

**PRICE SETTING IN A DIFFERENTIATED-PRODUCT DUOPOLY WITH  
ASYMMETRIC INFORMATION ABOUT DEMAND**

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ABSTRACT

This paper explores price-setting in a two-period duopoly model in which only one firm is uncertain about the degree of product differentiation and the intercept of the demand curve. In this context, the informed firm must choose whether to keep one step ahead of its rival obtaining more profits in the first period or to fool its rival into thinking that the demand is high to obtain more profits in the second period. Under certain conditions, the optimal prices will increase with demand uncertainty faced by the uninformed firm and are greater in this duopoly context than in a monopoly.

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## 1. INTRODUCTION

Previous theoretical research has analyzed the effect of demand uncertainty on pricing policies in oligopolistic markets (e.g., Klemperer and Meyer, 1986; Reisinger and Rensner, 2009). Several papers have paid attention to dynamic competition under demand uncertainty in markets with homogeneous products. To cite a few instances, Eden (1990, 2009) and Lucas and Woodford (1993) presented a model where symmetric firms gradually change prices over time in response to observed increases in cumulative aggregate sales. On the other hand, Riordan (1985) proposed a dynamic Cournot framework wherein firms actually draw inferences about the position of the demand curve from past observations on prices. Thus, this model analyses firms' perceived abilities to fool rivals into thinking that the demand curve is higher or lower than it really is.

Similarly, Mirman, Samuelson and Schlee (1994) developed a model with a similar framework to Riordan's. However, in this case, firms can influence the informational content of market data in order to increase future expected profits.

All these models studied firms' decisions when they offer homogeneous products. Nevertheless, research is scarce on oligopolistic markets where firms offer differentiated products and face demand uncertainty about the degree of substitutability between products (e.g., Harrington, 1992, 1995; Aghion, Espinosa and Jullien, 1993; Keller and Rady, 2003). These authors analyzed how learning behavior can substantially modify the outcome of competition in an oligopolistic industry facing demand uncertainty. In these papers, firms' actions provide not only current rewards, but also information about the underlying state of demand. Thus, each firm will choose its action depending on the value of that information. In other words, there is a conflict between short-term and long-term incentives and the equilibrium behavior must solve this conflict of incentives.

In all those papers, all the firms have the same information about the demand conditions, but in some markets, some firms have informational advantages because they have more experience in the market or lower costs of information than others. This type of advantages could affect significantly not only the better informed firms, but also the worse informed firms' behavior. In this context, we will expect the uninformed firm to use market outcomes in the last periods to infer the unknown market parameters, but the informed firms will change their current decisions to manipulate the information inferred by the uninformed firms.

In this paper, we consider a model where two firms offer a differentiated product in two periods and set their prices simultaneously in each period. One of the competitors is aware of the market demand conditions whereas the other one is not aware of the intercept of the demand curve and the degree of substitutability between products. For this reason, the uninformed firm is uncertain about the size of the market and about the distribution of consumers' reservation prices. Each firm can observe the prices of both products and its own demand quantity achieved in the first period before choosing its price in period 2. Therefore, the uninformed firm updates its knowledge on the demand parameters in the second period based on market data observed in period 1. However, this uninformed firm cannot infer the values of the demand parameters from the market data realized in the first period because this firm never observes the volume of sales of its rival<sup>2</sup>.

On the basis of the assumptions of the model, we show that demand uncertainty under asymmetric information can affect firms' responses to their rivals' changes in prices. For example, if the informed firm increases its price in the first period when the degree of

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<sup>2</sup> Riordan (1985) also presented a two-period duopoly model in which none of firms can observe the position of the demand curve because each firm never observes the previous output of its rival. For this reason, firms can only imperfectly infer the position of the demand curve from past observations on prices.

substitutability between products is higher than the expectation of its rival, the demand quantity of the uninformed firm achieved in period 1 will increase more than it expected. Hence, this uninformed firm will believe that the demand intercept is higher than expected in the first period and it will increase the price in the second period. However, if the informed firm increases its price in period 1 when the degree of substitutability between products is lower than that expected by its rival, the demand quantity of the uninformed firm achieved in period 1 will increase less than it expected and it will update a lower demand intercept and reduce its price in the second period.

Additionally, we find that the uninformed firm will mislead itself in the second period when the degree of substitutability between products is different from its expectation. For example, if this firm increases its price in period 1 when product differentiation is greater than expected, its realized demand will decrease less than it thought. Then, believing that the demand intercept is greater, this firm will increase its price in the second period and the informed firm will anticipate this behavior.

The model also shows that the higher the uncertainty about demand faced by the uninformed firm, the higher the price it set in period 1. Nevertheless, an increase in demand uncertainty affects the informed firm in two ways. On the one hand, it will find it easier to deceive its rival in the second period, but, on the other hand, the uninformed firm will also mislead itself due to its lack of information and the informed firm must anticipate it. Thus, the informed firm has to take into account both effects when demand uncertainty faced by its rival increases and their magnitude will depend on the degree of substitutability between products.

From the implications of our model, we can deduce the relationship between price dispersion in a market and demand uncertainty faced by the uninformed firm. In particular, when the optimal price set by the informed firm is higher than the optimal price set by the

uninformed one in the first period, price dispersion increases with demand uncertainty in markets for highly substitutable products, while the opposite occurs in markets for highly differentiated products. Likewise, the relationship between price dispersion and demand uncertainty is just the opposite when the optimal price set by the informed firm is lower than its rival's one.

Furthermore, if the uninformed firm's demand uncertainty is sufficiently high, the model predicts that the price set by the informed one in a duopoly market would be greater than the price it would set under a monopoly.

Our model differs from previous theoretical literature about oligopolistic markets with firms offering differentiated products under demand uncertainty in several points. First, we introduce asymmetric information in a dynamic game where two firms simultaneously set their prices in two periods, but where only one firm is aware of the degree of substitutability between products and of the intercept of the demand curve. As a result of this informational advantage, the best informed firm will choose its price in the first period to influence its rival's expectations about the demand parameters in the next period. This potential effect has interesting implications on price competition between firms. Hence, this paper could explain competition between firms in some real markets where some firms have more information than others.

Second, unlike previous models about learning by experimentation in oligopolistic markets with differentiated products (e.g., Aghion, Espinosa and Jullien, 1993; Harrington, 1995; Keller and Rady, 2003), the amount of information provided by market data cannot be influenced by firms' behaviour in the model presented here. The theoretical literature has used two types of strategies to introduce learning by experimentation. On the one hand, Aghion, Espinosa and Jullien (1993) and Keller and Rady (2003) considered that firms choose their prices in each period without observing a parameter of the demand functions,

which can only take two possible values, and a common random shock, which has a known distribution of probability. Under these assumptions, each firm can update its knowledge about the unknown parameter in the second period by observing the volume of its sales in the first period. On the other hand, Harrington (1995) proposed a similar duopolistic structure, but the unknown parameter can take values from a continuous interval. This paper shows that each firm can use the difference between its own sales and that of its rival in light of the price differential in the previous period as an unbiased predictor of the unknown parameter. In some markets, these informational assumptions are too strong.

In our model, the uninformed firm cannot observe its rival's volume of sales in the previous period<sup>3</sup> and each of the two unknown parameters can take values from a continuous interval. We find that, under certain standard assumptions, there will always be infinite values of one unknown parameter for each value of the other, which are consistent with market data observed by the uninformed firm in the first period. For this reason, the uninformed firm can never learn the true value of the unknown parameters from market data. Hence, the interaction between experimentation and competition is not considered in this model.

Finally, this paper adds different arguments to the price-increasing competition models. For instance, Chen and Riordan (2008) presented a discrete choice duopoly model of product differentiation in which the symmetric duopoly price can be higher than the monopoly price under certain conditions. However, Chen and Riordan did not take into account the effect of asymmetric information about market conditions. In our model, the

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<sup>3</sup> Firms' inability to observe their rivals' demand quantity realized in the prior periods is usual in some industries. For example, Kalnins (2006) describes hotel difficulties obtaining information about their rivals' occupancy rates in the USA.

best informed firm sets a higher price in a duopoly market than under monopoly if the demand uncertainty faced by the uninformed firm is higher than a certain threshold.

This paper is organized as follows. The next section describes the model, whereas the third analyses the equilibrium in a myopic game where both firms only compete in one period. Then, the fourth section deals with the equilibrium in the second period, which allows us to derive some economic predictions. Section 5 deduces the Bayesian Perfect Equilibrium of the game and illustrates the main implications of the model. Section 6 determines the conditions under which the effect of competition on prices is positive or negative, and finally, section 7 summarizes the main conclusions. The Appendix includes the proofs of each Proposition obtained.

## 2. THE MODEL

*Market structure.* Consider a duopoly lasting two periods. There are 2 risk-neutral firms in this market: firms i and u. Each firm has a constant unit cost of production equal to zero. The outputs of firms i and u at date t are denoted by  $q_t^i, q_t^u$ . Each firm simultaneously and independently chooses its price in each period. So  $p_t^i, p_t^u$  denote the prices of firms i and u at date t. Moreover, each firm sells a differentiated product. Over relevant ranges of output, the following system of linear inverse demand curves is assumed:

$$(1) \quad p_t^i = a - \frac{\beta}{\theta} q_t^i - \frac{\gamma}{\theta} q_t^u$$

$$(2) \quad p_t^u = a - \frac{\beta}{\theta} q_t^u - \frac{\gamma}{\theta} q_t^i$$

Where  $t = 1, 2$ ;  $\alpha, \beta, \gamma, \theta$  are the demand parameters, which are greater than zero, and  $\beta > \gamma$ , because if  $\beta = \gamma$ , both products are perfect substitutes<sup>4</sup>. As a higher value for  $\theta$  is associated with a higher cross-price elasticity in this specification, the substitutability of firms' products is increasing in  $\theta$ .

The values of  $a$  and  $\theta$  are drawn from the twice differentiable distribution functions  $F(a)$  and  $G(\theta)$ , with associated density functions,  $f(a)$  and  $g(\theta)$ , where  $\alpha \in [\underline{\alpha}, \bar{\alpha}]$  and  $\theta \in [0, \bar{\theta}]$ , where  $\bar{\alpha} > \underline{\alpha} \geq 0$  and  $\bar{\theta} > 0$ . The values of these parameters do not change over time. To avoid unnecessary complications, it is a requirement that support of  $\theta$  is sufficiently small such that no equilibria emerge in which a firm sells a negative quantity. It is assumed that,

$$(3) \quad E(\alpha) = \alpha^*$$

$$(4) \quad E(\theta) = 1$$

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<sup>4</sup> The system of linear demand curves specified is similar to that considered by Klemperer and Meyer (1986). They analysed the effect of two types of demand uncertainty on the strategic variable chosen by duopolists. First, they only included a common random additive shock in the intercept of the demand curves. Second, they specified a system of linear demand curves with fixed vertical intercept but with common multiplicative uncertainty about the slope and the degree of substitutability between products. The demand functions considered in our model simultaneously include both types of uncertainty in the same way as Reisinger and Rössner (2009). As Klemperer and Meyer (1986) showed, a rotation about a fixed vertical intercept for any demand curve represents a change in the total size of a market in which the distribution of consumers' reservation prices remains unchanged. On the other hand, they pointed out that a rotation about a fixed horizontal intercept represents a particular type of change in the distribution of reservation prices, with the total size of the market remaining unchanged. Finally, a vertically additive shift of the demand function is an intermediate case involving a change in both the size of the market and the distribution of the reservation prices. Reisinger and Rössner (2009) argued that introducing a shock to the intercept and a shock to the slope is more relevant in reality because uncertainty usually affects both market size and reservation price distribution.



Where  $E(\cdot)$  denotes the expectations about the demand parameters. Moreover, the random variables,  $a$  and  $\theta$ , are statistically independent and the probability distribution function of  $\theta$  is symmetric around 1, which is the average of  $\theta$ . From the definition of the variance of  $\theta$ ,  $\sigma_\theta^2$ , and from the assumptions (3) and (4), we obtain that  $E(\theta^2) = \sigma_\theta^2 + 1$ . As we have assumed that the distribution of  $\theta$  is symmetric,  $E(\theta - 1)^3 = 0$  and then,  $E(\theta^3) = 3E(\theta^2) - 2$ . The independence assumption of  $a$  and  $\theta$  implies that  $E(a\theta) = a^*$ ,  $E(a\theta^2) = a^* \cdot E(\theta^2)$  and  $E(a\theta^3) = a^* \cdot E(\theta^3)$ . All the information about the distributions of  $a$  and  $\theta$  is common knowledge.

*Firms' information in period 1.* Before choosing prices in the first period, it is assumed that firm i (the informed one) can observe all the demand parameters, including the realizations of  $a$  and  $\theta$ , but firm u (the uninformed one) cannot observe those realizations.

*Firms' information in period 2.* Before choosing its price in period 2, each firm observes the prices chosen in period 1,  $p_1^i, p_1^u$ , and its own demand quantity. Using this information, firm u updates its beliefs about the demand parameters,  $a$  and  $\theta$ , and firm i can infer its rival's volume of sales in the first period,  $q_1^u$ , because it knows all the demand parameters. However, firm u can never infer its rival's volume of sales<sup>5</sup>.

*Equilibrium: Definition and interpretation.* On the one hand, a strategy for firm i involves the specification of a price in period 1,  $p_1^i$ , and a function determining the period-2 price from its rival's realized quantity and from prices in period 1,  $p_2^i = \psi_2^i(q_1^u, p_1^i, p_1^u)$ . Firm i's price

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<sup>5</sup> Using the same specification of the demand curves, Klemperer and Meyer (1986) demonstrate that the optimal prices set by firms with constant marginal costs do not change with  $\theta$  for a given value of  $a$  because the distribution of reservation prices do not change with  $\theta$ . Hence, there are infinite values of  $\theta$  for each value of  $a$  which are consistent with the optimal prices set by firms in a given period. For this reason, firm u can never infer the values of  $a$  and  $\theta$  through observing the prices set by both firms in period 1.

in period 2 depends on its rival's demand and on both firms' prices in period 1 because firm  $u$  forms an expectation about the future demand curve from its realized demand and prices in the last period. Thus, firm  $i$  takes into account this expectation process when it chooses its price in period 2. On the other hand, a strategy for firm  $u$  is the specification of a price in period 1,  $p_1^u$ , and a function determining the period-2 price from firm  $u$ 's informational set in period 1,  $p_2^u = \psi_2^u[q_1^u, p_1^i, p_1^u]$ .

Then,  $[p_1^{i*}, \psi_2^{i*}(q_1^u, p_1^i, p_1^u)]$  is an equilibrium strategy for firm  $i$  if and only if it solves

$$\max_{p_1^i, \psi_2^i(q_1^u, p_1^i, p_1^u)} \pi^i = p_1^i q_1^i + \delta \psi_2^i(q_1^u, p_1^i, p_1^u) q_2^i$$

Where  $\delta \in [0,1]$  is the discount factor, which is common for both firms. Similarly,

$[p_1^{u*}, \psi_2^{u*}(q_1^u, p_1^i, p_1^u)]$  is an equilibrium strategy for firm  $u$  if and only if it solves

$$\max_{p_1^u, \psi_2^u(q_1^u, p_1^i, p_1^u)} \pi^u = E\{p_1^u q_1^u + \delta [\psi_2^u(q_1^u, p_1^i, p_1^u) q_2^u]\}$$

The expectation operator in the definition of equilibrium,  $E[.]$  is defined with respect to the distribution functions over  $(\alpha, \theta)$ . The concept of solution is a Perfect Bayesian Equilibrium. Both firms choose prices in a sequentially rational fashion. At period 2, each one chooses a price that maximizes expected period-2 profits, conditional on its observation of the period-1 prices and its own demand quantity (firm  $i$  also infers its rival's demand quantity). At period 1, each firm chooses a price that maximizes expected discounted profits, given its period-2 decision rule. This behaviour defines an equilibrium strategy.

### 3. MYOPIC PRICING

In preparation for the construction of Perfect Bayesian Equilibrium, this section introduces the benchmark of myopic behavior wherein we assume that firms only compete in one period.

In a one-shot game, firm  $i$  chooses a price to maximize its profits, that is,

$$\max_{p_1^i} \pi_1^i = p_1^i q_1^i$$

Using firm  $i$ 's demand equation (A.1) when  $t=1$  (see it in the Appendix), we can arrive at the reaction function of firm  $i$ ,

$$p_1^i = \frac{\alpha(\beta - \gamma)}{2\beta} + \frac{\gamma}{2\beta} p_1^u$$

As firm  $u$  does not know all the demand parameters, it expects that firm  $i$  faces the following problem:

$$\max_{p_1^i} E(\pi_1^i) = E(p_1^i q_1^i)$$

Now, we use firm  $i$ 's demand equation (A.1) when  $t=1$  (see it in the Appendix),  $E(\theta) = 1$  and the independence assumption of  $\alpha$  and  $\theta$  to achieve the expected reaction function of firm  $i$ , that is:

$$p_1^i = \frac{\alpha^*(\beta - \gamma)}{2\beta} + \frac{\gamma}{2\beta} p_1^u$$

Lastly, firm  $u$  chooses its price to maximize its expected profits in the following way:

$$\max_{p_1^u} E(\pi_1^u) = E(p_1^u q_1^u)$$

As before, using firm u's demand equation (A.2) when  $t=1$  (see it in the Appendix),  $E(\theta) = 1$  and the independence assumption of  $\alpha$  and  $\theta$ , the expected reaction function of firm u is the following:

$$p_1^u = \frac{\alpha^*(\beta - \gamma)}{2\beta} + \frac{\gamma}{2\beta} p_1^i$$

Firm i knows its rival believes that it will behave following its expected reaction function. Then, substituting the expected reaction function of firm i into the uninformed firm's reaction function, we calculate the optimal price set by firm u.

$$p_1^{u*} = \frac{\alpha^*(\beta - \gamma)}{2\beta - \gamma}$$

If we substitute this optimal price into the true reaction function of firm i, the optimal price set by this informed firm will be:

$$p_1^{i*} = \frac{(\beta - \gamma)[(2\beta - \gamma)\alpha + \gamma\alpha^*]}{2\beta(2\beta - \gamma)}$$

Which yields firm i the payoff

$$\pi_1^{i*} = \frac{(\beta - \gamma)\theta[(2\beta - \gamma)\alpha + \gamma\alpha^*]^2}{4\beta(\beta + \gamma)(2\beta - \gamma)^2}$$

While firm u's payoff is:

$$\pi_1^{u*} = \frac{\alpha^*(\beta - \gamma)\theta[(4\beta^2 - \gamma^2)\alpha - (2\beta^2 - \gamma^2)\alpha^*]}{2\beta(\beta + \gamma)(2\beta - \gamma)^2}$$

In this context, the optimal prices set by each firm will not depend on  $\theta$ . Furthermore, it is

easy to show that  $\frac{\partial p_1^{i*}}{\partial \alpha^*} > 0$ ,  $\frac{\partial p_1^{u*}}{\partial \alpha^*} > 0$  and  $\frac{\partial p_1^{i*}}{\partial \alpha^*} < \frac{\partial p_1^{u*}}{\partial \alpha^*}$ . Thus, if  $p_1^{i*} > p_1^{u*}$ , price dispersion

between firms will decrease with  $\alpha^*$ , while the opposite will occur if  $p_1^{i*} < p_1^{u*}$ .

#### 4. EQUILIBRIUM IN THE SECOND PERIOD GAME

The first step of the analysis is to characterize the second-period decision problem of each firm. On the one hand, firm  $i$  maximizes its profits in the second period,

$$(5) \max_{p_2^i} \pi_2^i = p_2^i q_2^i$$

However, as firm  $u$  does not know the demand conditions, it expects that firm  $i$  is facing the following problem:

$$(6) \max_{p_2^i} E(\pi_2^i) = E(p_2^i q_2^i)$$

On the other hand, firm  $u$  maximizes its expected profits in the second period,

$$(7) \max_{p_2^u} E(\pi_2^u) = E(p_2^u q_2^u)$$

We assume that the demand equations (1) and (2) are fulfilled. Solving problems (5), (6) and (7), we can obtain the reaction functions of firms  $i$  and  $u$ , respectively. Specifically, the first-order condition of problem (6) allows us to determine the expected reaction function of firm  $i$ , which must be substituted in the reaction function of firm  $u$  to obtain its optimal price for period 2 in equilibrium. Then, this price of firm  $u$  is included in the reaction function of firm  $i$  and we obtain its optimal price for period 2 in equilibrium. We can then analyse the following Proposition:

*PROPOSITION 1. Firm  $i$  can affect its rival's optimal price in period 2 by changing its price in the first period provided that the degree of substitutability between products is different from that expected by firm  $u$ .*

In other words, let  $p_2^{u*}$  be the equilibrium price in the second period for firm  $u$ . Then,

$$(8) \quad \frac{\partial p_2^{u*}}{\partial p_1^i} \begin{cases} \geq 0 \\ \leq 0 \end{cases} \text{ if and only if } \theta \begin{cases} \geq 1 \\ \leq 1 \end{cases}$$

The solution of the maximization of problems (5)-(7) and the demonstration of this Proposition are included in the Appendix, but its intuition is very simple. If firm i increased its price in the first period when  $\theta$  is higher than the expectation of its rival, the demand quantity of firm u realized in the first period would increase more than it expected. Hence, firm u would believe that the demand intercept is higher than expected in the first period and it would increase the price in period 2. The opposite would occur if firm i decreased its price when  $\theta$  is higher than 1. Likewise, if firm i decreased its price in the first period when  $\theta$  is lower than its rival's expectation, the demand quantity of firm u realized in period 1 would decrease less than it expected and once again, it would believe that the demand intercept is greater than expected. As a result, firm u would increase its price in the second period. The opposite would occur if firm i increased its price when  $\theta$  is lower than 1. In a nutshell, firm i would want to deceive its rival in period 2 by increasing its price in period 1 when  $\theta$  is greater than 1 and by decreasing its price when  $\theta$  is lower than 1. By doing so, firm i will face a less competitive rival in the second period.

From this Proposition, we can analyse the informed firm's incentive to fool its rival by changing its price in period 1 in order to obtain more profits in the second period. This incentive can be measured by the discounted increase in firm i's profits in period 2 due to the rise in the price of the uninformed firm induced by the change in the price set by firm i in the first period, minus this discounted increase in firm i's profits expected by firm u, that

is,  $\delta \left| \frac{\partial \pi_2^{i*}}{\partial p_2^u} \cdot \frac{\partial p_2^{u*}}{\partial p_1^i} \right| - \delta \left| E \left( \frac{\partial \pi_2^{i*}}{\partial p_2^u} \cdot \frac{\partial p_2^{u*}}{\partial p_1^i} \right) \right|$ ,<sup>6</sup> where  $\pi_2^{i*}$  is the profit obtained by firm i in the

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<sup>6</sup> From Proposition 1, firm i wants to increase its price when  $\theta$  is greater than 1, whereas it wants to decrease its price when  $\theta$  is lower than 1 to face a less competitive rival in period 2. As I am measuring firm i's profits from these changes in its price, I use the absolute values.

second period when both firms choose their equilibrium prices. If firm u could perfectly anticipate the values of  $a$  and  $\theta$ , which will occur when  $\theta = 1$ , its rival would not manage to fool it. In this case, firm u's expectations about the change in firm i's profits due to a change in firm u's price in period 2 induced by a change in firm i's price would coincide with the real change in firm i's profits. Thus, firm i can deceive its rival only when both expressions are different.

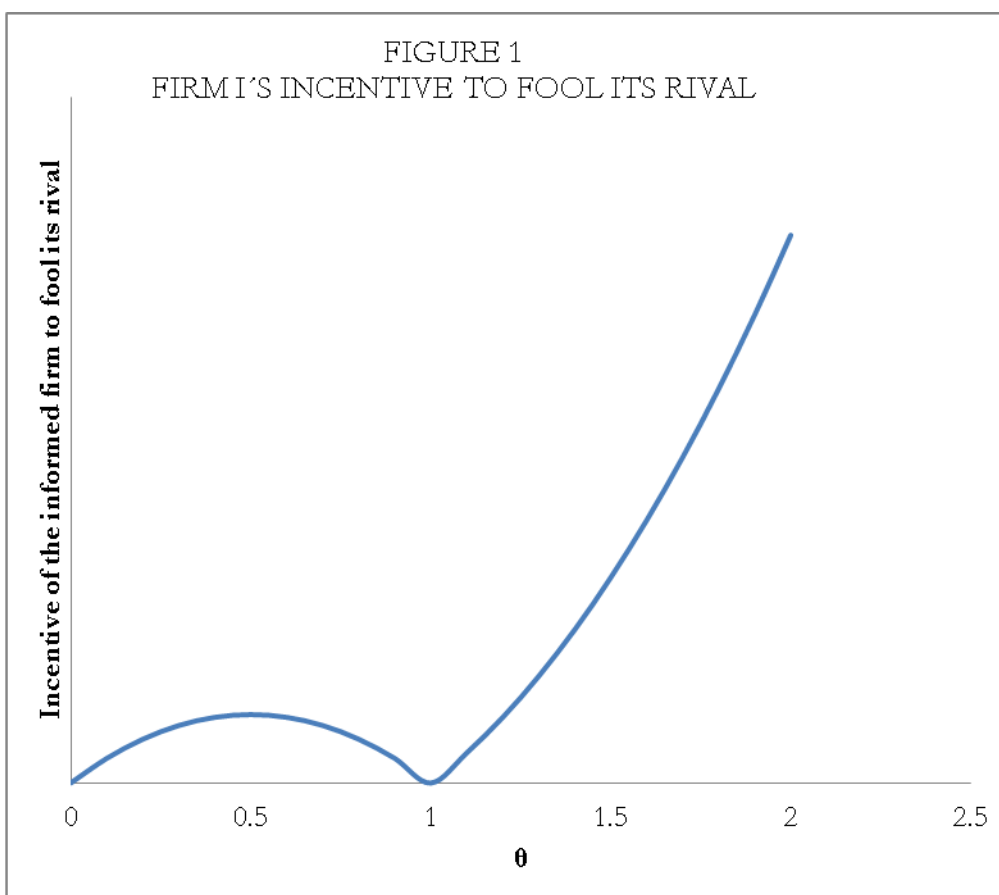
Using (A.1) for  $t=2$  and (A.16), we obtain this incentive for each optimal price set by firm i in the second period, that is,

$$(9) \quad \delta \left| \frac{\partial \pi_2^{i*}}{\partial p_2^u} \cdot \frac{\partial p_2^{u*}}{\partial p_1^i} \right| - \delta \left| E \left( \frac{\partial \pi_2^{i*}}{\partial p_2^u} \cdot \frac{\partial p_2^{u*}}{\partial p_1^i} \right) \right| = \delta p_2^{i*} \frac{\gamma^2 |\theta^2 - \theta|}{(\beta^2 - \gamma^2)(2\beta - \gamma)} - K$$

Where  $K = \delta \left| E \left( \frac{\partial \pi_2^{i*}}{\partial p_2^u} \cdot \frac{\partial p_2^{u*}}{\partial p_1^i} \right) \right|$ , which does not depend on  $\theta$ . The relationship between this incentive and  $\theta$  is represented in Figure 1 for arbitrary values of the remaining demand parameters, and it is a concave function with respect to  $\theta$  when  $\theta$  is lower than 1 and a convex function when  $\theta$  is greater than 1.

When  $0 < \theta < 1$ , firm i wants to decrease its price in period 1 to face a less competitive rival in period 2 as Proposition 1 shows. When  $\theta$  increases in this region, there are two opposite effects. On the one hand, the fall in the price set by the informed firm in period 1 will decrease the uninformed firm's sales closer to its own expectation as  $\theta$  increases in this region, and then, firm u will have a lower incentive to increase its price in the second period. On the other hand, the higher the degree of substitutability between products, the greater the increase in firm i's demand quantity achieved in the second period caused by the rise in the uninformed firm's price. When  $0 < \theta < 0.5$ , the second effect prevails over the first, but the opposite occurs when  $0.5 < \theta < 1$ , as we can observe in Figure 1.

When  $\theta > 1$ , firm  $i$  wants to increase its price in period 1 to fool its rival in period 2. In this region, we also find two effects as  $\theta$  goes up. First, when firm  $i$  increases its price in period 1, the difference between firm  $u$ 's demand quantity in this period and its expectation will increase with  $\theta$ . Therefore, firm  $u$ 's deception will increase with  $\theta$  in this region. Second, the increase in firm  $i$ 's demand quantity prompted by the rise in firm  $u$ 's price in period 2 will increase with  $\theta$ . Now, both effects are mutually reinforcing and for this reason, Figure 1 shows that firm  $i$ 's incentive to fool its rival grows at an increasing rate as the degree of substitutability between products rises in this region.



Firm  $i$  must take into account that the uninformed firm's choice in the first period will affect its own beliefs about the demand parameters provided that  $\theta$  is different from 1. As



a result, firm  $i$  must anticipate its rival's self-deception and adjust its price in period 2 when firm  $u$ 's price changes in period 1, as Proposition 2 shows.

PROPOSITION 2. *Changes in firm  $u$ 's price in period 1 will affect the optimal price set by firm  $i$  in the second period provided that the degree of substitutability between products is different from that expected by firm  $u$ .*

In other words, let  $p_2^{i*}$  be the equilibrium price in the second period for firm  $i$ . Then,

$$(10) \quad \frac{\partial p_2^{i*}}{\partial p_1^u} \begin{matrix} \geq \\ \leq \end{matrix} 0 \text{ if and only if } \theta \begin{matrix} \leq \\ \geq \end{matrix} 1$$

The demonstration of this Proposition can also be found in the Appendix, but its intuition is similar to the previous one. In particular, if firm  $u$  increased its price in the first period when  $\theta$  is greater than 1, its demand quantity achieved in period 1 would reduce more than it expected. Then, firm  $u$  would infer that the demand intercept is lower than it thought and it would reduce its price in period 2. Firm  $i$  would anticipate its rival's behaviour and would reduce its price in the second period. The opposite occurs when  $\theta$  is lower than 1.

## 5. PERFECT BAYESIAN EQUILIBRIUM

The following step is to analyse the decisions of firms in the first period, given the decision rules in the second period. The problem of firm  $i$  in the first period will be:

$$(11) \quad \max_{p_1^i} \pi^i = p_1^i q_1^i + \delta \pi_2^{i*}$$

The first-order condition for this problem will be:

$$(12) \quad \frac{\partial \pi^i}{\partial p_1^i} = q_1^{i*} + p_1^{i*} \frac{\partial q_1^i}{\partial p_1^i} + \delta \frac{\partial \pi_2^{i*}}{\partial p_1^i} = 0$$

Where  $q_1^{i*}$  is the demand quantity for firm  $i$  in period 1 when both firms fix the equilibrium prices. We can apply the envelope theorem, that is,  $\frac{\partial \pi_2^{i*}}{\partial p_1^i} = p_2^{i*} \frac{\partial q_2^{i*}}{\partial p_1^i}$ , where  $q_2^{i*}$  is the demand quantity for firm  $i$  in period 2 when both firms choose the equilibrium prices.

However, firm  $u$  expects that firm  $i$  faces the following maximization problem:

$$(13) \max_{p_1^i} E(\pi^i) = E(p_1^i q_1^i) + \delta E(\pi_2^{i*})$$

Then, the expected first-order condition of firm  $i$  will be:

$$(14) \frac{\partial E(\pi^i)}{\partial p_1^i} = E \left[ q_1^{i*} + p_1^{i*} \frac{\partial q_1^i}{\partial p_1^i} + \delta \frac{\partial \pi_2^{i*}}{\partial p_1^i} \right] = 0$$

Once again, a similar version of the envelope theorem can be applied, that is,

$$E \left( \frac{\partial \pi_2^{i*}}{\partial p_1^i} \right) = E \left( p_2^{i*} \frac{\partial q_2^{i*}}{\partial p_1^i} \right).$$

Finally, the problem of the risk-neutral firm  $u$  in period 1 will be:

$$(15) \max_{p_1^u} E(\pi^u) = E(p_1^u q_1^u) + \delta E(\pi_2^{u*})$$

Where  $\pi_2^{u*}$  is the profit obtained by firm  $u$  in the second period when both firms choose their equilibrium prices. Now, the expected first-order condition of firm  $u$  will be:

$$(16) \frac{\partial E(\pi^u)}{\partial p_1^u} = E \left[ q_1^{u*} + p_1^{u*} \frac{\partial q_1^u}{\partial p_1^u} + \delta \frac{\partial \pi_2^{u*}}{\partial p_1^u} \right] = 0$$

Where  $q_1^{u*}$  is the demand quantity for firm  $u$  in period 1 when both firms fix the equilibrium prices. If we subtract (14) from (12), we arrive at the following expression:

$$(17) \left[ q_1^{i*} - E(q_1^{i*}) + p_1^{i*} \frac{\partial q_1^i}{\partial p_1^i} - E \left( p_1^{i*} \frac{\partial q_1^i}{\partial p_1^i} \right) \right] + \left[ \delta \frac{\partial \pi_2^{i*}}{\partial p_2^{u*}} \frac{\partial p_2^{u*}}{\partial p_1^i} - \delta E \left( \frac{\partial \pi_2^{i*}}{\partial p_2^{u*}} \frac{\partial p_2^{u*}}{\partial p_1^i} \right) \right] = 0$$

Where  $\frac{\partial \pi_2^{i*}}{\partial p_2^{u*}} \frac{\partial p_2^{u*}}{\partial p_1^i} = \frac{\partial \pi_2^{i*}}{\partial p_1^i}$  and  $E\left(\frac{\partial \pi_2^{i*}}{\partial p_2^{u*}} \frac{\partial p_2^{u*}}{\partial p_1^i}\right) = E\left(\frac{\partial \pi_2^{i*}}{\partial p_1^i}\right)$ . On the one hand, the first term in brackets in (17) measures firm  $i$ 's incentive to surprise its rival in the first period, that is, this expression is the difference between the true change in firm  $i$ 's profits in period 1 caused by a change in its price and that expected by firm  $u$ . On the other hand, the second term in brackets represents firm  $i$ 's incentive to fool its rival in the second period by changing its price in the first one as we saw in the last Section (see equation (9)). Both incentives must be equal in equilibrium.

The solution of the maximization problems (11), (13) and (15) is included in the Appendix and we can use firm  $i$ 's solution in (A.26) to obtain the effect of an increase in demand uncertainty faced by firm  $u$  on the optimal price set by firm  $i$  through the following Proposition:

**PROPOSITION 3.** *The higher the demand uncertainty faced by firm  $u$ , the greater the optimal price set by firm  $i$  in period 1, except for intermediate values of  $\theta$ .*

In other words,  $\frac{\partial p_1^{i*}}{\partial \sigma_\theta^2}$  depends on  $\theta$  in the following manner:

$$(18) \quad \frac{\partial p_1^{i*}}{\partial \sigma_\theta^2} > 0 \text{ if } 0 < \theta < 1 + \frac{2(2\beta - \gamma)}{\gamma\sqrt{\delta}}$$

$$(19) \quad \frac{\partial p_1^{i*}}{\partial \sigma_\theta^2} < 0 \text{ if } 1 + \frac{2(2\beta - \gamma)}{\gamma\sqrt{\delta}} < \theta < 1 + \frac{\beta(2\beta - \gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}}$$

$$(20) \quad \frac{\partial p_1^{i*}}{\partial \sigma_\theta^2} > 0 \text{ if } \theta > 1 + \frac{\beta(2\beta - \gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}}$$

$$(21) \quad \frac{\partial p_1^{i*}}{\partial \sigma_\theta^2} = 0 \text{ if } \theta = 1 + \frac{2(2\beta - \gamma)}{\gamma\sqrt{\delta}} \text{ or } \theta = 1 + \frac{\beta(2\beta - \gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}}$$

The proof of this Proposition is included in the Appendix, but Figure 2 represents the different regions obtained.

To understand the intuition of this Proposition, we need to turn back to firm  $i$ 's incentive to mislead its rival in the second period by changing its price in period 1 (Proposition 1)<sup>7</sup>. In particular, an increase in demand uncertainty faced by the uninformed firm will affect this incentive in two ways. First, the greater the demand uncertainty faced by firm  $u$ , the greater the effect of a change in firm  $i$ 's price in period 1 on firm  $u$ 's behaviour in period 2 and then, firm  $i$ 's incentive to fool its rival will increase with demand uncertainty (uncertainty effect). Secondly, as the relationship between  $\alpha$  and  $\theta$  is decreasingly convex (see equation A.3 in the Appendix), a mean-preserving spread of  $\theta$  will increase firm  $u$ 's expectation about the demand intercept in the second period. Thus, firm  $i$ 's incentive to mislead its rival will decrease with demand uncertainty because firm  $u$ 's self-deception will increase (expectation effect).

We can distinguish several regions. When  $\theta < 1$ , the second effect is greater than the first one because the demand intercept expected by firm  $u$  will increase more for low values of  $\theta$ , given the high convexity of the relationship between  $\alpha$  and  $\theta$  in this region. Thus, the rise in firm  $u$ 's uncertainty will bring down firm  $i$ 's incentive to mislead its rival by decreasing its price in the second period as shown in (18). Due to continuity, this also occurs for some values of  $\theta$  greater than 1. When  $\theta > 1$ , once again, the convexity of the relationship between  $\alpha$  and  $\theta$  is so high for low values of  $\theta$ , that the expectation effect will dominate the uncertainty one. Hence, an increase in demand uncertainty will bring down firm  $i$ 's incentive to deceive its rival by raising its price in period 1 as shown in (19). However, the uncertainty effect will dominate the expectation one for sufficiently high values of  $\theta$  because the convexity of the relationship between  $\alpha$  and  $\theta$  (equation A.3) is

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<sup>7</sup> Remember that firm  $i$  would want to fool its rival in the second period by decreasing its price in period 1 when  $\theta$  is lower than 1 and by increasing it when  $\theta$  is greater than 1.

lower. In this case, a rise in the variability of  $\theta$  will increase firm  $i$ 's incentive to raise its price in period 1 in order to mislead its rival in period 2 as shown in (20).

FIGURE 2

EFFECTS OF DEMAND UNCERTAINTY FACED BY FIRM  $u$  ON THE OPTIMAL PRICE SET BY FIRM  $i$

$$\begin{array}{c}
 \begin{array}{ccc}
 + & - & + \\
 \hline
 1 + \frac{2(2\beta - \gamma)}{\gamma\sqrt{\delta}} & & 1 + \frac{\beta(2\beta - \gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}} \\
 \hline
 & & \frac{\partial p_i^*}{\partial \sigma_\theta^2} \\
 & & \theta
 \end{array}
 \end{array}$$

The next step is to analyse the effect of an increase in demand uncertainty faced by firm  $u$  on its optimal price set in the first period. Using (A.25), we find that this effect is unambiguous, as the following Proposition shows:

PROPOSITION 4. *The higher the demand uncertainty faced by the uninformed firm, the greater its optimal price in period 1.*

In other words,

$$(22) \quad \frac{\partial p_i^*}{\partial \sigma_\theta^2} > 0 \quad \forall \theta$$

The Appendix includes the proof of this Proposition. Once again, we have to focus our attention on firm  $i$ 's incentive to mislead its rival in period 2 to understand the intuition of this Proposition. As Figure 1 shows, a reduction in  $\theta$  below its expectation would increase firm  $i$ 's incentive to deceive its rival by decreasing its price in period 1, but to a lesser extent than a rise in  $\theta$  above its expectation of the same magnitude would increase firm  $i$ 's incentive to deceive firm  $u$  by increasing its price in the first period. Thus, a mean-

preserving spread of  $\theta$  will provide firm  $i$  with incentives, on average, to inflate its price. When the uninformed firm is a monopolist in the market, the uncertainty about  $\theta$  does not affect the optimal price set by this firm because changes in  $\theta$  only generate isoelastic demand shifts<sup>8</sup>. However, under the presence of an informed firm and the possibility that this firm can influence firm  $u$ 's beliefs about the demand parameters for the next period, the higher the uncertainty about  $\theta$ , the higher the price set by the uninformed firm because it expects to face, on average, a less competitive rival.

Next, we analyse the effect of an increase in demand uncertainty faced by firm  $u$  on price dispersion in this model. As shown in the Appendix, it is easy to see that firm  $i$  will set a higher price than firm  $u$  in the first period provided that the realized intercept of the demand curve is sufficiently high and the degree of substitutability between products is sufficiently low. But even when the demand intercept is sufficiently low, firm  $i$  will also set a greater price than its competitor in period 1 to fool it in the second period if both products are substitute enough. Otherwise, the informed firm will set a lower price than the uninformed one in period 1. The following Proposition shows the relationship between price dispersion and demand uncertainty for each case.

**PROPOSITION 5.** *If the optimal price set by the informed firm is higher than that of its rival in the first period, price dispersion between firms will increase with demand uncertainty when products are sufficiently substitute and will decrease in other case. The opposite will occur when the optimal price set by firm  $i$  is lower than the optimal price set by firm  $u$  in period 1.*

In other words,  $\frac{\partial |p_1^{i*} - p_1^{u*}|}{\partial \sigma_\theta^2}$  depends on  $\theta$  in the following manner.

$$(23) \text{ If } \alpha > \lambda(\theta)\alpha^* \text{ and } \theta \leq 1 + \frac{\beta(2\beta - \gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}}, \text{ then } p_1^{i*} \geq p_1^{u*} \text{ and } \frac{\partial |p_1^{i*} - p_1^{u*}|}{\partial \sigma_\theta^2} < 0$$

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<sup>8</sup> For example, see Klemperer and Meyer (1986).

$$(24) \text{ If } a < \lambda(\theta)a^* \text{ and } \theta \cong 1 + \frac{\beta(2\beta-\gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}}, \text{ then } p_1^{i*} \cong p_1^{u*} \text{ and } \frac{\partial |p_1^{i*} - p_1^{u*}|}{\partial \sigma_\theta^2} > 0$$

where

$$\lambda(\theta) = \frac{[8\beta(2\beta-\gamma)^2(2\beta+\gamma) + 8\delta\beta^2(2\beta-\gamma)(4\beta^2 + \beta\gamma - 2\gamma^2)\sigma_\theta^2 + \delta^2\gamma^3(2\beta^2 - \gamma^2)\sigma_\theta^4][4\beta(2\beta-\gamma)^3 + \delta\gamma^3(\beta-\gamma)(\theta-1)^2]}{2\beta(2\beta-\gamma)[4(2\beta-\gamma)^2(4\beta^2 - \gamma^2) + \delta(8\beta^4 - 8\beta^2\gamma^2 + \gamma^4)\sigma_\theta^2][4\beta(2\beta-\gamma)^2 + \delta\gamma^2(2\beta-\gamma)(\theta-1) + \delta\gamma^3\theta(\theta-1)]}$$

. The proof of this Proposition can be found in the Appendix. It shows that the relationship between demand uncertainty and price dispersion depends on the relative prices of both firms and on the true degree of substitutability between products. From previous Propositions it is clear that if  $\theta$  is sufficiently high, firm  $i$ 's incentive to deceive its rival by increasing its price in the first period will increase with the level of uncertainty. As firm  $u$  lacks this incentive in period 1, the optimal price set by firm  $i$  will increase with demand uncertainty more than the optimal price set by firm  $u$ . The opposite will occur when  $\theta$  is sufficiently low. For this reason, when the degree of substitutability between products is high enough, price dispersion increases with demand uncertainty provided that the informed firm sets a higher price than the uninformed one, but price dispersion decreases as demand uncertainty increases when firm  $i$  sets a lower price than its rival. When the degree of substitutability between products is sufficiently low, the relationship between price dispersion and demand uncertainty is just the opposite.

## 6. THE EFFECT OF COMPETITION ON PRICES

Finally, it is interesting to compare the price set by the informed firm in period 1 to the price set by this firm in a monopoly market. This comparison can be helpful to understand the consequences of a new entrant under certain circumstances. For example, when a new firm enters the market, the incumbent usually has better information about consumers because it has more experience in the market. We can then obtain some useful predictions

about prices under these conditions, comparing the price set by an informed monopolist to the price set by the informed firm in this model.

LEMMA 1. *When the realized demand parameters are sufficiently close to their expectations, the optimal price set by the informed firm in period 1 in this duopoly context is higher than the optimal price set by this firm in a monopoly market provided that demand uncertainty faced by the uninformed firm is high enough.*

In other words, within certain intervals of both unknown parameters around their averages,  $\alpha \in (\alpha^* - \varepsilon, \alpha^* + \varepsilon)$ ,  $\theta \in (1 - \omega, 1 + \omega)$ , where  $\varepsilon$  and  $\omega$  are sufficiently low, there exists a threshold,  $\overline{\sigma_\theta^2}$ , for  $\sigma_\theta^2$ , such that

$$p_1^{i*} \underset{\leq}{\overset{\geq}} p_1^{iM} \text{ when } \sigma_\theta^2 \underset{\leq}{\overset{\geq}} \overline{\sigma_\theta^2}$$

where  $p_1^{iM}$  is the optimal price set by an informed monopolist in the first period under the demand conditions given by (1) and (2). If we denote the total market demand in each period as  $q_t$ , we can prove that  $p_1^{iM} = \frac{\alpha}{2}$  by substituting  $p_t$  for  $p_t^i$  and  $p_t^u$  and  $q_t$  for  $q_t^i + q_t^u$  when  $t = 1$  in the linear demand curves (1) and (2) and solving the resulting maximization problem of the monopolist. It is easy to see that  $p_1^{iM}$  is greater than  $p_1^{i*}$  when  $\alpha = \alpha^*$ ,  $\theta = 1$  and  $\sigma_\theta^2 = 0$ . Since  $p_1^{i*}$  is a continuous function with respect to  $\alpha$  and  $\theta$  around the averages of both unknown parameters,  $p_1^{iM}$  does not depend on  $\sigma_\theta^2$  and  $\frac{\partial p_1^{i*}}{\partial \sigma_\theta^2} > 0$  when  $\theta$  is around its average as Proposition 3 shows, the demonstration of

Lemma 1 is obvious.

To end this Section, the following Lemma shows the same comparison for firm u, but this result is more general because it does not depend on  $\theta$ .

LEMMA 2. *The uninformed firm sets a higher price in period 1 than in a monopoly situation if the demand uncertainty is sufficiently high.*



In other words, there exists a threshold,  $\overline{\sigma_\theta^2}$ , for  $\sigma_\theta^2$ , such that,

$$\forall \theta, p_1^{u*} \underset{\leq}{\geq} p_1^{uM} \text{ when } \sigma_\theta^2 \underset{\leq}{\geq} \overline{\sigma_\theta^2}$$

where  $p_1^{uM}$  is the optimal price set by an uninformed monopolist in the first period under the demand conditions given by (1) and (2). Since  $p_1^{uM} = \frac{a^*}{2}$ ,  $p_1^{uM}$  is greater than  $p_1^{u*}$  when

$\sigma_\theta^2 = 0$  and  $\frac{\partial p_1^{u*}}{\partial \sigma_\theta^2} > 0$  from Proposition 4, the demonstration of Lemma 2 is also obvious.

## 7. CONCLUSION

This paper presents a two-stage game model where two firms offer differentiated products and one of them faces demand uncertainty, which affects both the intercept and the slope of the demand curve. In each period, each firm sets its price simultaneously and non-cooperatively.

Previous theoretical papers have studied firms' decisions in oligopolistic markets under demand uncertainty, but their assumptions about the information regarding market conditions are very restrictive for some situations. In particular, some authors assume that each firm can observe its rivals' prices and volume of sales in the last periods before choosing quantities or prices. Others consider that all the firms in the market have the same information about the demand conditions. Using more plausible assumptions for some markets, this model can help to explain some empirical puzzles.

First, some empirical literature has shown that firms responses' to changes in prices of their rivals depend on the degree of product differentiation between competing goods (Pels and Rietveld 2004; Ward et al. 2002). For example, Pels and Rietveld (2004) studied the pricing policies of airlines on the London-Paris route and observed that some of them

lower their fares when other competitors raise theirs, whereas others might follow the price movements of a competitor. Nonetheless, they can only turn to the random nature of demand in this context to explain their results. We show that each firm's response in period 2 to its rival's changes in prices in the first period will depend on whether the degree of substitutability between products is greater or lower than its expectation.

Second, the empirical evidence about the effect of the Internet on price dispersion is diverse. While some authors suggested that price dispersion is higher in online markets than in offline ones, others estimated the opposite (Clay et al. 2002; Haynes and Thompson 2008; Orlov 2011). If there are some firms with better information about market conditions than others and demand uncertainty is higher in online markets than in offline ones, at least in the first stage of the Internet, the model presented here can help to explain these mixed results. In particular, it is shown that the relationship between demand uncertainty and price dispersion depends on the relative prices of the informed and the uninformed competitors and on the degree of substitutability between products. For example, if the informed firm sets a higher price than the uninformed one, price dispersion will increase with demand uncertainty in markets for highly substitutable products, while the opposite will occur in markets for highly differentiated products.

Finally, this model can provide a more plausible explanation for the recent price-increasing competition evidence obtained by some empirical papers in the food industry (e.g., Ward et al., 2002; Thomadsen, 2007). For example, Ward et al. (2002) found that the entry of new private labels raised prices of national brands in the food industry, and Thomadsen (2007) obtained that prices may be higher under duopoly competition than under monopoly in the fast-food industry. In fact, it is very unlikely that a new entrant in a market has the same information about potential consumers as the incumbent, as other theoretical models assume. We show that even when the expectations about the demand parameters are close

to their realized values, the incumbent, which is better informed on market conditions, will set a higher price with the new competitor than without it if demand uncertainty faced by the new entrant is sufficiently high. This model generalizes this result to a situation in which the new entrant has more information about market conditions than the incumbent.

We end by pointing out some limitations of the model. Firstly, the results of this paper may depend on the functional forms of the demand and cost curves. Similarly, the assumption of statistical independence between unknown demand parameters can be too restrictive in some contexts. Furthermore, some firms might have imperfect but better information about market conditions than others. In this case, the quality of private information about demand parameters would be higher for the best informed firms. Although the robustness of the predictions of this model to these alternative assumptions is an open question for future research, it could help to explain pricing policies in some contexts. Specifically, the implications of the model can be fulfilled in markets where one firm has much more experience than its rivals and the latter have imperfect information about the demand conditions.

## APPENDIX

*Proof of Proposition 1.* Following the backward induction method, we begin with the analysis of the equilibrium in the second period. Starting with firm u, it chooses its price to maximize its profit in the second period,

$$\max_{p_2^u} E(\pi_2^u) = E(p_2^u q_2^u)$$

From the system of linear inverse demand curves given in equations (1) and (2), we obtain

$$(A.1) \quad q_t^i = \frac{\alpha\theta}{(\beta+\gamma)} - \frac{\beta\theta}{(\beta^2-\gamma^2)} p_t^i + \frac{\gamma\theta}{(\beta^2-\gamma^2)} p_t^u$$

$$(A.2) \quad q_t^u = \frac{\alpha\theta}{(\beta+\gamma)} - \frac{\beta\theta}{(\beta^2-\gamma^2)} p_t^u + \frac{\gamma\theta}{(\beta^2-\gamma^2)} p_t^i$$

where  $t = 1, 2$ . At the end of period 1, firm u observes both prices and the realization of its own demand quantity. From these market data, firm u infers the relationship between  $\alpha$  and  $\theta$ . In particular, when  $t = 1$ , we can rearrange the equation (A.2) to obtain  $\alpha$  depending on  $p_1^i, p_1^u$  and  $q_1^u$ ,

$$(A.3) \quad \alpha = \frac{(\beta+\gamma)}{\theta} q_1^u + \frac{\beta}{(\beta-\gamma)} p_1^u - \frac{\gamma}{(\beta-\gamma)} p_1^i$$

Using this relationship,  $E(\theta) = 1$ , and the demand equation (A.2) when  $t=2$ , the problem of the risk-neutral firm u in the second period will be the following:

(A.4)

$$\max_{p_2^u} E(\pi_2^u) = q_1^u p_2^u + \frac{\beta}{(\beta^2-\gamma^2)} p_1^u p_2^u - \frac{\gamma}{(\beta^2-\gamma^2)} p_1^i p_2^u - \frac{\beta}{(\beta^2-\gamma^2)} p_2^{u2} + \frac{\gamma}{(\beta^2-\gamma^2)} p_2^i p_2^u$$

The first-order condition is

$$(A.5) \quad \frac{\partial E(\pi_2^u)}{\partial p_2^u} = q_1^u + \frac{\beta}{(\beta^2-\gamma^2)} p_1^u - \frac{\gamma}{(\beta^2-\gamma^2)} p_1^i - \frac{2\beta}{(\beta^2-\gamma^2)} p_2^u + \frac{\gamma}{(\beta^2-\gamma^2)} p_2^i = 0$$

Then, the expected reaction function of firm u in the second period is

$$(A.6) \quad p_2^u = \frac{(\beta^2-\gamma^2)}{2\beta} q_1^u + \frac{1}{2} p_1^u - \frac{\gamma}{2\beta} p_1^i + \frac{\gamma}{2\beta} p_2^i$$

Firm i knows that firm u behaves according to this reaction function. Firm u expects that firm i will face the following problem:

$$\max_{p_2^i} E(\pi_2^i) = E(p_2^i q_2^i)$$

Then, using (A.3), firm  $i$ 's demand equation (A.1) when  $t=2$  and  $E(\theta) = \mathbf{1}$ , the expected problem of firm  $i$  in period 2 will be

$$(A.7) \quad \max_{p_2^i} E(\pi_2^i) = q_1^u p_2^i + \frac{\beta}{(\beta^2 - \gamma^2)} p_1^u p_2^i - \frac{\gamma}{(\beta^2 - \gamma^2)} p_1^i p_2^i - \frac{\beta}{(\beta^2 - \gamma^2)} p_2^{i2} + \frac{\gamma}{(\beta^2 - \gamma^2)} p_2^u p_2^i$$

The expected first-order condition of firm  $i$  is

$$(A.8) \quad \frac{\partial E(\pi_2^i)}{\partial p_2^i} = q_1^u + \frac{\beta}{(\beta^2 - \gamma^2)} p_1^u - \frac{\gamma}{(\beta^2 - \gamma^2)} p_1^i - \frac{2\beta}{(\beta^2 - \gamma^2)} p_2^i + \frac{\gamma}{(\beta^2 - \gamma^2)} p_2^u = 0$$

Then, the expected reaction function of firm  $i$  is

$$(A.9) \quad p_2^i = \frac{(\beta^2 - \gamma^2)}{2\beta} q_1^u + \frac{1}{2} p_1^u - \frac{\gamma}{2\beta} p_1^i + \frac{\gamma}{2\beta} p_2^u$$

We can substitute  $p_2^i$  from equation (A.9) into (A.6) to obtain the optimal price set by firm  $u$  in the second period,  $p_2^{u*}$ ,

$$(A.10) \quad p_2^{u*} = \frac{(\beta^2 - \gamma^2)}{(2\beta - \gamma)} q_1^u + \frac{\beta}{(2\beta - \gamma)} p_1^u - \frac{\gamma}{(2\beta - \gamma)} p_1^i$$

As firm  $i$  knows  $a$  and  $\theta$ , it chooses its price to maximize its profit in period 2,

$$\max_{p_2^i} \pi_2^i = p_2^i q_2^i$$

Then if we use the demand equation (A.1) when  $t=2$ , the problem of this firm will be:

$$(A.11) \quad \max_{p_2^i} \pi_2^i = \frac{a\theta}{(\beta + \gamma)} p_2^i - \frac{\beta\theta}{(\beta^2 - \gamma^2)} p_2^{i2} + \frac{\gamma\theta}{(\beta^2 - \gamma^2)} p_2^u p_2^i$$

The first-order condition of firm  $i$  will be:

$$(A.12) \quad \frac{\partial \pi_2^i}{\partial p_2^i} = \frac{a\theta}{(\beta + \gamma)} - \frac{2\beta\theta}{(\beta^2 - \gamma^2)} p_2^i + \frac{\gamma\theta}{(\beta^2 - \gamma^2)} p_2^u = 0$$

Thus, the reaction function of firm  $i$  will be:

$$(A.13) \quad p_2^i = \frac{(\beta-\gamma)a}{2\beta} + \frac{\gamma}{2\beta} p_2^u$$

Firm i knows that its rival chooses its price following equation (A.10). Thus, if  $p_2^{u*}$  from this equation is included in (A.13), the equilibrium price of firm i is obtained:

$$(A.14) \quad p_2^{i*} = \frac{(\beta-\gamma)a}{2\beta} + \frac{\gamma(\beta^2-\gamma^2)}{2\beta(2\beta-\gamma)} q_1^u + \frac{\gamma}{2(2\beta-\gamma)} p_1^u - \frac{\gamma^2}{2\beta(2\beta-\gamma)} p_1^i$$

As firm i knows  $a$ ,  $\theta$ ,  $p_1^i$ ,  $p_1^u$  and  $q_1^i$  before choosing its price in the second period, it can infer  $q_1^u$  using the system of linear inverse demand curves. Now, from (A.10), we can analyse the change in the optimal price set by the uninformed firm in the second period due to a change in the informed firm's price in the first period given  $p_1^u$ :

$$(A.15) \quad \frac{\partial p_2^{u*}}{\partial p_1^i} = \frac{(\beta^2-\gamma^2)}{(2\beta-\gamma)} \frac{\partial q_1^u}{\partial p_1^i} - \frac{\gamma}{(2\beta-\gamma)}$$

Equation (A.2) when  $t=1$  is used to calculate  $\frac{\partial q_1^u}{\partial p_1^i}$  given  $p_1^u$  and this is included in (A.15).

Hence, we have the following result:

$$(A.16) \quad \frac{\partial p_2^{u*}}{\partial p_1^i} = \frac{\gamma(\theta-1)}{(2\beta-\gamma)}$$

As  $\beta > \gamma$ , Proposition 1 has been demonstrated.

*Proof of Proposition 2.* We can now calculate the reaction of the informed firm's price in the second period due to a change in the uninformed firm's price in period 1 from equation

(A.14) given  $p_1^i$ ,

$$(A.17) \quad \frac{\partial p_2^{i*}}{\partial p_1^u} = \frac{\gamma(\beta^2-\gamma^2)}{2\beta(2\beta-\gamma)} \frac{\partial q_1^u}{\partial p_1^u} + \frac{\gamma}{2(2\beta-\gamma)}$$

Using firm u's demand equation (A.2) when  $t=1$  to obtain  $\frac{\partial q_1^u}{\partial p_1^u}$  given  $p_1^i$ , we arrive at the following result.

$$(A.18) \quad \frac{\partial p_2^{i*}}{\partial p_1^u} = \frac{\gamma(1-\theta)}{2(2\beta-\gamma)}$$

As  $\beta > \gamma$ , Proposition 2 has been demonstrated.

*Perfect Bayesian Equilibrium.* Now, we obtain the equilibrium prices in the first period from the maximization problems of firms i and u, that is, from (11), (13) and (15). First of all, we obtain the reaction functions of both firms. Starting with the informed firm, if we substitute firm i's demand equation (A.1) for  $t=1$  into (11) and use the envelope theorem, the following first-order condition for firm i will be obtained,

$$(A.19) \quad \frac{a\theta(\beta-\gamma)}{(\beta^2-\gamma^2)} - \frac{2\beta\theta}{(\beta^2-\gamma^2)} p_1^i + \frac{\gamma\theta}{(\beta^2-\gamma^2)} p_1^u + \delta \left[ \frac{(\beta-\gamma)a}{2\beta} + \frac{\gamma(\beta^2-\gamma^2)}{2\beta(2\beta-\gamma)} q_1^u + \frac{\gamma}{2(2\beta-\gamma)} p_1^u - \frac{\gamma^2}{2\beta(2\beta-\gamma)} p_1^i \right] \left[ -\frac{\beta\theta}{(\beta^2-\gamma^2)} \frac{\partial p_2^{i*}}{\partial p_1^i} + \frac{\gamma\theta}{(\beta^2-\gamma^2)} \frac{\partial p_2^{u*}}{\partial p_1^i} \right] = 0$$

We can calculate  $\frac{\partial p_2^{i*}}{\partial p_1^i}$  and  $\frac{\partial p_2^{u*}}{\partial p_1^i}$  from (A.14) and (A.10) and  $q_1^u$  can be substituted by the expression (A.2) for  $t=1$ . After these substitutions and some operations, the reaction function of firm i in the first period will be:

$$(A.20) \quad p_1^i = \frac{m}{n} + \frac{s}{n} p_1^u$$

where,

$$m = 4\beta(\beta-\gamma)(2\beta-\gamma)^2 a + \delta\gamma^2(\beta-\gamma)(2\beta-\gamma+\gamma\theta)a(\theta-1)$$

$$n = 8\beta^2(2\beta-\gamma)^2 - \delta\gamma^4(\theta-1)^2$$

$$s = 4\beta\gamma(2\beta-\gamma)^2 - \delta\beta\gamma^3(\theta-1)^2$$

Now, we proceed with solving the informed firm problem as expected by firm u. Assuming that  $E(\theta) = 1$ , substituting firm i's demand equation (A.1) for  $t=1$  into (13) and using the envelope theorem, the expected first-order condition for firm i will be:

(A.21)

$$\frac{E(a\theta)(\beta-\gamma)}{(\beta^2-\gamma^2)} - \frac{2\beta}{(\beta^2-\gamma^2)}p_1^i + \frac{\gamma}{(\beta^2-\gamma^2)}p_1^u + \delta E \left\{ \left[ \frac{(\beta-\gamma)a}{2\beta} + \frac{\gamma(\beta^2-\gamma^2)}{2\beta(2\beta-\gamma)}q_1^u + \frac{\gamma}{2(2\beta-\gamma)}p_1^u - \frac{\gamma^2}{2\beta(2\beta-\gamma)}p_1^i \right] \left[ -\frac{\beta\theta}{(\beta^2-\gamma^2)}\frac{\partial p_2^{i*}}{\partial p_1^i} + \frac{\gamma\theta}{(\beta^2-\gamma^2)}\frac{\partial p_2^{u*}}{\partial p_1^i} \right] \right\} = 0$$

By calculating  $\frac{\partial p_2^{i*}}{\partial p_1^i}$  and  $\frac{\partial p_2^{u*}}{\partial p_1^i}$  from (A.14) and (A.10), substituting  $q_1^u$  by the expression (A.2)

for  $t=1$  and assuming that  $E(\theta) = 1$ ,  $E(a) = a^*$  and that  $a$  and  $\theta$  are statistically independent, the expected reaction function of firm i in the first period can be expressed as:

$$(A.22) \quad p_1^i = \frac{m'}{n'} + \frac{s'}{n'}p_1^u$$

where,

$$m' = 4\beta(\beta-\gamma)(2\beta-\gamma)^2a^* + \delta\gamma^2(\beta-\gamma)(2\beta+\gamma)a^*\sigma_\theta^2$$

$$n' = 8\beta^2(2\beta-\gamma)^2 - \delta\gamma^4\sigma_\theta^2$$

$$s' = 4\beta\gamma(2\beta-\gamma)^2 - \delta\beta\gamma^3\sigma_\theta^2$$

Finally, we solve the uninformed firm's problem in period 1 given by (15). Substituting  $E(\theta) = 1$  and firm u's demand equation (A.2) for  $t=1$  into (15) and using the envelope theorem, the first-order condition for this risk-neutral firm will be:

$$(A.23) \quad \frac{E(a\theta)(\beta-\gamma)}{(\beta^2-\gamma^2)} - \frac{2\beta}{(\beta^2-\gamma^2)}p_1^u + \frac{\gamma}{(\beta^2-\gamma^2)}p_1^i +$$



$$\delta E \left\{ \left[ \frac{(\beta^2 - \gamma^2)}{(2\beta - \gamma)} q_1^u + \frac{\beta}{(2\beta - \gamma)} p_1^u - \frac{\gamma}{(2\beta - \gamma)} p_1^i \right] \left[ -\frac{\beta\theta}{(\beta^2 - \gamma^2)} \frac{\partial p_2^{u*}}{\partial p_1^u} + \frac{\gamma\theta}{(\beta^2 - \gamma^2)} \frac{\partial p_2^{i*}}{\partial p_1^i} \right] \right\} = 0$$

By calculating  $\frac{\partial p_2^{i*}}{\partial p_1^i}$  and  $\frac{\partial p_2^{u*}}{\partial p_1^u}$  from (A.14) and (A.10), respectively, substituting  $q_1^u$  by the expression (A.2) for  $t=1$  and assuming that  $E(\theta) = 1$ ,  $E(a) = a^*$  and that  $a$  and  $\theta$  are statistically independent, the expected reaction function of firm  $u$  in the first period can be expressed as:

$$(A.24) \quad p_1^u = \frac{t}{u} + \frac{v}{u} p_1^i$$

where,

$$t = 2(\beta - \gamma)(2\beta - \gamma)^2 a^* + 2\delta(\beta - \gamma)(2\beta^2 - \gamma^2) a^* \sigma_\theta^2$$

$$u = 4\beta(2\beta - \gamma)^2 + \delta\beta(2\beta^2 - \gamma^2) \sigma_\theta^2$$

$$v = 2\gamma(2\beta - \gamma)^2 + \delta\gamma(2\beta^2 - \gamma^2) \sigma_\theta^2$$

As firm  $u$  expects firm  $i$  to behave as obtained in equation (A.22), this is substituted into firm  $u$ 's reaction function in (A.24) and the equilibrium price chosen by this uninformed firm in the first period is the following:

$$(A.25) \quad p_1^{u*} = \frac{tn' + vm'}{un' - vs'}$$

By including this price in the reaction function of firm  $i$  from equation (A.20), its equilibrium price set in period 1 can be expressed as:

$$(A.26) \quad p_1^{i*} = \frac{m(un' - vs') + s(tn' + vm')}{n(un' - vs')}$$

*Proof of Proposition 3.* First, it is necessary to obtain the derivative of the optimal price set by firm  $i$  in period 1 with respect to the variance of  $\theta$  from (A.26). After some simplifications, the following expression is obtained:

$$(A.27) \quad \frac{\partial p_i^{i*}}{\partial \sigma_\theta^2} = \frac{2\delta\gamma(\beta-\gamma)(2\beta-\gamma)\alpha^*K[32\beta^2(2\beta-\gamma)^4 - 4\delta\gamma^2(2\beta-\gamma)^2(2\beta^2+\gamma^2)(\theta-1)^2 + \delta^2\gamma^6(\theta-1)^4]}{[n(\alpha n^2 - \nu s^2)]^2}$$

where,

$$K = 8\beta K_1^3 K_2 K_3 + \delta^2 \gamma^3 K_4 K_5 \sigma_\theta^4 + 8\delta \gamma^3 K_1^2 K_4 K_6 \sigma_\theta^2$$

$$K_1 = 2\beta - \gamma$$

$$K_2 = 2\beta + \gamma$$

$$K_3 = 24\beta^4 - 8\beta^3\gamma - 12\beta^2\gamma^2 + 8\beta\gamma^3 - \gamma^4$$

$$K_4 = 2\beta^2 - \gamma^2$$

$$K_5 = 8\beta^4 - 8\beta^2\gamma^2 + \gamma^4$$

$$K_6 = 4\beta^2 - \gamma^2$$

As  $\beta > \gamma > 0$ , then,  $K_1, K_2, K_3, K_4, K_5, K_6 > 0$  and  $K > 0$ . Thus, the sign of  $\frac{\partial p_i^{i*}}{\partial \sigma_\theta^2}$

depends on the sign of

$$32\beta^2(2\beta - \gamma)^4 - 4\delta\gamma^2(2\beta - \gamma)^2(2\beta^2 + \gamma^2)(\theta - 1)^2 + \delta^2\gamma^6(\theta - 1)^4. \quad \text{If we}$$

substitute  $(\theta - 1)^2$  by  $X$  and  $(\theta - 1)^4$  by  $X^2$  in the last expression and solving it for zero,

$$(A.28) \quad 32\beta^2(2\beta - \gamma)^4 - 4\delta\gamma^2(2\beta - \gamma)^2(2\beta^2 + \gamma^2)X + \delta^2\gamma^6X^2 = 0$$

It is clear that the solutions of this equation are the following:

$$(A.29) \quad X_1 = \frac{8\beta^2(2\beta-\gamma)^2}{\delta\gamma^4}$$

$$(A.30) \quad X_2 = \frac{4(2\beta-\gamma)^2}{\delta\gamma^2}$$

If we substitute these values of  $X$  in  $X = (\theta - 1)^2$ , we obtain the only positive values of  $\theta$  which satisfies equation (A.28), that is:

$$(A.31) \quad \theta_1 = 1 + \frac{2(2\beta-\gamma)}{\gamma\sqrt{\delta}}$$

$$(A.32) \quad \theta_2 = 1 + \frac{\beta(2\beta-\gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}}$$

Thus, part (21) of Proposition 3 has been proven. It is clear that the sign on the left-hand side of (A.28) is positive when  $0 < \theta < \theta_1$  or  $\theta > \theta_2$  and negative when  $\theta_1 < \theta < \theta_2$ . Hence, Proposition 3 has been proven.

*Proof of Proposition 4.* The effect of an increase in the variance of  $\theta$  on the optimum price set by firm u in period 1 is directly derived from (A.25). After some simplifications, the following result is obtained:

$$(A.33)$$

$$\frac{\partial p_1^{u*}}{\partial \sigma_\theta^2} = \frac{16\delta\beta^2(\beta-\gamma)K_1^4K_2[4\beta K_1(4\beta^2+\beta\gamma-2\gamma^2)-K_2]a^*+16\delta^2\beta\gamma^3(\beta-\gamma)K_1^3K_4K_6a^*\sigma_\theta^2+2\delta^3\beta\gamma^3(\beta-\gamma)K_1K_4K_5a^*\sigma_\theta^4}{(un^*-us^*)^2}$$

The denominator of (A.33) is positive and the numerator is compounded by three terms, all of which are positive because  $\beta > \gamma$ . Thus, Proposition 4 has been proven.

*Price Difference.* Here, we obtain the difference between the optimal prices set by both firms in period 1 from (A.25) and (A.26),

(A.34)

$$p_1^{i*} - p_1^{u*} = -\frac{(\beta-\gamma)a^*[8\beta K_1^3 K_2 + 8\delta\beta^2 K_1(4\beta^2 + \beta\gamma - 2\gamma^2)\sigma_\theta^2 + \delta^2\gamma^3 K_4\sigma_\theta^4][4\beta K_1^3 + \delta\gamma^3(\beta-\gamma)(\theta-1)^2]}{2\beta K_1[4K_1^2 K_6 + \delta K_5\sigma_\theta^2][8\beta^2 K_1^2 - \delta\gamma^4(\theta-1)^2]} + \frac{(\beta-\gamma)a[4\beta K_1^2 + \delta\gamma^2 K_1(\theta-1) + \delta\gamma^3\theta(\theta-1)]}{[8\beta^2 K_1^2 - \delta\gamma^4(\theta-1)^2]}$$

If  $a > \lambda(\theta)a^*$  and  $\theta \leq 1 + \frac{\beta(2\beta-\gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}}$ , then,  $p_1^{i*} \geq p_1^{u*}$ , but if  $a < \lambda(\theta)a^*$  and

$\theta \geq 1 + \frac{\beta(2\beta-\gamma)\sqrt{\delta}}{\gamma^2\sqrt{\delta}}$ , then,  $p_1^{i*} \leq p_1^{u*}$ .

*Proof of Proposition 5.* Now, we can obtain the effect of an increase in demand uncertainty faced by firm u on the difference between prices set in period 1, that is:

(A.35)

$$\frac{\partial(p_1^{i*} - p_1^{u*})}{\partial\sigma_\theta^2} = -\frac{(\beta-\gamma)a^*[4\beta K_1^3 + \delta\gamma^3(\beta+\gamma)(\theta-1)^2][8\delta\beta K_1^3 K_2 K_3 + 8\delta^2\gamma^3 K_1^2 K_4 K_6\sigma_\theta^2 + \delta^3\gamma^3 K_4 K_5\sigma_\theta^4]}{2\beta K_1[8\beta^2 K_1^2 - \delta\gamma^4(\theta-1)^2][4K_1^2 K_6 + \delta K_5\sigma_\theta^2]^2}$$

The sign of this derivative is the opposite of the sign of  $[8\beta^2 K_1^2 - \delta\gamma^4(\theta-1)^2]$ . Thus

Proposition 5 has been proven.

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