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Abstract

The unequivocal majority of a social choice rule F is the minimum number of agents that must agree on their best alternative in order to guarantee that this alternative is the only one prescribed by F. If the unequivocal majority of F is larger than the minimum possible value, then some of the alternatives prescribed by F are undesirable (there exists a different alternative which is the most preferred by more than 50% of the agents). Moreover, the larger the unequivocal majority of F, the worse these alternatives are (since the proportion of agents that prefer the same different alternative increases). We show that the smallest unequivocal majority compatible with Maskin-monotonicity is $n - \left\lfloor \frac{n-1}{m} \right\rfloor$, where $n \geq 3$ is the number of agents and $m \geq 3$ is the number of alternatives. This value represents no less than $66.\hat{6}\%$ of the population.

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1 Introduction

One of the central questions of the social choice theory concerns the design and implementation of collective decisions. Consider a society with $n \geq 3$ agents and $m \geq 3$ alternatives. Suppose that the goals of the group of agents can be summarized in a social choice rule, i.e., a correspondence that prescribes the socially desirable alternatives as a function of the individual preference relations. The problem is then to design social choice rules that fulfill a variety of desirable properties.

In this paper, we focus on the key necessary condition for Nash implementability of a social choice rule: Maskin-monotonicity.[1] This condition not only is one of the crucial concepts in implementation theory, but it is a desirable property in itself that can be justified from a normative point of view: it argues that no alternative can be dropped from being chosen unless for some agent its desirability deteriorates.

We are also interested in a property that is related with the majority required to ensure that a given alternative is chosen. More precisely, we define the unequivocal majority of a social choice rule F as the minimum number of agents that must agree on their best alternative in order to guarantee that this alternative is the only one prescribed by F. The minimum possible unequivocal majority is equal to $\left\lfloor \frac{n}{2} + 1 \right\rfloor$. If the unequivocal majority of a social choice rule F is larger than this minimum value, then some of the alternatives prescribed by F are such that there exists a different alternative which is the most preferred by more than 50% of the agents. Moreover, the larger the unequivocal majority of F, the greater is this proportion of agents.[2] For this reason, we would like that a social choice rule has an unequivocal majority as small as possible.

Our main result shows that the smallest unequivocal majority compatible with Maskin-monotonicity is $n - \left\lfloor \frac{n-1}{m} \right\rfloor$. This value is equal to the minimal number required for a majority that ensures the non-existence of cycles in pair-wise comparisons (see Greenberg, 1979).

Since $n - \left\lfloor \frac{n-1}{m} \right\rfloor > \left\lfloor \frac{n}{2} + 1 \right\rfloor$, an obvious implication of our result is that

[1] See, e.g., Maskin (1999) and Repullo (1987).

[2] If the unequivocal majority of a social choice rule F is n_F, then there are some situations in which an alternative a is the most preferred one by $n_F - 1$ agents, but there is some other alternative b which is among the ones prescribed by F. If n_F is larger than $\left\lfloor \frac{n}{2} + 1 \right\rfloor$, then $n_F - 1$ represents more than 50% of the agents.

there is no Condorcet consistent social choice rule satisfying Maskin-monotonicity.[3] Sen (1995) proposed to evaluate the extent to which a social choice rule may fail Maskin-monotonicity by identifying the minimal way in which it has to be enlarged so as to satisfy this property.[4] Our result implies that the minimal monotonic extension of any Condorcet consistent social choice rule has an unequivocal majority equal to $n - \lfloor \frac{n-1}{m} \rfloor$.[5] In other words, in some situations in which a Condorcet winner exists, the set of alternatives that are considered eligible by any Maskin-monotonic social choice rule F must be enlarged to include some other alternatives. How bad these other alternatives need to be? Our result shows that some of these alternatives must be such that $n - \lfloor \frac{n-1}{m} \rfloor - 1$ agents (i.e., no less than $66.\hat{6}\%$ of the population) prefer the same different alternative. This is the minimum price to pay for achieving Maskin-monotonicity.[6]

The paper is organized as follows: Section 2 provides definitions, Section 3 states the result, and Section 4 provides the conclusions.

2 Definitions

Let N be a set of $n \geq 3$ agents and let A be a set of $m \geq 3$ alternatives. Each agent $i \in N$ has a (strict) preference relation, P_i, defined over the set of alternatives. Let $\mathbb{P}$ be the class of all possible (strict) preference relations on A. An admissible profile of preference relations is denoted by $P = (P_i)_{i \in N} \in \mathbb{P}^n$.

Let 2^A denote the set of all subsets of A. A **social choice rule** (SCR) is a correspondence $F : \mathbb{P}^n \rightarrow 2^A \backslash \{\emptyset\}$, which associates each possible profile of preference relations P a non-empty subset of alternatives $F(P) \subseteq A$.

A SCR F is **efficient** if it always selects Pareto-efficient alternatives, i.e., for all $P \in \mathbb{P}^n$ and $a \in F(P)$, there is no $b \in A$ such that $bP_i a$ for all $i \in N$.

The SCR F is **unanimous** if it only chooses the unanimously best alternative whenever it exists, i.e., for all $P \in \mathbb{P}^n$ and $a \in A$ such that $aP_i b$

[3]A Condorcet winner is an alternative that is not defeated by any other alternative in pairwise comparisons. A social choice rule is Condorcet consitent if it only selects the Condorcet-winner whenever it exists.

[4]See also Thomson (1999) and Erdem and Sanver (2005).

[5]In general, the minimal monotonic extension of any social choice rule has an unequivocal majority equal or grater than $n - \lfloor \frac{n-1}{m} \rfloor$.

[6]That is, any Maskin-monotonic social choice rule must sometimes select alternatives that would not be Pareto-efficient in a reduced setting with $n - \lfloor \frac{n-1}{m} \rfloor - 1$ agents.

for all $b \in A\backslash\{a\}$ and $i \in N$, $F(P) = \{a\}$. Note that unanimity is a weaker requirement than efficiency.

An agent $i \in N$ is a **dictator** for the SCR F if, for all profile of preference relations $P \in \mathbb{P}^n$, there is some $a \in F(P)$ which is the most preferred alternative for i at P. A SCR that admits a dictator is called **dictatorial**.[7]

A SCR F is supposed to represent the objectives of a social planner. In many situations the planner cannot achieve directly the outcomes recommended by F. To obtain the alternatives prescribed by F in a decentralized way, the planner must design a mechanism which implements it. From Maskin (1999) we know that Maskin-monotonicity is a necessary condition for the Nash implementability of a SCR.

Definition 1 *A SCR F satisfies **Maskin-monotonicity** when, for all $P, \hat{P} \in \mathbb{P}^n$ and $a \in F(P)$, if $a \notin F(\hat{P})$ then there exist some $i \in N$ and $b \in A$ such that aP_ib and $b\hat{P}_ia$.*

Roughly speaking, this condition says that if an alternative a is selected by F for some profile of preference relations P, then a must be also selected for any other profile of preference relations $\hat{P}$ where no alternative has risen in any agent's preference ranking with respect to a. Maskin-monotonicity not only is one of the key concepts in implementation theory, but it is a desirable property in itself.

A social choice function (SCF), $f : \mathbb{P}^n \rightarrow A$, is a SCR that assigns a single alternative $f(P) \in A$ to every profile of preference relations $P \in \mathbb{P}^n$. A result parallel to Arrow's impossibility theorem states that any efficient SCF that satisfies Maskin-monotonicity is dictatorial.[8] Fortunately, this negative result can be avoided if we consider correspondences instead of functions. Consider, for example, the SCR $\tilde{F}$ that for each profile of preference functions selects all the alternatives $a \in A$ such that: (1) a is the best alternative for one agent at least and, (2) a is not the worst alternative for $(n-1)$ agents. It is easy to show that this SCR is efficient, Maskin-monotonic and non-dictatorial.

[7] This notion is usually defined for single-valued SCRs. Nevertheless, it may also make sense for a multivalued SCR F, since any of the alternatives prescribed by F is considered socially optimal (in particular, if F is dictatorial, it allows for the possibility of always choosing the most preferred alternative for the dictator).

[8] If all possible preference relations (not necessarily strict) are admissible, then the negative result is even stronger: in this case any efficient SCR does not satisfy Maskin-monotonicity (see Appendix). This incompatibility vanishes if we consider weak-efficiency instead of efficiency.

Given a profile of preference relations $P \in \mathbb{P}^n$ and an alternative $a \in A$, let $n_a^P \leq n$ be the number of agents for whom a is the most preferred alternative:

$$n_a^P = \#\{i \in N : aP_ib, \forall b \in A \backslash \{a\}\} \tag{1}$$

Definition 2 *The **unequivocal majority** of a SCR F, n_F, is the minimum number of agents that must agree on their most preferred alternative in order to guarantee that F will select that (and only that) alternative, i.e.:*

$$\begin{aligned} n_F = \quad & \min \check{n} \\ & s.t.\ \check{n} \in \aleph_F \end{aligned}$$

where $\aleph_F = \{\hat{n} \leq n : \forall P \in \mathbb{P}^n, \forall a \in A, if\ n_a^P \geq \hat{n}\ then\ F(P) = \{a\}\}$.

Note that a SCR F has an unequivocal majority if and only if it is unanimous (i.e., $\aleph_F \neq \emptyset$ if and only if F is unanimous).

3 Results

For all $x \in \mathbb{R}$, let $\lfloor x \rfloor$ denote the largest integer smaller or equal than x. Clearly, the unequivocal majority of a SCR (whenever it exists) is always greater or equal than $\left\lfloor \frac{n}{2} + 1 \right\rfloor$ and smaller or equal than n. The question that we want to answer in this section is: which is the smallest unequivocal majority compatible with Maskin-monotonicity?

Our first result establishes a necessary condition for a Maskin-monotonic SCR F having a given unequivocal majority. Roughly speaking, this condition says that, if F is Maskin-monotonic and has an unequivocal majority smaller or equal than k, then any alternative selected by F should be Pareto-efficient in any "reduced" setting with k agents.

Lemma 1 *Let F be a SCR satisfying Maskin-monotonicity and with an unequivocal majority smaller or equal than k. Then, for all $P \in \mathbb{P}^n$ and $a \in F(P)$, there is no alternative $b \in A$ which is preferred to a by k agents.*

Proof. Let F be a SCR satisfying Maskin-monotonicity and such that $n_F \leq k \leq n$. Suppose by contradiction that there exist $P \in \mathbb{P}^n$, $a \in F(P)$ and $b \in A$ such that b is preferred to a by k agents. In particular, suppose without loss of generality that bP_ia for all $i \in \{1, ..., k\}$. Let $\hat{P} \in \mathbb{P}^n$ be such that:

(1) for all $i \in \{1, ..., k\}$ and $c \in A \backslash \{b\}$, $b\hat{P}_i c$,
(2) for all $i \in \{1, ..., k\}$ and $c \in A$ such that $aP_i c$, $a\hat{P}_i c$, and
(3) for all $i \in \{k+1, ..., n\}$, $\hat{P}_i = P_i$.

Since $n_F \leq k$, from (1) we have $F(\hat{P}) = \{b\}$. On the other hand, from (2) and (3), there is no $j \in N$ and $c \in A$ such that $aP_j c$ and $c\hat{P}_j a$. Then, by Maskin-monotonicity, we have $a \in F(\hat{P})$, which is a contradiction. ■

The next result defines a lower bound for the unequivocal majority of any Maskin-monotonic SCR. This lower bound depends on the number of agents and the number of alternatives.

Lemma 2 *Let F be a SCR with an unequivocal majority equal to n_F. If F satisfies Maskin-monotonicity then $n_F \geq n - \left\lfloor \frac{n-1}{m} \right\rfloor$.*

Proof. Suppose by contradiction that there exists a SCR F that satisfies Maskin-monotonicity and such that $n_F \leq n - \left\lfloor \frac{n-1}{m} \right\rfloor - 1$. Suppose first that $n \leq m$ (note that in this case $\left\lfloor \frac{n-1}{m} \right\rfloor = 0$). For all $P \in \mathbb{P}^n$, $i \in N$, and $a \in A$, let $1 \leq p_a^{P_i} \leq m$ denote the position of alternative a in the ranking of alternatives generated by P_i (i.e., $p_a^{P_i} = 1$ if a is the most preferred alternative for agent i, $p_a^{P_i} = 2$ if a is the second most preferred alternative for agent i, and so on). Let $P \in \mathbb{P}^n$ be a profile of preference relations such that:
(1) $p_a^{P_1} = 1$, $p_a^{P_2} = 2$, ..., $p_a^{P_n} = n$,
(2) $p_b^{P_1} = n$ and, for all $i \in N \backslash \{1\}$, $p_b^{P_i} = p_a^{P_i} + 1$,
(3) $p_c^{P_2} = n$ and, for all $i \in N \backslash \{2\}$, $p_c^{P_i} = p_b^{P_i} + 1$, and so on.

Table I shows as an example the case in which $n = 4 \leq m$.

Agent 1	Agent 2	Agent 3	Agent 4
a	b	c	d
d	a	b	c
c	d	a	b
b	c	d	a
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Table I

The profile of preference relations P defined above is such that, for all $a \in A$, there is some $b \in A$ which is strictly preferred to a by $n-1$ agents. Then, from Lemma 1, we have $F(P) = \emptyset$, which is a contradiction.

Suppose now that $m < n$. Let $P \in \mathbb{P}^n$ be a profile of preference relations defined as follows. The preference relations of agents 1, ..., m are such that:

(1) $p_a^{P_1} = 1$, $p_a^{P_2} = 2$, ..., $p_a^{P_m} = m$,

(2) $p_b^{P_1} = m$ and $p_b^{P_i} = p_a^{P_i} + 1$ for all $i \in \{1, ..., m\} \backslash \{1\}$,

(3) $p_c^{P_2} = m$ and $p_c^{P_i} = p_b^{P_i} + 1$ for all $i \in \{1, ..., m\} \backslash \{2\}$, and so on.

The preference relation of any agent $i \in \{m+1, ..., n\}$ is the same that the preference relation of agent $i - m$ (i.e., $P_i = P_{i-m}$ for all $i \in \{m + 1, ..., n\}$). Table II shows as an example the case in which $m = 4$ and $n = 7$.

Agent 1	Agent 2	Agent 3	Agent 4	Agent 5	Agent 6	Agent 7
a	b	c	d	a	b	c
d	a	b	c	d	a	b
c	d	a	b	c	d	a
b	c	d	a	b	c	d

Table II

Note that the profile of preference relations P defined above is such that, for all $a \in A$, there is some $b \in A$ which is strictly preferred to a by $(m - 1) \left\lfloor \frac{n}{m} \right\rfloor + \max\{0, n - m \left\lfloor \frac{n}{m} \right\rfloor - 1\} = n - \left\lfloor \frac{n-1}{m} \right\rfloor - 1$ agents. Then, from Lemma 1, we have $F(P) = \emptyset$, which is a contradiction. ■

Now, we can state the main result of the paper:

Theorem 1 *The smallest unequivocal majority which is compatible with Maskin-monotonicity is* $n^* = n - \left\lfloor \frac{n-1}{m} \right\rfloor$.[9]

Proof. From Lemma 2 we know that any Maskin-monotonic SCR F is such that $n_F \geq n - \left\lfloor \frac{n-1}{m} \right\rfloor$. Next we show that there is some SCR F satisfying Maskin-monotonicity and such that $n_F = n - \left\lfloor \frac{n-1}{m} \right\rfloor$. Let F^* be a SCR such that, for all $P \in \mathbb{P}^n$:

$$F^*(P) = \{a \in A : \nexists b \in A \text{ that is preferred to } a \text{ by } n - \left\lfloor \frac{n-1}{m} \right\rfloor \text{ agents at } P\}$$

[9] Of course, the fact that a SCR has an unequivocal majority equal to n^* does not imply that it satisfies Maskin-monotonicity. For example, Baharad and Nitzan (2003) showed that the unequivocal majority of the Borda rule is $n - \left\lfloor \frac{n-1}{m} \right\rfloor$ (and it is well-known that this SCR does not satisfy Maskin-monotonicity).

Next we show that it is well-defined, satisfies Maskin-monotonicity and $n_{F^*} = n - \left\lfloor \frac{n-1}{m} \right\rfloor$.

Step 1. $F^*(P) \neq \emptyset$ for all $P \in \mathbb{P}^n$. To see this note that for all $P \in \mathbb{P}^n$ there is some $a \in A$ which is the most preferred alternative for at least $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ agents (and therefore $a \in F^*(P)$).

Step 2. F^* satisfies Maskin-monotonicity. Let P, $\hat{P} \in \mathbb{P}^n$ and $a \in A$ be such that $a \in F^*(P)$ and $a \notin F^*(\hat{P})$. Since $a \notin F^*(\hat{P})$, there exists $b \in A$ which is preferred to a for at least $n - \left\lfloor \frac{n-1}{m} \right\rfloor$ agents at $\hat{P}$. However, since $a \in F^*(P)$, b is not preferred to a for $n - \left\lfloor \frac{n-1}{m} \right\rfloor$ agents at P. Therefore, since $n < 2(n - \left\lfloor \frac{n-1}{m} \right\rfloor)$, there exist some $i \in N$ for whom aP_ib and $b\ \hat{P}_ia$.

Step 3. $n_{F^*} = n - \left\lfloor \frac{n-1}{m} \right\rfloor$. Let $P \in \mathbb{P}^n$ be such that there is some $a \in A$ which is the most preferred alternative for at least $n - \left\lfloor \frac{n-1}{m} \right\rfloor$ agents. Since $\frac{n}{2} < n - \left\lfloor \frac{n-1}{m} \right\rfloor$, we have $F^*(P) = \{a\}$, and therefore $n_{F^*} \leq n - \left\lfloor \frac{n-1}{m} \right\rfloor$. Moreover, from Lemma 2 we have $n_{F^*} > n - \left\lfloor \frac{n-1}{m} \right\rfloor - 1$. ■

It is easy to see that the SCR F^* defined in the proof of Theorem 1 not only is Maskin-monotonic and has the smallest unequivocal majority compatible with this condition, but it also is efficient and, when $m < n$, non-dictatorial.[10]

Remark 1 *Note that* $n - \left\lfloor \frac{n-1}{3} \right\rfloor \leq n^* \leq n$. *Hence, since* $\underset{n\to\infty}{Lim} \frac{n - \left\lfloor \frac{n-1}{3} \right\rfloor}{n} = \frac{2}{3}$, *if F is a Maskin-monotonic SCR, the minimum percentage of agents that must agree on their best alternative in order to guarantee that F will choose only that alternative is always greater than* $66.\hat{6}\%$. *This implies that some of the alternatives selected by F are such that there exists a different alternative which is preferred by a wide majority of agents (no less than* $66.\hat{6}\%$*). This undesirable property is the price to pay for achieving Maskin-monotonicity.*

Corollary 1 *Any Maskin-monotonic SCR F is such that, for some $P \in \mathbb{P}^n$ and $a \in F(P)$, there is another alternative that is preferred to a by at least* $n - \left\lfloor \frac{n-1}{m} \right\rfloor - 1$ *agents (i.e., no less than* $66.\hat{6}\%$ *of the population).*

[10]If $n \leq m$, F^* selects all Pareto-efficient allocations, and therefore all agents are dictators. Nevertheless, in this case there exist some other Maskin-monotonic, efficient and non-dictatorial SCRs that have an unequivocal majority equal to n^*, like the SCR $\tilde{F}$ defined in the previous section. Note also that any Maskin-monotonic SCR F that has an unequivocal majority equal to n^* is such that $F \subseteq F^*$.

Remark 2 *Let $k \in \{\lfloor \frac{n}{2}+1 \rfloor, ..., n\}$. We say that an alternative $a \in A$ is a* **k-*Condorcet winner*** *at $P \in \mathbb{P}^n$ if there is no other alternative in A that is preferred to a by at least k agents. A SCR is* **k-*Condorcet consistent*** *if it only chooses k-Condorcet winners whenever they exist. Greenberg (1979) showed that a necessary and sufficient condition that for every profile of preference relations there exists a k-Condorcet winner is that $k \geq n - \lfloor \frac{n-1}{m} \rfloor$ (see also Weber, 1993).*[11] *Hence, the minimal number required for a majority that ensures the non-existence of cycles in pair-wise comparisons is equal to the minimal unequivocal majority which is compatible with Maskin-monotonicity. The following result is an immediate corollary of Theorem 1.*

Corollary 2 *A SCR satisfying Maskin-monotonicity and k-Condorcet consistency exists if and only if $k \geq n - \lfloor \frac{n-1}{m} \rfloor$.*[12]

4 Conclusion

The unequivocal majority of a social choice rule F is the minimum number of agents that must agree on their most preferred alternative in order to guarantee that this alternative is chosen. The larger unequivocal majority of F, the more undesirable are some of the alternatives prescribed by F. We have shown that the smallest unequivocal majority compatible with Maskin-monotonicity is $n - \lfloor \frac{n-1}{m} \rfloor$. This value represents no less than $66.\hat{6}\%$ of the population. We have proposed a social choice rule that not only is Maskin-monotonic and has the smallest unequivocal majority, but it is also efficient and non-dictatorial.

[11] Note that this result (together with Lemma 1) could be used to provide an alternative proof of Lemma 2

[12] When $k = \lfloor \frac{n}{2}+1 \rfloor$, a k-Condorcet consistent SCR is simply called Condorcet consistent. Hence, this result implies that there is no Condorcet consistent SCR satisfying Maskin-monotonicity.

Appendix

We show that if all possible preference relations (not necessarily strict) are admissible, then there is an incompatibility between efficiency and Maskin-monotonicity.

Proposition 1 *Let $\Re$ be the domain of all possible preference relations (not necessarily strict) on A. There is no efficient SCR $F : \Re^n \to 2^A \backslash \{\emptyset\}$ satisfying Maskin-monotonicity.*

Proof. Suppose by contradiction that there exists some SCR $F : \Re^n \to 2^A \backslash \{\emptyset\}$ which is efficient and satisfies Maskin-monotonicity. Let $R \in \Re^n$ be such that there are $a, b, c \in A$ satisfying:

(1) for all $i \in N$ and all $d \in A \backslash \{a, b, c\}$, $aP_i d$, $bP_i d$ and $cP_i d$ (i.e., a, b and c are the three most preferred alternatives for all agents),

(2) $aP_1 bP_1 c$, $bP_2 aP_2 c$ and $cP_3 aI_3 b$ (where I_i denotes the indifference relation) and

(3) for all $i \in N \backslash \{1, 2, 3\}$, $aI_i bI_i c$.

Step 1. $a \notin F(R)$. Suppose on the contrary that $a \in F(R)$. Let $\hat{R} \in \Re^n$ be such that $a\hat{I}_1 b\hat{P}_1 c\hat{P}_1 d$ for all $d \in A \backslash \{a, b, c\}$, while $\hat{R}_i = R_i$ for all $i \in N \backslash \{1\}$). By Maskin-monotonicity we have $a \in F(\hat{R})$. However, a is not Pareto-efficient at $\hat{R}$, which is a contradiction.

Step 2. $b \notin F(R)$. Suppose on the contrary that $b \in F(R)$. Let $\tilde{R} \in \Re^n$ be such that $a\hat{I}_2 b\tilde{P}_2 c\tilde{P}_2 d$ for all $d \in A \backslash \{a, b, c\}$, while $\tilde{R}_i = R_i$ for all $i \in N \backslash \{2\}$). By Maskin-monotonicity we have $b \in F(\tilde{R})$. However, b is not Pareto-efficient at $\tilde{R}$, which is a contradiction.

Step 3. $c \notin F(R)$. Suppose on the contrary that $c \in F(R)$. Let $\check{R} \in \Re^n$ be such that $a\check{I}_3 bI_3 c\check{P}_3 d$ for all $d \in A \backslash \{a, b, c\}$, while $\check{R}_i = R_i$ for all $i \in N \backslash \{2\}$). By Maskin-monotonicity we have $c \in F(\check{R})$. However, c is not Pareto-efficient at $\check{R}$, which is a contradiction.

From Steps 1-3 and the fact that any $d \in A \backslash \{a, b, c\}$ is not Pareto-efficient at R we have $F(R) = \emptyset$, which is a contradiction. ■

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