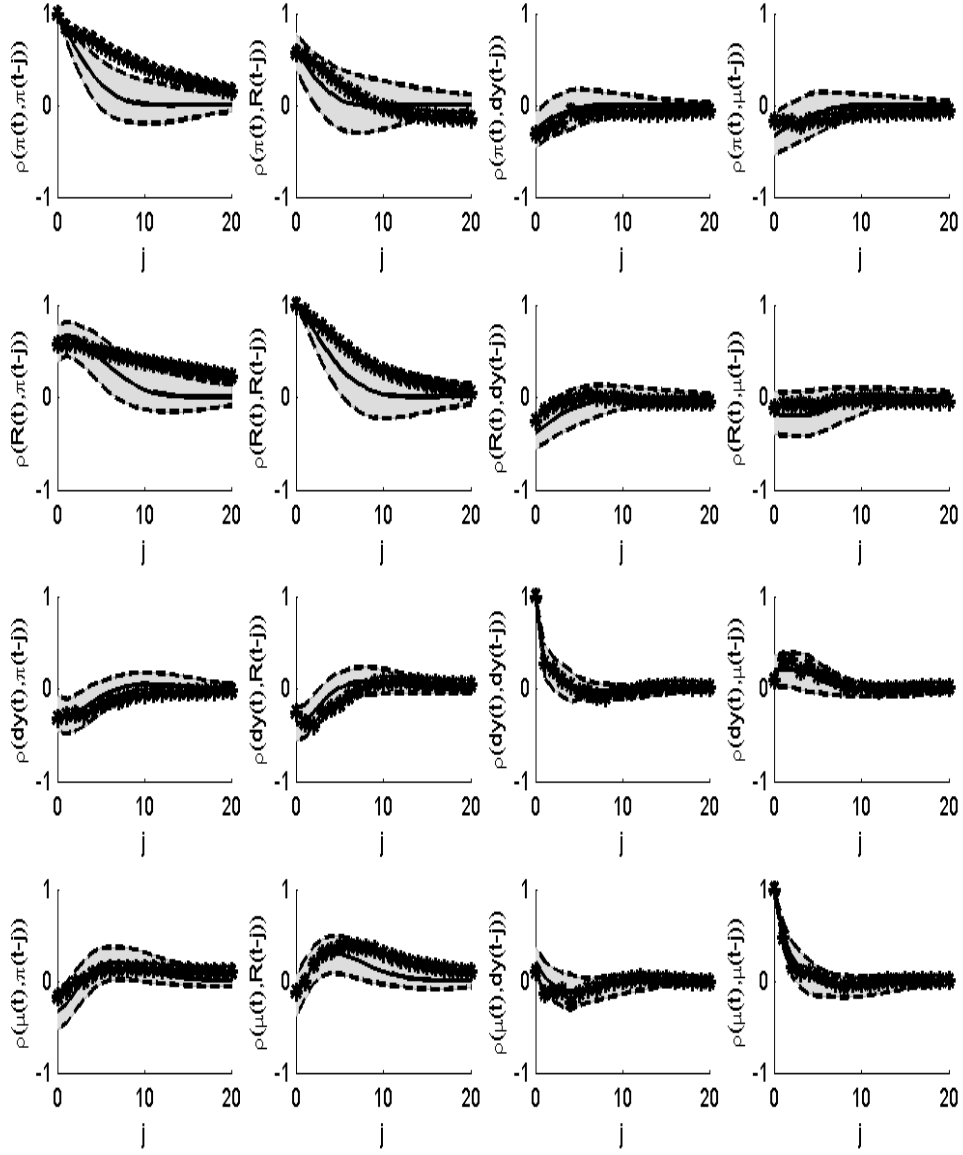


Figure 3: Dynamic correlations (1968-1994). US data (\*) and estimated model (solid lines with shaded area showing the 5%-95% confidence interval).

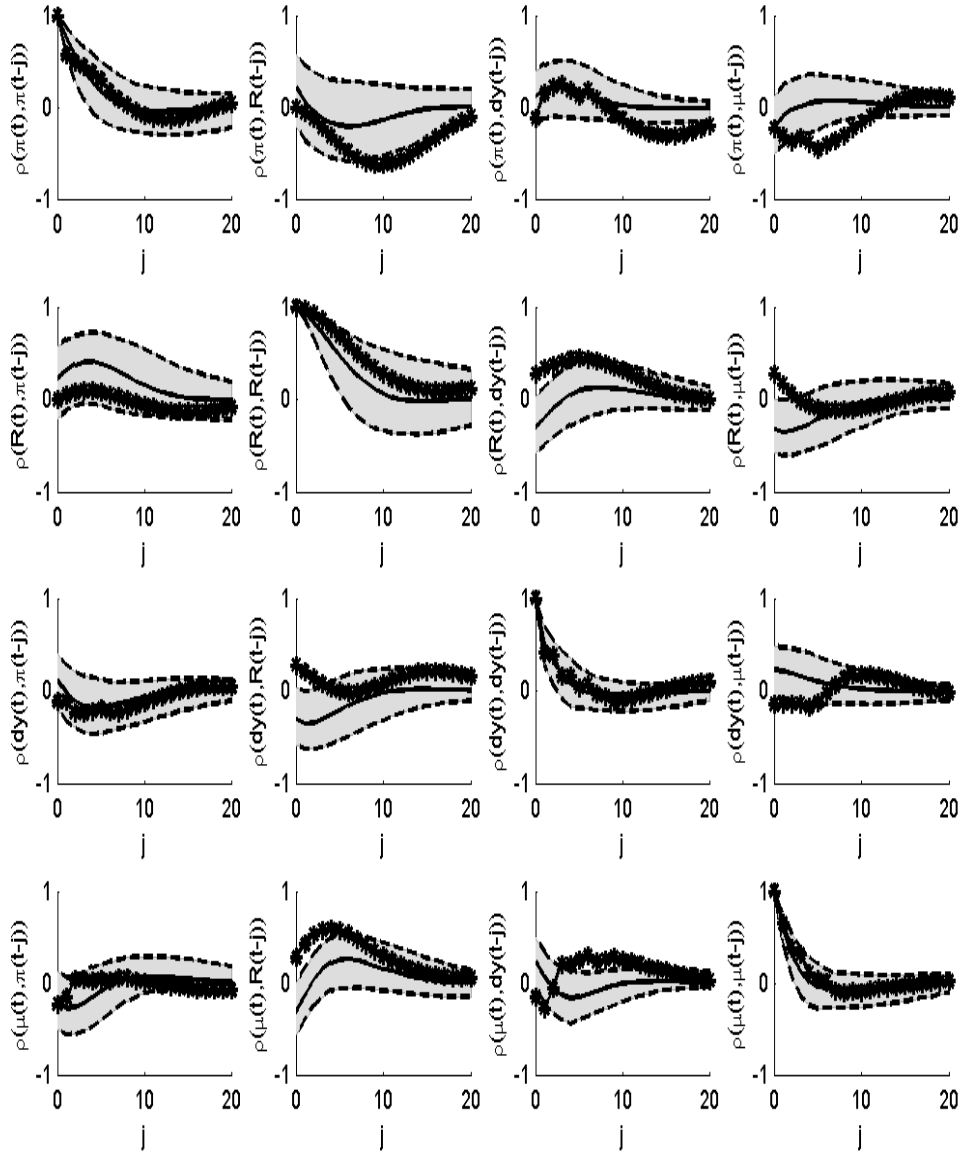


Figure 4: Dynamic correlations (1995-2011). US data (\*) and estimated model (solid lines with shaded area showing the 5%-95% confidence interval).

## 5 What did it change from 1995 onwards?

In this section, we analyze the structural equations and the sources of variability to pin down the elements that can better illustrate the observed changes that have occurred in recent US business cycles.

### 5.1 Inflation dynamics

Price inflation dynamics are governed in the model by the New Keynesian Phillips Curve (NKPC):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} + \pi_3 m c_t + \varepsilon_t^p,$$

where  $\pi_1 = \frac{\iota_p}{1+\beta\iota_p}$ ,  $\pi_2 = \frac{\bar{\beta}}{1+\beta\iota_p}$ , and  $\pi_3 = \frac{1}{1+\beta\iota_p} \left[ \frac{(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)} \right]$  and  $m c_t$  denotes the log deviation of the real marginal cost with respect to the steady-state level. The structural analysis of inflation dynamics can be illustrated by examining the NKPC across both sub-samples. Hence, the estimates of the structural parameters give the following NKPC over the first sub-sample (1968:1-1994:4)

$$\begin{aligned} \pi_t &= 0.25\pi_{t-1} + 0.74E_t\pi_{t+1} + 0.0270m c_t + \varepsilon_t^p, \\ \varepsilon_t^p &= 0.88\varepsilon_{t-1}^p - 0.68\eta_{t-1}^p + \eta_t^p, \quad std(\eta^p) = 0.15\%, \end{aligned}$$

whereas for the second sub-sample (1995:1-2011:4) the estimated NKPC is

$$\begin{aligned} \pi_t &= 0.22\pi_{t-1} + 0.78E_t\pi_{t+1} + 0.0125m c_t + \varepsilon_t^p, \\ \varepsilon_t^p &= 0.81\varepsilon_{t-1}^p - 0.61\eta_{t-1}^p + \eta_t^p, \quad std(\eta^p) = 0.11\%. \end{aligned}$$

The backward-looking component of inflation is slightly lower after 1995, which indicates that the endogenous inflation inertia has diminished a bit after 1995. This result is based on the decrease of nominal inertia described by the price indexation parameter (i.e.,  $\iota_p$  falls from 0.34 to 0.28 as reported in Table 1A). Remarkably, the backward-looking dynamics of price inflation are much weaker in our estimated NKPC than in the estimation of CSS (2012).

The estimate of the slope coefficient falls substantially in the second period, as it comes down to less than half of the value found in the first sub-sample. This result is consistent with the increase in the estimate of the Calvo sticky-price probability. As shown in Table 1,  $\xi_p = 0.62$  in the period before 1995, and  $\xi_p = 0.75$  in the period after 1995. So, the average number of months without optimal pricing increases from 7.9 months to 12 months. As argued by Smets and Wouters (2007),



the price stability period of the Great Moderation (of real variables) may explain the increase in price stickiness associated with lower menu costs.

As for the exogenous variability of inflation, the comparison of the estimated NKPCs shows that price mark-up shocks are less volatile and less persistent in the second sub-sample. The autoregressive coefficient falls by around 8% while the standard deviation of the innovations is 27% lower.

Summarizing, the decline in inflation volatility after 1995 can be explained by two factors: i) stickier prices turn inflation less sensitive to real marginal cost fluctuations and ii) firms receive lower and, more important, less persistent mark-up pricing shocks.<sup>11</sup>

## 5.2 Money market

### *Money demand*

The introduction of the transactions-facilitating role of money can shed some light on the possible influence of variations of money demand behavior to explaining the changes of inflation and interest rate dynamics. Making the first difference on the money demand equation (3) gives

$$\mu_t - \pi_t = (\lambda_m/\gamma) (\mu_{t-1} - \pi_{t-1}) + (1 - \lambda_m/\gamma) \Delta c_t - \frac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}} \Delta R_t - a_2 (1 - \lambda_m/\gamma) \Delta \varepsilon_t^X.$$

Using the estimates reported in Tables 1A and 1B, the pre-1995 subsample is characterized by the following money demand behavior

$$\begin{aligned} \mu_t - \pi_t &= 0.52 (\mu_{t-1} - \pi_{t-1}) + 0.48 \Delta c_t - 8.63 \Delta R_t - 0.37 \Delta \varepsilon_t^X, \\ \varepsilon_t^X &= 0.93 \varepsilon_{t-1}^X + \eta_t^X, \quad std(\eta^X) = 5.48\%, \end{aligned}$$

whereas in the post-1995 subsample

$$\begin{aligned} \mu_t - \pi_t &= 0.46 (\mu_{t-1} - \pi_{t-1}) + 0.54 \Delta c_t - 19.85 \Delta R_t - 0.41 \Delta \varepsilon_t^X, \\ \varepsilon_t^X &= 0.96 \varepsilon_{t-1}^X + \eta_t^X, \quad std(\eta^X) = 2.70\%. \end{aligned}$$

It is quite noticeable from the comparison that the semi-elasticity of real money with respect to the nominal interest rate has increased dramatically in the second period. Both the decline of monetary

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<sup>11</sup>The qualification "more important", associated with less persistent mark-up shocks, made in this sentence is rather relevant because intertemporal rational agents decisions are severely affected by persistent shocks.

habits  $\lambda_m$  and the deep fall of (inflation and) the nominal interest rate in the steady state,  $R^{ss}$ , are the two model elements behind this change. The resulting dynamics of money demand imply that in equilibrium nominal interest rates are quite insensitive to changes in either consumption, inflation, nominal money growth or exogenous perturbations. For example, after a mark-up shock that raises inflation, the required increase in the nominal interest to adjust down real money demand would be quantitatively much smaller. Hence, the increase in interest-rate semi-elasticity of money demand can serve to explain both the low volatility of the nominal interest rate and the reduction in its cyclical correlation with inflation, observed after 1995.

*Money supply (monetary policy)*

Regarding money supply behavior, the Taylor-style rule (8) incorporates responses of the nominal interest rate to nominal money growth as part of a stabilizing systematic monetary policy. Before 1995, the estimated monetary policy rule is

$$\begin{aligned} R_t &= 0.66R_{t-1} + 0.64\pi_t + 0.05(y_t - y_t^p) + 0.14\mu_t + \varepsilon_t^R, \\ \varepsilon_t^R &= 0.27\varepsilon_{t-1}^R + \eta_t^R, \quad \text{std}(\eta^R) = 0.43\%. \end{aligned}$$

Meanwhile, in the second sub-sample that begins in 1995 the estimated monetary policy is

$$\begin{aligned} R_t &= 0.79R_{t-1} + 0.33\pi_t + 0.02(y_t - y_t^p) + 0.09\mu_t + \varepsilon_t^R, \\ \varepsilon_t^R &= 0.60\varepsilon_{t-1}^R + \eta_t^R, \quad \text{std}(\eta^R) = 0.18\%. \end{aligned}$$

The comparison highlights some relevant changes. First, the nominal interest rate adjusts more gradually during the period of the great inflation moderation, as the smoothing (inertia) coefficient increases from 0.66 to 0.79. Second, the response coefficients to inflation, the output gap, and money growth are lower after 1995, which might somewhat reflect a sense of a loose central-bank policy during the period of inflation moderation. In other words, monetary policy becomes more discretionary and less rule-oriented, as the systematic behavior weakens.<sup>12</sup> By contrast, CSS (2012) find a more anti-inflationary monetary-policy rule as one of the factors behind the return of the Gibson paradox. These different results might be partially explained by the different sample periods used in the two papers.<sup>13</sup>

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<sup>12</sup>Contributing to this line of argument, Taylor (2012) claims that, from 2003, Fed's monetary policy deviated significantly from a Taylor (1983)-type rule to become quite discretionary.

<sup>13</sup>In particular, our paper considers observations after 2007, ignored by CSS (2012), which capture the rather loose

### 5.3 Real wage dynamics

Wage setting behavior and nominal rigidities *à la* Calvo (1983) lead to the following expression for the dynamic evolution of real wages

$$w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 (w_t - mrs_t) + \varepsilon_t^w,$$

where  $w_1 = \frac{1}{1+\beta}$ ,  $w_2 = \frac{1+\bar{\beta}\iota_w}{1+\beta}$ ,  $w_3 = \frac{\iota_w}{1+\beta}$ , and  $w_4 = \frac{1}{1+\beta} \left[ \frac{(1-\bar{\beta}\xi_w)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)} \right]$ . In the estimation, the curvature of the Kimball labor aggregator is fixed at  $\varepsilon_w = 10.0$  and the steady-state wage mark-up is  $\phi_w = 1.5$ , following Smets and Wouters (2007). The wage mark-up,  $w_t - mrs_t$ , measured as the log difference between the real wage and the marginal rate of substitution between working and consuming is the key determinant of real wage fluctuations. The estimates before 1995 imply

$$\begin{aligned} w_t &= 0.50w_{t-1} + 0.50(E_t w_{t+1} + E_t \pi_{t+1}) - 0.80\pi_t + 0.30\pi_{t-1} - 0.0094(w_t - mrs_t) + \varepsilon_t^w, \\ \varepsilon_t^w &= 0.87\varepsilon_{t-1}^w - 0.59\eta_{t-1}^w + \eta_t^w, \quad std(\eta^w) = 0.18\%. \end{aligned}$$

Meanwhile, in the sample after 1995 the estimated real wage equation is

$$\begin{aligned} w_t &= 0.50w_{t-1} + 0.50(E_t w_{t+1} + E_t \pi_{t+1}) - 0.72\pi_t + 0.22\pi_{t-1} - 0.0079(w_t - mrs_t) + \varepsilon_t^w, \\ \varepsilon_t^w &= 0.44\varepsilon_{t-1}^w - 0.47\eta_{t-1}^w + \eta_t^w, \quad std(\eta^w) = 0.40\%. \end{aligned}$$

The structural components of real wage dynamics (backward/forward looking coefficients, slope coefficient) show slight shifts after 1995. The estimates of wage-stickiness,  $\xi_w$ , barely change across samples, while wage indexation on lagged inflation,  $\iota_w$ , falls in the second subsample (see Table 2 for the numbers). Nevertheless, significant differences are observed in the exogenous process that collects wage mark-up shocks. The coefficient of autocorrelation falls from 0.87 to 0.44. The moving-average coefficient is also lower, while the innovations have a higher standard deviation after 1995. The sizable reduction of persistence in wage mark-up shocks has dramatic effects on the sources of business cycle variability after 1995, as documented next.

### 5.4 Sources of variability

Both the impulse-response functions and the variance decomposition provide model-based information about how the exogenous sources of variability shape the business cycle fluctuations.

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monetary policy implemented by the Fed since the subprime mortgage crisis.

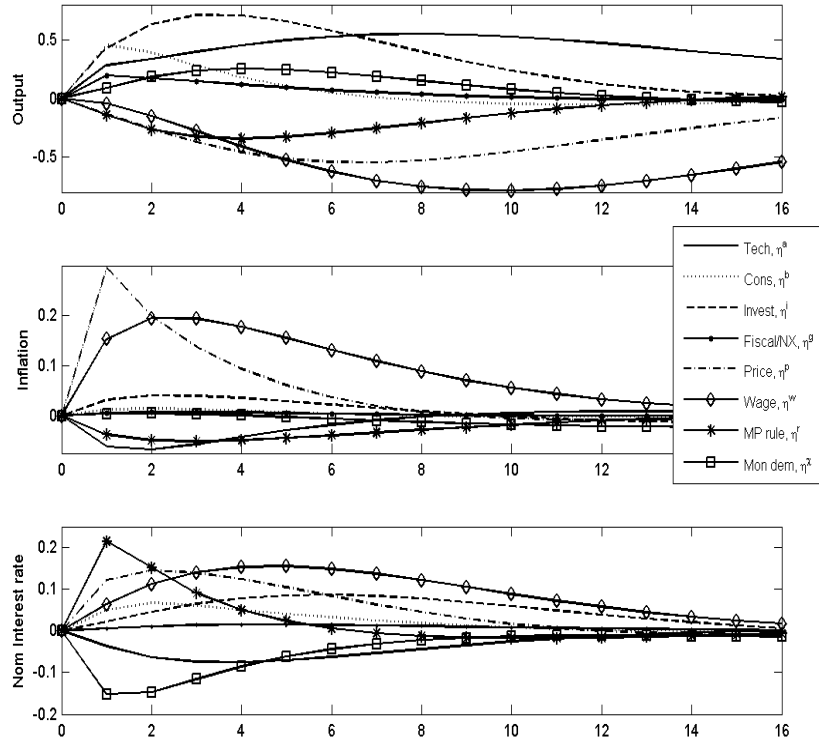


Figure 5: Impulse-response functions. Pre-1995 subsample.

Figures 5 and 6 display the responses of output, inflation and the nominal interest rate to the eight exogenous shocks in the pre-1995 and post-1995 periods, respectively. The size of the shocks has been normalized at the estimated standard deviation of their innovations.

The monetary policy shock to the extended Taylor (1993)-type rule (8) drives a negative comovement between inflation and the nominal interest rate. A positive  $\varepsilon_t^R$  raises the nominal interest rate that increases productivity and reduces marginal costs through the demand-side contraction. In turn, prices and the rate of inflation fall with higher nominal interest rates. By contrast, all the other seven shocks display a positive comovement between inflation and the nominal interest rates (see Figures 5 and 6). In the pre-1995 subsample, the negative comovement induced by monetary policy shocks is relatively weak. As a result, cyclical fluctuations of inflation and the nominal interest rate are dominated by the remaining shocks and give a moderately high coefficient of correlation (higher than 0.5 as shown in Figures 2 and 3). In particular, supply-side shocks on

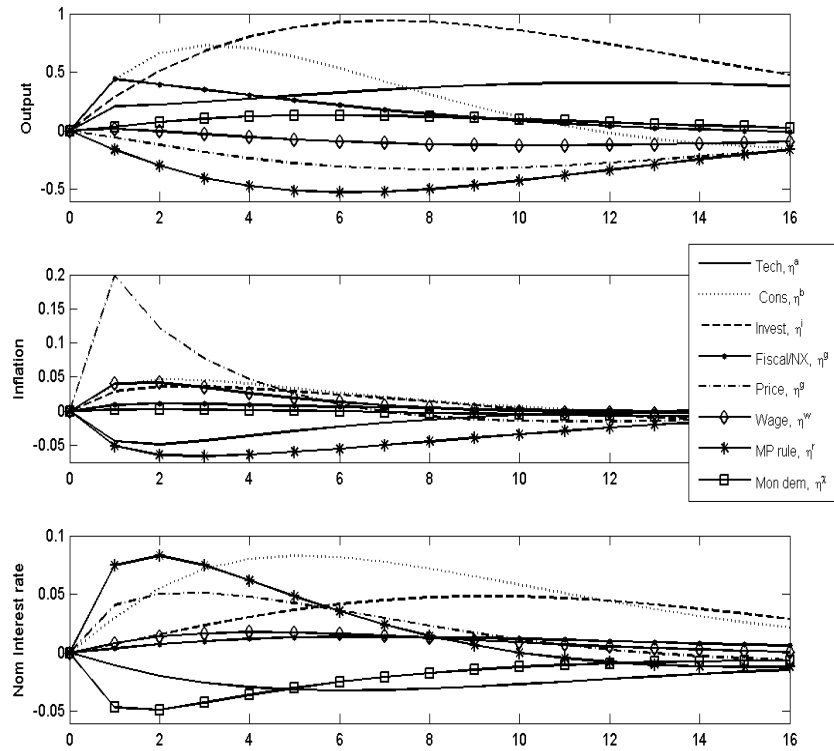


Figure 6: Impulse-response functions. Post-1995 subsample.

both price mark-up and wage mark-up are very influential on inflation and output. After 1995, the impulse-response functions show that the effects of wage mark-up shocks are clearly mitigated (compare diamond-marked lines in Figures 5 and 6). The loss of influence of wage mark-up shocks, can explain both the reduction of volatility on interest rates and inflation as well as the presence of the Gibson paradox. By contrast, demand-side shocks (such as those on consumption spending) induce much stronger responses of output and the nominal interest rate during this recent period.

Table 3. Variance decomposition, %

Innovations	Pre-1995 model					Post-1995 model				
	$y$	$\Delta y$	$R$	$\pi$	$\mu$	$y$	$\Delta y$	$R$	$\pi$	$\mu$
Technology, $\eta^a$	21.3	8.7	6.5	4.4	2.8	20.2	6.9	7.5	6.8	2.4
Consumption pref., $\eta^b$	2.4	18.5	3.2	0.2	0.5	11.6	31.3	36.8	9.1	3.6
Investment, $\eta^i$	14.8	21.3	11.2	2.9	2.0	42.0	25.4	19.2	9.1	2.7
Fiscal/Net exports, $\eta^g$	5.7	29.8	2.7	0.7	0.6	3.0	20.0	1.3	0.4	0.2
Price-push, $\eta^p$	12.9	5.7	15.7	36.6	9.4	8.6	3.8	9.0	46.5	8.4
Wage-push, $\eta^w$	37.3	8.8	31.2	49.2	12.5	1.0	0.6	1.6	5.0	1.2
MP rule, $\eta^R$	4.0	5.3	15.4	4.3	40.3	12.9	11.4	17.4	21.8	65.8
Money demand, $\eta^x$	1.7	1.8	14.3	1.8	31.9	0.6	0.6	7.2	1.2	15.7

In the variance decomposition of the estimated model (Table 3), the percentages of monetary policy shocks explaining inflation fluctuations increase in the second sub-sample (from 4% to 22%). As discussed above, monetary policy shocks increases the negative comovement variability, which helps explaining the Gibson paradox. However, the most significant change in the sources of variability is the dramatic decline in the participation of wage-push shocks. As Table 3 reports, these shocks were responsible for 49.2% of the variability of inflation and 31.2% of the variability of the nominal interest rate before 1995. The percentages fall below 5% from 1995 onwards. By contrast, demand-side shocks determine a much greater portion of business cycle fluctuations after 1995 when consumption and investment shocks explain more than 50% of output and the nominal interest rate variability. Hence, both the estimated variance decomposition and the impulse-response function analysis indicate a replacement from price/wage mark-up shocks to demand-side shocks on consumption, investment, and the monetary policy rule as the main sources of variability in recent US business cycles.

## 6 Counterfactual experiments

The previous section has discussed the different driving forces explaining the monetary dynamic changes since 1995. In this section, we carry out a large number of counterfactual experiments in order to assess the relative importance of these sources. In all these experiments we consider the

model parameter estimates obtained from the post-1995 period as the benchmark parameter values and we recalculate the variance/covariance matrix by considering alternative sets of parameters values obtained from the pre-1995 estimated model. In doing so, we want to answer the question of what would happen after 1995 if the values a given group of parameters were the same as the ones estimated for the pre-1995 sample? Eighteen counterfactual experiments were conducted. The following are the parameters changing in each of them: (i)  $r_\pi$ ,  $r_y$ ,  $r_\mu$  and  $\rho$ , which describe the systematic part of monetary policy, (ii) monetary policy shock parameters ( $\rho_R$ ,  $\sigma_R$ ), (iii) all monetary policy rule parameters ( $r_\pi$ ,  $r_y$ ,  $r_\mu$ ,  $\rho$ ,  $\rho_R$ ,  $\sigma_R$ ), (iv) money demand technology parameters ( $\lambda_m$ ,  $a_2$ ,  $R^{ss}$ ), (v) money demand shock parameters ( $\rho_\chi$ ,  $\sigma_\chi$ ), (vi) all money demand parameters ( $\lambda_m$ ,  $a_2$ ,  $R^{ss}$ ,  $\rho_\chi$ ,  $\sigma_\chi$ ), (vii) price setting parameters ( $\xi_p$ ,  $\iota_p$ ,  $\bar{\beta}$ ), (viii) price mark-up shock parameters ( $\sigma_p$ ,  $\rho_p$ ,  $\mu_p$ ), (ix) all New Keynesian Phillips curve parameters ( $\xi_p$ ,  $\iota_p$ ,  $\bar{\beta}$ ,  $\sigma_p$ ,  $\rho_p$ ,  $\mu_p$ ), (x) wage setting parameters ( $\xi_w$ ,  $\iota_w$ ,  $\bar{\beta}$ ), (xi) wage mark-up shock parameters ( $\sigma_w$ ,  $\rho_w$ ,  $\mu_w$ ), (xii) all real wage parameters ( $\xi_w$ ,  $\iota_w$ ,  $\bar{\beta}$ ,  $\sigma_w$ ,  $\rho_w$ ,  $\mu_w$ ), (xiii) consumption preference parameters ( $\lambda$ ,  $\sigma_c$ ), (xiv) consumption preference shock parameters ( $\rho_b$ ,  $\sigma_b$ ), (xv) all consumption parameters ( $\lambda$ ,  $\sigma_c$ ,  $\rho_b$ ,  $\sigma_b$ ), (xvi) investment technology parameters ( $\varphi$ ,  $\psi$ ), (xvii) investment shock parameters ( $\rho_i$ ,  $\sigma_i$ ), and (xviii) all investment parameters ( $\varphi$ ,  $\psi$ ,  $\rho_i$ ,  $\sigma_i$ ).

Table 4. Change in second-moment statistics, Post-95 minus Pre-95

	US data	Baseline model	MP Rule [i,ii,iii]'	Money demand [iv,v,vi]'	Prices [vii,viii,ix]'	Wages [x,xi,xii]'
Standard deviations:						
			-0.32	-0.27	-0.20	-0.26
$\sigma(\pi_t)$	-0.38	-0.27	-0.25	-0.27	<b>-0.12</b>	<b>-0.13</b>
			-0.32	-0.27	<b>-0.13</b>	<b>-0.12</b>
			-0.29	-0.25	-0.32	-0.33
$\sigma(R_t)$	-0.27	-0.34	-0.26	-0.30	-0.29	-0.27
			-0.25	<b>-0.19</b>	-0.30	-0.26
			-0.83	-1.03	-0.77	-0.77
$\sigma(\mu_t)$	-0.76	-0.75	<b>+0.49</b>	-0.45	-0.62	-0.62
			<b>-0.01</b>	-0.51	-0.70	-0.64
Autocorrelations:						
			-0.10	-0.05	+0.00	-0.04
$\rho(\pi_t, \pi_{t-1})$	-0.34	-0.05	-0.03	-0.04	-0.07	<b>+0.03</b>
			-0.10	-0.04	-0.05	<b>+0.03</b>
			+0.07	+0.06	+0.06	+0.06
$\rho(R_t, R_{t-1})$	+0.05	+0.07	-0.01	+0.05	+0.06	+0.07
			+0.05	+0.05	+0.06	+0.07
			+0.16	+0.08	+0.09	+0.09
$\rho(\mu_t, \mu_{t-1})$	+0.17	+0.10	<b>-0.12</b>	<b>-0.01</b>	+0.13	+0.14
			<b>-0.10</b>	<b>-0.05</b>	+0.11	+0.14
Correlations:						
			-0.24	-0.33	-0.36	-0.39
$\rho(\pi_t, R_t)$	-0.45	-0.39	-0.59	-0.40	-0.25	<b>-0.18</b>
			-0.32	-0.32	-0.31	<b>-0.18</b>
			+0.27	+0.35	+0.44	+0.38
$\rho(\pi_t, \Delta y_t)$	+0.25	+0.34	+0.41	+0.39	+0.20	+0.21
			+0.28	+0.35	+0.35	+0.21



Table 4 shows the change in some selected second-moment statistics between the pre-95 and the post-95 sub-samples in actual data, in the baseline model and in each counterfactual experiment.<sup>14</sup> Bold characters highlight the main sources for these second-moment statistics changes. Results show that the fall of inflation volatility,  $\sigma(\pi_t)$ , is mainly explained by changes in both the parameters describing wage mark-up shocks ( $\sigma_w, \rho_w, \mu_w$ ) and price mark-up shocks ( $\sigma_p, \rho_p, \mu_p$ ). Meanwhile, the milder fluctuations of the nominal interest rate and the nominal money growth observed in the most recent period are due to changes in money demand parameters ( $\lambda_m, a_2, R^{ss}, \rho_\chi, \sigma_\chi$ ) and changes in monetary policy shock parameters ( $\rho_R, \sigma_R$ ), respectively.

Experiment (xi) suggests that the changes in the wage mark-up shock parameters are partially responsible for the fall of inflation persistence. Moreover, the changes in monetary policy shock parameters and money demand parameters help to explain the small increase in nominal money growth persistence.

Finally, the changes in the wage mark-up shocks largely explain the Gibson paradox and the fall of both inflation volatility and inflation persistence. These results are in contrast with CSS (2012) where a more anti-inflationary policy rule and a decline of price indexation are the two sources for the re-emergence of the Gibson paradox and the fall of inflation persistence. Our counterfactual experiments show that, with a medium-scale DSGE monetary model, the Gibson paradox, the fall of inflation persistence and many of the changes observed since 1995 are mostly explained by exogenous changes in price/wage dynamics: the significant decline in persistence of wage mark-up shocks and the lower volatility of price mark-up shocks

## 7 Conclusions

This paper builds a full-fledged estimated DSGE model with money to study the driving forces of the recent U.S. inflation moderation period and other stylized facts, such as the fall of inflation persistence and the weakening of inflation and nominal interest rate correlation. Compared to standard DSGE models, the model incorporates a transaction-facilitating demand for money, a real-balance effect on consumption, transaction costs as a small percentage of the overall household

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<sup>14</sup>The latest six experiments involving parameters describing consumption and investment demands do not imply significant changes in second-moments statistics. For the sake of brevity, these experiment results are excluded from Table 4. They are available from the authors upon request.

spending, and a Taylor-type monetary policy rule that reacts to changes in nominal money growth.

Our findings suggest a combination of lower exogenous variability and, to a lower extent, structural changes in money demand, monetary policy and firms' pricing behavior as the main driving forces of the changes observed in the U.S. business cycle since 1995. Regarding the exogenous variability, the supply-side shocks on both prices and wages, that were dominant in the cyclical variability in the 70's and 80's, lose much of their significance after 1995. Their explanatory power for business cycle fluctuations has been mostly replaced in the recent period by either investment spending or interest-rate shocks. The structural analysis of the money market shows that the estimated interest-rate elasticity of money demand more than doubles in the recent period the value estimated for the earlier sample period. The lower average rates of return explain this higher responsiveness of money demand to changes in the nominal interest rates. Other non-modeled factors could be the greater accessibility of households to a variety of money-like assets, and the financial innovation of the period. We have also found a swing in monetary policy towards a more conservative and gradual strategy after 1995. The Fed's response coefficients to inflation, the output gap and money growth have been considerably lower after 1995 than what they had been before. Finally, the estimates of private sector decision making show little differences across periods. The most remarkable one is that the level of price stickiness rises in the post-95 sample period. It helps to characterize the observed lower inflation volatility as firms attenuate the reaction of prices to changes in the marginal costs. This factor and the substantial decline of persistence in wage-push shocks are key elements explaining the inflation moderation.

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## Appendix

### I. Household's optimizing program with transactions-facilitating money

Recalling the specification of Smets and Wouters (2007), and adding a consumption preference shock  $\varepsilon_t^b$ , the instantaneous utility of the representative  $j$ -th household is

$$\left[ \exp\left(\varepsilon_t^b\right) \frac{1}{1-\sigma_c} (C_t(j) - \lambda C_{t-1})^{1-\sigma_c} \right] \exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} (L_t(j))^{1+\sigma_l}\right),$$

where  $\sigma_c, \sigma_l > 0$ ,  $C_t(j)$  is current consumption of the  $j$ -indexed representative household,  $C_{t-1}$  is lagged aggregate consumption, and  $L_t(j)$  is the supply of household-specific labor. In addition, the budget constraint incorporates (real) spending on transaction costs,  $H_t(j)$ , and the possibility of a net increase in real money balances,  $\frac{M_t(j)}{P_t} - \frac{M_{t-1}(j)}{P_t}$ . For period  $t$ , the budget constraint is written as follows

$$\begin{aligned} \frac{W_t(j)L_t(j)}{P_t} + \frac{R_t^k Z_t(j)K_{t-1}(j)}{P_t} - a(Z_t(j))K_{t-1}(j) + \frac{Div_t}{P_t} - T_t = \\ C_t(j) + I_t(j) + \frac{B_t(j)}{(1+R_t)P_t} - \frac{B_{t-1}(j)}{P_t} + \frac{M_t(j)}{P_t} - \frac{M_{t-1}(j)}{P_t} + H_t(j), \end{aligned}$$

which is equivalent to

$$\begin{aligned} \frac{W_t(j)L_t(j)}{P_t} + \frac{R_t^k Z_t(j)K_{t-1}(j)}{P_t} - a(Z_t(j))K_{t-1}(j) + \frac{Div_t}{P_t} - T_t = \\ C_t(j) + I_t(j) + \frac{B_t(j)}{(1+R_t)P_t} - \frac{B_{t-1}(j)}{P_{t-1}}(1 + \pi_t)^{-1} + \frac{M_t(j)}{P_t} - \frac{M_{t-1}(j)}{(1+\pi_t)P_{t-1}} + H_t(j), \end{aligned}$$

where  $\pi_t = (P_t/P_{t-1}) - 1$  is the rate of inflation between periods  $t - 1$  and  $t$ . Following Casares (2007), transaction costs (in real terms) are determined by the functional form

$$H_t(j) = a_0 + a_1 C_t(j) \left( \frac{C_t(j)}{\exp(\varepsilon_t^X) \left( \frac{M_t(j)}{P_t} - \lambda_m \frac{M_{t-1}}{P_{t-1}} \right)} \right)^{\frac{a_2}{1-a_2}}, \quad (A1)$$

where  $a_0, a_1 > 0$ ,  $0 < a_2 < 1$ ,  $0 < \lambda_m < 1$  and  $\varepsilon_t^X$  is a money-augmenting AR(1) shock. The partial derivatives are

$$\begin{aligned} H_{C_t(j)} &= \frac{a_1}{1-a_2} \left( \frac{C_t(j)}{\exp(\varepsilon_t^X) \left( \frac{M_t(j)}{P_t} - \lambda_m \frac{M_{t-1}}{P_{t-1}} \right)} \right)^{\frac{a_2}{1-a_2}}, \\ H_{\frac{M_t(j)}{P_t}} &= -\frac{a_1 a_2}{1-a_2} \exp(\varepsilon_t^X) \left( \frac{C_t(j)}{\exp(\varepsilon_t^X) \left( \frac{M_t(j)}{P_t} - \lambda_m \frac{M_{t-1}}{P_{t-1}} \right)} \right)^{\frac{1}{1-a_2}}, \end{aligned}$$

which satisfy the desirable properties:  $H_{C_t(j)} > 0$ ,  $H_{C_t(j)C_t(j)} > 0$ ,  $H_{\frac{M_t(j)}{P_t}} < 0$ ,  $H_{\frac{M_t(j)}{P_t} \frac{M_t(j)}{P_t}} > 0$  and  $H_{C_t(j) \frac{M_t(j)}{P_t}} < 0$ .

The first order conditions for consumption, real money, labor supply, and bonds that result from the household optimizing program are

$$\begin{aligned} \exp\left(\varepsilon_t^b\right) (C_t(j) - \lambda C_{t-1})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} (L_t^s(j))^{1 + \sigma_l}\right) - \Xi_t (1 + H_{C_t(j)}) &= 0, & (C_t^{foc}(j)) \\ -\Xi_t \left(1 + H_{\frac{M_t(j)}{P_t}}\right) + \beta E_t \Xi_{t+1} (1 + \pi_{t+1})^{-1} &= 0, & \left(\left(\frac{M_t(j)}{P_t}\right)^{foc}\right) \\ \exp\left(\varepsilon_t^b\right) \frac{(C_t(j) - \lambda C_{t-1})^{1 - \sigma_c}}{1 - \sigma_c} \exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} (L_t(j))^{1 + \sigma_l}\right) (\sigma_c - 1) (L_t(j))^{\sigma_l} + \Xi_t \frac{W_t(j)}{P_t} &= 0, & (L_t^{foc}(j)) \\ -\Xi_t (1 + R_t)^{-1} + \beta E_t \Xi_{t+1} (1 + \pi_{t+1})^{-1} &= 0, & \left(\left(\frac{B_t(j)}{P_t}\right)^{foc}\right) \end{aligned}$$

where  $\Xi_t$  is the Lagrange multiplier of the budget constraint in period  $t$ , and  $H_{C_t(j)}$  and  $H_{\frac{M_t(j)}{P_t}}$  are the partial derivatives of the transaction costs function with respect to consumption and real money balances, respectively.

#### Transactions costs

In equilibrium, the representative household assumption implies that aggregate and household-level amounts of consumption or money demand are identical. Hence, the value of log fluctuations of transaction costs around the steady-state,  $h_t$ , is provided by loglinearizing (A1) to obtain<sup>15</sup>

$$h_t = \frac{1 - (a_0/H)}{1 - a_2} \left( c_t - a_2 \left( \frac{1}{1 - \lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1 - \lambda_m/\gamma} m_{t-1} \right) - a_2 \varepsilon_t^\lambda \right). \quad (\text{A2})$$

#### Money demand

Plugging the expression for  $\beta E_t \Xi_{t+1} (1 + \pi_{t+1})^{-1}$  obtained from  $\left(\frac{B_t(j)}{P_t}\right)^{foc}$  in  $\left(\frac{M_t(j)}{P_t}\right)^{foc}$  yields the money demand equation

$$-H_{\frac{M_t(j)}{P_t}} = \frac{R_t}{1 + R_t},$$

that presents a standard microeconomic optimality condition that equates the marginal return of monetary services ( $-H_{\frac{M_t(j)}{P_t}}$ ) to the marginal (opportunity) cost of money holdings ( $\frac{R_t}{1 + R_t}$ ). After loglinearizing, it gives

$$\log H_{\frac{M_t}{P_t}} - \log H_{\frac{M}{P}} = -\frac{1}{R^{ss}} R_t, \quad (\text{A3})$$

where  $R^{ss}$  is the steady-state nominal interest rate. In equilibrium, the log deviations of the marginal

<sup>15</sup> Lower-case variables denote log deviations from steady state of the corresponding upper-case variable.

transactions-facilitating service of money is given by

$$\log H \frac{M_t}{P_t} - \log H \frac{M}{P} = -\frac{1}{1-a_2} c_t + \frac{1}{1-a_2} \left( \frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) + \frac{a_2}{1-a_2} \varepsilon_t^\chi.$$

The fluctuations of the marginal service of transactions-facilitating money can be taken from equation (A4) and substituted in (A3) to yield the semi-log real money demand equation

$$m_t = (\lambda_m/\gamma) m_{t-1} + (1 - \lambda_m/\gamma) c_t - \frac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}} R_t - a_2 (1 - \lambda_m/\gamma) \varepsilon_t^\chi, \quad (\text{A5})$$

where  $\gamma$  is the steady-state output growth.

### Consumption

The loglinear version of  $\left(\frac{B_t(j)}{P_t}\right)^{foc}$  is

$$\log \Xi_t = E_t \log \Xi_{t+1} + (R_t - E_t \pi_{t+1}). \quad (\text{A6})$$

We need one expression for  $\log \Xi_t$  that can be found by loglinearizing  $C_t^{foc}(j)$ . It leads to (for simplicity, the constant terms were dropped)

$$\log \Xi_t = -\sigma_c \left( \frac{1}{1-\lambda/\gamma} c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1} \right) + \frac{(\sigma_c-1)w^{ss}}{\phi_w(1-\lambda/\gamma)c^{ss}} \log L_t(j) - H_{C(j)} \log H_{C_t(j)} + \varepsilon_t^b. \quad (\text{A7})$$

Taking both  $\log \Xi_t$  and the corresponding expression for  $\log \Xi_{t+1}$  from (A6), and inserting them both in (A7) result in

$$\begin{aligned} & -\sigma_c \left( \frac{1}{1-\lambda/\gamma} c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1} \right) + \frac{(\sigma_c-1)w^{ss}}{\phi_w(1-\lambda/\gamma)c^{ss}} \log L_t(j) - H_c^{ss} \log H_{C_t(j)} = \\ & E_t \left( -\sigma_c \left( \frac{1}{1-\lambda/\gamma} c_{t+1} - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_t \right) + \frac{(\sigma_c-1)w^{ss}}{\phi_w(1-\lambda/\gamma)c^{ss}} \log L_{t+1}(j) - H_c^{ss} \log H_{C_{t+1}(j)} \right) \\ & + (R_t - E_t \pi_{t+1}) - (1 - \rho_b) \varepsilon_t^b, \end{aligned}$$

where terms can be rearranged for the IS-type consumption curve

$$\begin{aligned} c_t &= \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \frac{1}{1+\lambda/\gamma} E_t c_{t+1} + \frac{(\sigma_c-1)w^{ss}}{\sigma_c \phi_w (1+\lambda/\gamma) c^{ss}} (l_t - E_t l_{t+1}) \\ & - \frac{(1-\lambda/\gamma)H_C^{ss}}{\sigma_c(1+\lambda/\gamma)} \left( \log H_{C_t} - E_t \log H_{C_{t+1}} \right) - \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)} (R_t - E_t \pi_{t+1}) + \frac{(1-\rho_b)(1-\lambda/\gamma)}{\sigma_c(1+\lambda/\gamma)} \varepsilon_t^b. \quad (\text{A8}) \end{aligned}$$

The consumption marginal transaction costs in loglinear terms is

$$\log H_{C_t} - \log H_C = \frac{a_2}{1-a_2} c_t - \frac{a_2}{1-a_2} \left( \frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - \frac{a_2}{1-a_2} \varepsilon_t^\chi. \quad (\text{A9})$$

Inserting (A9) and the corresponding expression for period  $t + 1$  into equation (A8) gives rise to the consumption equation with real-money balance effects

$$(1 + c_4) c_t = c_1 c_{t-1} + c_2 E_t c_{t+1} + c_3 (l_t - E_t l_{t+1}) + c_4 \left( \frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - c_4 \left( \frac{1}{1-\lambda_m/\gamma} E_t m_{t+1} - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_t \right) - c_5 (R_t - E_t \pi_{t+1}) + c_4 (1 - \rho_\chi) \varepsilon_t^\chi + c_5 (1 - \rho_b) \varepsilon_t^b, \quad (\text{A10})$$

where  $c_1 = \frac{\lambda/\gamma}{1+\lambda/\gamma}$ ,  $c_2 = \left( \frac{1}{1+\lambda/\gamma} + c_4 \right)$ ,  $c_3 = \frac{(\sigma_c - 1)w^{ss}}{\sigma_c(1+\lambda/\gamma)\phi_w c^{ss}}$ ,  $c_4 = \frac{(1-\lambda/\gamma)H_C a_2}{\sigma_c(1+\lambda/\gamma)(1-a_2)}$ , and  $c_5 = \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}$ .

*Money-adjusted marginal rate of substitution*

In the labor supply first-order condition,  $(L_t^{foc}(j))$ , the marginal rate of substitution between consumption and hours worked appears adjusted by the introduction of transaction costs. It yields

$$-\frac{U_{L_t(j)}}{U_{C_t(j)}/(1 + H_{C_t(j)})} = (L_t(j))^{\sigma_l} (C_t(j) - \lambda C_{t-1}) (1 + H_{C_t(j)}),$$

which in a log-linear approximation, using the marginal transaction cost introduced above, and

aggregating across households becomes

$$mrs_t = \sigma_l l_t + \left( \frac{1}{1-\lambda/\gamma} + \frac{H_C a_2}{1-a_2} \right) c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1} - \frac{H_C a_2}{1-a_2} \left( \frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - \frac{H_C a_2}{1-a_2} \varepsilon_t^\chi.$$



II. Set of model parameters:

Table A. Model parameter description

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$\varphi$	Elasticity of the cost of adjusting capital
$\lambda$	Consumption habits
$\sigma_c$	Inverse of the elasticity of intertemporal substitution in utility function
$\sigma_l$	Inverse of the elasticity of labor supply with respect to the real wage
$\xi_p$	Calvo probability of price stickiness
$\xi_w$	Calvo probability of wage stickiness
$\iota_w$	Wage indexation to lagged wage inflation
$\iota_p$	Price indexation to lagged price inflation
$\psi$	Elasticity of capital utilization adjustment cost
$\Phi$	One plus steady-state fixed cost to total cost ratio (price mark-up)
$\rho$	Smoothing coefficient in monetary policy rule
$r_\pi$	Inflation coefficient in monetary policy rule
$r_Y$	Output gap coefficient in monetary policy rule
$r_\mu$	Money-growth coefficient in monetary policy rule
$\pi$	Steady-state rate of inflation
$100(\beta^{-1}-1)$	Steady-state rate of discount
$l$	Steady-state labor
$100(\gamma - 1)$	One plus steady-state rate of output growth
$\alpha$	Capital share in production function
$\lambda_m$	Monetary habits
$a_0$	Fixed transaction costs
$a_1$	Scale parameter of variable transaction costs
$a_2$	Elasticity parameter of transaction costs function

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Table A. (*Continued*)

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$\sigma_a$	Standard deviation of productivity innovation
$\sigma_b$	Standard deviation of risk premium innovation
$\sigma_g$	Standard deviation of exogenous spending innovation
$\sigma_i$	Standard deviation of investment-specific innovation
$\sigma_R$	Standard deviation of monetary policy rule innovation
$\sigma_p$	Standard deviation of price mark-up innovation
$\sigma_w$	Standard deviation of wage mark-up innovation
$\sigma_\chi$	Standard deviation of money demand innovation
$\rho_a$	Autoregressive coefficient of productivity shock
$\rho_b$	Autoregressive coefficient of risk premium shock
$\rho_g$	Autoregressive coefficient of exogenous spending shock
$\rho_i$	Autoregressive coefficient of investment-specific shock
$\rho_R$	Autoregressive coefficient of policy rule shock
$\rho_p$	Autoregressive coefficient of price mark-up shock
$\mu_p$	Moving-average coefficient of price mark-up shock
$\rho_w$	Autoregressive coefficient of wage mark-up shock
$\mu_w$	Moving-average coefficient of wage mark-up shock
$\rho_p$	Autoregressive coefficient of money demand shock

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III. Set of log-linearized dynamic equations:

Real money demand equation:

$$m_t = (\lambda_m/\gamma) m_{t-1} + (1 - \lambda_m/\gamma) c_t - \frac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}} R_t - a_2 (1 - \lambda_m/\gamma) \varepsilon_t^\chi. \quad (\text{A11})$$

Transaction costs equation:

$$h_t = \frac{1-(a_0/H)}{1-a_2} \left( c_t - a_2 \left( \frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - a_2 \varepsilon_t^\chi \right). \quad (\text{A12})$$

Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + h_y h_t + g_y \varepsilon_t^g, \quad (\text{A13})$$

where  $c_y = \frac{C}{Y} = 1 - g_y - i_y$ ,  $i_y = \frac{I}{Y} = (\gamma - 1 + \delta) \frac{K}{Y}$ ,  $z_y = r^k \frac{K}{Y}$ , and  $h_y = \frac{H}{Y}$  are steady-state ratios.

As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at  $\delta = 0.025$  and  $g_y = 0.18$ .

Consumption equation:

$$(1 + c_4) c_t = c_1 c_{t-1} + c_2 E_t c_{t+1} + c_3 (l_t - E_t l_{t+1}) + c_4 \left( \frac{1}{1 - \lambda_m / \gamma} m_t - \frac{\lambda_m / \gamma}{1 - \lambda_m / \gamma} m_{t-1} \right) - c_4 \left( \frac{1}{1 - \lambda_m / \gamma} E_t m_{t+1} - \frac{\lambda_m / \gamma}{1 - \lambda_m / \gamma} m_t \right) - c_5 (R_t - E_t \pi_{t+1}) + c_4 (1 - \rho_\chi) \varepsilon_t^\chi + c_5 (1 - \rho_b) \varepsilon_t^b, \quad (\text{A14})$$

where  $c_1 = \frac{\lambda/\gamma}{1+\lambda/\gamma}$ ,  $c_2 = \left( \frac{1}{1+\lambda/\gamma} + c_4 \right)$ ,  $c_3 = \frac{(\sigma_c - 1)w^{ss}}{\sigma_c(1+\lambda/\gamma)\phi_w c^{ss}}$ ,  $c_4 = \frac{(1-\lambda/\gamma)H_C a_2}{\sigma_c(1+\lambda/\gamma)(1-a_2)}$ , and  $c_5 = \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}$

Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \quad (\text{A15})$$

where  $i_1 = \frac{1}{1+\bar{\beta}}$ , and  $i_2 = \frac{1}{(1+\bar{\beta})\gamma^2\varphi}$  with  $\bar{\beta} = \beta\gamma^{(1-\sigma_c)}$ .

Arbitrage condition (value of capital,  $q_t$ ):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} \varepsilon_t^b, \quad (\text{A16})$$

where  $q_1 = \bar{\beta}\gamma^{-1}(1 - \delta) = \frac{(1-\delta)}{(r^k+1-\delta)}$ .

Log-linearized aggregate production function:

$$y_t = \phi_p (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a), \quad (\text{A17})$$

where  $\phi_p = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$  and  $\alpha$  is the capital-share in the production function.<sup>16</sup>

Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \quad (\text{A18})$$

Capital utilization:

$$z_t = z_1 r_t^k, \quad (\text{A19})$$

where  $z_1 = \frac{1-\psi}{\psi}$ .

Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i, \quad (\text{A20})$$

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<sup>16</sup>From the zero profit condition in steady-state, it should be noticed that  $\phi_p$  also represents the value of the steady-state price mark-up.

where  $k_1 = \frac{1-\delta}{\gamma}$  and  $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right) (1 + \bar{\beta}) \gamma^2 \varphi$ .

Log fluctuations of the real marginal cost:

$$mc_t = w_t - \alpha (k_t^s - l_t) - \varepsilon_t^a. \quad (\text{A21})$$

New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} + \pi_3 mc_t + \varepsilon_t^p, \quad (\text{A22})$$

where  $\pi_1 = \frac{\iota_p}{1+\bar{\beta}\iota_p}$ ,  $\pi_2 = \frac{\bar{\beta}}{1+\bar{\beta}\iota_p}$ , and  $\pi_3 = \frac{1}{1+\bar{\beta}\iota_p} \left[ \frac{(1-\bar{\beta}\xi_p)(1-\xi_p)}{\xi_p((\phi_p-1)\varepsilon_p+1)} \right]$ . The coefficient of the curvature of the Kimball goods market aggregator, included in the definition of  $A$ , is fixed in the estimation procedure at  $\varepsilon_p = 10$  as in Smets and Wouters (2007).

Optimal demand for capital by firms:

$$-(k_t^s - l_t) + w_t = r_t^k. \quad (\text{A23})$$

Wage markup equation:

$$w_t - mrs_t = w_t - \left( \sigma_l l_t + \left( \frac{1}{1-\lambda/\gamma} + \frac{H_C a_2}{1-a_2} \right) c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1} - \frac{H_C a_2}{1-a_2} \left( \frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - \frac{H_C a_2}{1-a_2} \varepsilon_t^\chi \right). \quad (\text{A24})$$

Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 (w_t - mrs_t) + \varepsilon_t^w, \quad (\text{A25})$$

where  $w_1 = \frac{1}{1+\bar{\beta}}$ ,  $w_2 = \frac{1+\bar{\beta}\iota_w}{1+\bar{\beta}}$ ,  $w_3 = \frac{\iota_w}{1+\bar{\beta}}$ , and  $w_4 = \frac{1}{1+\bar{\beta}} \left[ \frac{(1-\bar{\beta}\xi_w)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)} \right]$  with the curvature of the Kimball labor aggregator fixed at  $\varepsilon_w = 10.0$  and a steady-state wage mark-up fixed at  $\phi_w = 1.5$  as in Smets and Wouters (2007).

Monetary policy rule, a Taylor-type rule including responses to the rate of nominal money growth:

$$R_t = \rho R_{t-1} + (1 - \rho) [r_\pi \pi_t + r_y (y_t - y_t^p) + r_\mu \mu_t] + \varepsilon_t^R. \quad (\text{A26})$$

Relationship between nominal money growth, inflation and real money dynamics

$$\mu_t - \pi_t = m_t - m_{t-1}. \quad (\text{A27})$$

Block of potential variables (with  $^p$  superscript), obtained when assuming flexible prices, flexible wages and shutting down price mark-up and wage indexation shocks. Flexible-price condition (no

price mark-up fluctuations,  $mpl_t^p = w_t^p$ ):

$$\alpha (k_t^{s,p} - l_t^p) + \varepsilon_t^a = w_t^p. \quad (\text{A28})$$

Flexible-wage condition (no wage mark-up fluctuations,  $w_t^p = mrs_t^p$ ):

$$w_t^p = \sigma_l l_t^p + \left( \frac{1}{1-\lambda/\gamma} + \frac{H_C a_2}{1-a_2} \right) c_t^p - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1}^p - \frac{H_C a_2}{1-a_2} \left( \frac{1}{1-\lambda_m/\gamma} m_t^p - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1}^p \right) - \frac{H_C a_2}{1-a_2} \varepsilon_t^\chi \quad (\text{A29})$$

Potential real money equation

$$m_t^p = (\lambda_m/\gamma) m_{t-1}^p + (1 - \lambda_m/\gamma) c_t^p - \frac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}} R_t^p - a_2 (1 - \lambda_m/\gamma) \varepsilon_t^\chi. \quad (\text{A30})$$

Potential transaction costs equation:

$$h_t^p = \frac{1-(a_0/H)}{1-a_2} \left( c_t^p - a_2 \left( \frac{1}{1-\lambda_m/\gamma} m_t^p - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1}^p \right) - a_2 \varepsilon_t^\chi \right). \quad (\text{A31})$$

Potential aggregate resource constraint:

$$y_t^p = c_y c_t^p + i_y i_t^p + z_y z_t^p + h_y h_t^p + \varepsilon_t^g \quad (\text{A32})$$

Potential consumption equation:

$$\begin{aligned} (1 + c_4) c_t^p &= c_1 c_{t-1}^p + c_2 E_t c_{t+1}^p + c_3 (l_t^p - E_t l_{t+1}^p) \\ &+ c_4 \left( \frac{1}{1-\lambda_m/\gamma} m_t^p - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1}^p \right) - c_4 \left( \frac{1}{1-\lambda_m/\gamma} E_t m_{t+1}^p - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_t^p \right) \\ &- c_5 (R_t^p - E_t \pi_{t+1}^p) + c_4 (1 - \rho_\chi) \varepsilon_t^\chi + c_5 (1 - \rho_b) \varepsilon_t^b, \end{aligned} \quad (\text{A33})$$

Potential investment equation:

$$i_t^p = i_1 i_{t-1}^p + (1 - i_1) E_t i_{t+1}^p + i_2 q_t^p + \varepsilon_t^i. \quad (\text{A34})$$

Arbitrage condition (value of potential capital,  $q_t^p$ ):

$$q_t^p = q_1 E_t q_{t+1}^p + (1 - q_1) E_t r_{t+1}^{k,p} - (R_t^p - E_t \pi_{t+1}^p) + c_3^{-1} \varepsilon_t^b. \quad (\text{A35})$$

Log-linearized potential aggregate production function:

$$y_t^p = \phi_p (\alpha k_t^{s,p} + (1 - \alpha) l_t^p + \varepsilon_t^a). \quad (\text{A36})$$

Potential capital (with one period time-to-build):

$$k_t^{s,p} = k_{t-1}^p + z_t^p. \quad (\text{A37})$$

Potential capital utilization:

$$z_t^p = z_1 r_t^{k,p}. \quad (\text{A38})$$

Potential capital accumulation equation:

$$k_t^p = k_1 k_{t-1}^p + (1 - k_1) i_t^p + k_2 \varepsilon_t^i. \quad (\text{A39})$$

Potential demand for capital by firms ( $r_t^{k,p}$  is the potential log of the rental rate of capital):

$$-(k_t^{s,p} - l_t^p) + w_t^p = r_t^{k,p}. \quad (\text{A40})$$

Monetary policy rule (under flexible prices and flexible wages):

$$R_t^p = \rho R_{t-1}^p + (1 - \rho)[r_\pi \pi_t^p + r_\mu \mu_t^p] + \varepsilon_t^R. \quad (\text{A41})$$

Potential nominal money growth, inflation and real money dynamics:

$$\mu_t^p - \pi_t^p = m_t^p - m_{t-1}^p. \quad (\text{A42})$$

#### IV. Equations-and-variables summary

- Set of equations:

Equations (A11)-(A42) determine solution paths for 32 endogenous variables.

- Set of variables:

Endogenous variables (32):  $y_t, c_t, i_t, z_t, l_t, R_t, \pi_t, mc_t, mrs_t, q_t, r_t^k, k_t^s, k_t, \mu_t, m_t, h_t, w_t, y_t^p, c_t^p, i_t^p, z_t^p, l_t^p, R_t^p, \pi_t^p, q_t^p, r_t^{k,p}, k_t^{s,p}, k_t^p, \mu_t^p, m_t^p, h_t^p$ , and  $w_t^p$ .

Predetermined variables (13):  $c_{t-1}, i_{t-1}, k_{t-1}, \pi_{t-1}, w_{t-1}, R_{t-1}, m_{t-1}, y_{t-1}, c_{t-1}^p, i_{t-1}^p, m_{t-1}^p, k_{t-1}^p$ , and  $R_{t-1}^p$ .

Exogenous variables (8): AR(1) technology shock  $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$ , AR(1) risk premium shock  $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$ , AR(1) exogenous spending shock cross-correlated to technology innovations  $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a$ , AR(1) investment shock  $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$ , AR(1) monetary policy shock  $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$ , ARMA(1,1) price mark-up shock  $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$ , ARMA(1,1) wage mark-up shock  $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$ , and AR(1) money demand shock  $\varepsilon_t^X = \rho_X \varepsilon_{t-1}^X + \eta_t^X$ .