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A case for Ambiguity

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Abstract

An expert wishes to be approved by a decision maker, who is outcome concerned. She then has an incentive to send an informative message. But if there is more than one expert competing for the receiver's approval and the latter doubts about the objectives of experts, they each have an incentive to fool the receiver and look as the only truthful expert in the population. If they succeed, no truthful equilibrium exists. In this scenario, we show that it may be in the decision maker's interest to be ambiguous about his motives as, if prospering, he could guarantee revelation of information by (at least) outcome concerned experts.

Keywords: Multiple experts; approval; two sided incomplete information
JEL: D78; D82; D83

1 Introduction

Consider the case of a local authority that plans, let us say, the expansion of a town. The authority would like the city to grow in the most sustainable way. However, it has no information on details such as the use of renewable energies, most efficient construction materials, design of green areas, etc., that depend on the state of the science (unknown or uncomprehensible to the authority), which determines the optimality of a project. The local government may then open a formal competitive process through which private sector firms can apply for the contracting authority work. If private sector firms are aware of the intentions of the local authority and want the award of the contract, they have a reason to meet the standards of the decision maker and provide him with useful information. However, if the number of experts competing for the contract is higher that one, they each have a current incentive to fool the local authority and make him believe they are

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the only trustworthy firms in the economy. In this situation, can information be transmitted?

This paper presents a sender-receiver game that captures the specifics of this situation. We assume that the sender has valuable information to the receiver who, based on that information, take an action that determines the welfare of both. One important distinction between our model and the standard sender-receiver game, is that we explicitly assume that the expert cares about the receiver's approval. This feature is known to the receiver. The assumption that the sender is concerned about whether her advise is followed by the decision maker or not intends to capture the reality of many one shot games in which experts (let us say, firms) want to conform to the preferences of the decision maker (local authority), possibly because their prize (contract award) depends on the receiver's opinion or evaluation of her performance. The decision maker is however uncertain about the ultimate motives of the sender: Whether she is good, i.e., she wants the contract award but additionally, is outcome concerned; or opportunistic, i.e., she exclusively cares about getting the contract. The receiver is assumed to have standard preferences, i.e., he cares about the policy implemented.

In this scenario, the sender has an incentive to reveal her information, as it is in her interest to be approved by the decision maker and in the receiver's to get information. Now, consider that new experts enter the game and that they all compete for the contract award. If competition reduces the senders' payoffs, experts have now an incentive to fool the decision maker and make him believe they each are the only trustworthy sender in the economy. If competition is low, an expert always manages not to reveal information but to look as if she were indeed being truthful. However, if competition is tougher, a single expert cannot always unilaterally affect the decision maker's behavior. In this sense, competition does not necessarily yield more information transmission. We next extend the model and introduce uncertainty on the receiver's motives. In particular, we consider that the senders think that the decision maker may be biased in favor of one decision. This assumption captures the, quite often, reality of many countries where the interests of decision makers are obscure. We show that, under this new scenario, there always exists an equilibrium in which (at least) good senders are truthful. The reason is that uncertainty allows the senders to differentiate their speeches according to the type of receiver they want to cater. This is possible even though they share the belief on the type of the decision maker (but do have different payoff structures). This result has an interesting reading: If a decision maker that cares about information were given the opportunity to speak about his motives, he would choose not to clearly state them. A case for ambiguity.

This paper explores the problem of eliciting information, which has been the subject of a large literature. Examples are Crawford and Sobel (1982), Morris (2001) or Dewatripont and Tirole (2005). A central feature to all these papers is the lack of congruence between the interest of the sender and that of the receiver. In contrast to this literature, we propose a game in which experts care intrinsically about whether their advice is followed or not. Schultz (1996) and

Heidhues and Lagerlof (2003), among others, consider the problem of asymmetries in information between voters and parties, where the assumption that senders (parties) care about the receivers' approval (citizens' vote) is in the game. On a different context, Prendergast (1993) illustrates an (endogenous) incentive for subordinates to conform to the opinion of the manager when remuneration depends on subjective evaluation procedures. Our contribution to this literature is to consider competition between (more than two) experts and to introduce uncertainty about the receiver's motives. Note also that in the present paper, senders are exclusively interested in their current payoffs and do not try to influence the decision maker's future beliefs about their motives. In this sense, we depart from the literature on career concerns.

Another important feature of the present paper is to consider that, besides the sender, the receiver may also have private information. To the best of our knowledge, there are few papers dealing with this type of consideration. Examples are Prendergast (1993), Watson (1996) and Olszewski (2004), who propose models in which the sender is unsure about the signal the receiver has (on the state of the world). Both, Olszewski (2004) and Watson (1996), consider cheap talk games and show that, even with divergent preferences, if the receiver has private information and the sender is either sufficiently concerned for honesty (Olszewski (2004)) or sufficiently confused about her own information (Watson (1996)), honest communication is an equilibrium of the game. In contrast to these papers, our model is not a cheap talk game; hence, with one single expert (as they assume), honest communication is an equilibrium in our case. Finally, Andina-Díaz (2009) considers a model in which the sender doubts about the motives of the receiver, which is the exact feature this paper presents. Despite it, there are important differences between Andina-Díaz (2009) and the present paper. Among them, we now consider a one-shoot game with competing experts and focus on the problem of truthful information transmission; whereas Andina-Díaz (2009) presents a two-period game with one single expert, which serves the author to analyze the incentives of players to strategically communicate.

Last, we consider advice from multiple experts, as Austen-Smith (1993) or Krishna and Morgan (2001), who both propose models of two experts. They consider games in which experts want to influence the decision maker in a way that is not necessarily in the latter's interest and study whether the ability of receiving information from various sources improves as respect to a situation of a single expert. In contrast to these papers, in the present model, expert do not want to pull the receiver in a certain direction but compete for his approval. Hence, it is not clear that competition yields more information.

The rest of the paper is organized as follows. In Section 2 we propose the basic model and analyze the case of one-sided incomplete information both, with and without competition between experts. In Section 3 we introduce uncertainty about the receiver's motives and study the equilibria of the game with and without competition. In Section 4 we compare the two scenarios and analyze the benefit of competition. Finally, Section 5 concludes.

2 A basic model

There is a sender S and a receiver R. The sender has information on an underlying state of the world w, which is unknown to the receiver. The state of the world is $w \in \{0, 1\}$, and the prior probability on the true state being 0 is $\theta \in (0, 1)$.

The sender observes the state and sends a message $m \in \{0, 1\}$ to the receiver who, upon observing the message, takes an action $a \in \{0, 1\}$. The sender gains utility 0 from the approval of the receiver, which occurs when the policy prescribed by the sender is implemented by the receiver. Otherwise, she gets utility -1. This feature is known to the receiver. The receiver is however uncertain about the ultimate objective of the sender. Specifically, with probability $\alpha \in (0, 1)$ the sender is good, i.e., outcome concerned, in which case she cares about approval as well as about the policy implemented. This fact is represented by the extra utility term $-\lambda(a - w)^2$, where $\lambda > 0$ measures how strong are outcome concerns of S. With probability $1 - \alpha$ the sender is bad, i.e., opportunistic, in which case she exclusively cares about being approved by the receiver.

The receiver is assumed to have standard preferences, i.e., he cares about the state of the world and her decision. For simplicity, we assume that her utility is given by $-(a - w)^2$. The receiver would like the sender to reveal the state to him. It occurs in equilibrium, as it is in the interest of the sender to be approved by the receiver.

We restrict our attention to equilibria in pure strategies.

Proposition 1. *In the game with one-sided incomplete information and one single expert, there is a truthful equilibrium in which both types of expert are sincere.*

Proof. Let us conjecture an equilibrium in which, for all $w \in \{0, 1\}$, $m(w) = w$ for both types of sender. By Bayes' rule, the receiver trusts the sender's message; hence, for all $m \in \{0, 1\}$, $a(m) = m$. Then, the good sender optimally chooses to be truthful, which implies a payoff of 0; whereas cheating implies a payoff of $-\lambda$. Similarly, the bad sender does not find it strictly profitable to deviate from truth-telling as, for all $m \in \{0, 1\}$, her payoff is 0. $\square$

The equilibrium is not unique. In fact, there are two other types of equilibria: one in which the good sender is truthful and the bad type is not; another one in which both types of expert are uninformative. However, the equilibria in which the good expert reveals her information Pareto dominates the other equilibria. [1]

Competing experts

Next, we explore the case of n competing senders, which we denote by S_1, S_2, to S_n, and their messages by m_1, m_2, to m_n, respectively. Each sender's type is her private information and types are drawn independently. Consequently, a competing

[1] The bad sender obtains a payoff of 0 in any equilibria; whereas the good expert obtains 0 in any equilibrium in which she is truthful and $-\lambda$ in any one in which she is not.

expert does not know to which extend her opponent is exclusively opportunistic. We assume that the n senders compete simultaneously for the approval of the receiver, which goes for the sender that matches the action of the receiver. In case $l \leq n$ experts' messages match this action, we assume the receiver flips a coin so each of those senders has $1/l$ probability of having her report accepted. In other words, those senders that please the receiver get a payoff of $(1 - \frac{1}{l})(-1)$, plus $-\lambda(a - w)^2$ if the sender is good. All the other features of the model remain the same.

For simplicity, we focus on symmetric equilibria, i.e., equilibria in which all the experts of a type send the same report. We explore whether truth-telling is an equilibrium with competing experts and obtain that this is not necessarily the case. More precisely, we obtain that competition reduces the senders' payoffs, which increases the incentives of experts to look as the only reliable expert in the population. If competition is soft ($n = 2$), experts can always cheat the receiver and look this way. Hence, in this case, no truthful equilibrium exist. However, as competition becomes tougher ($n \geq 3$), the ability to show as the only sincere expert diminishes. If this is the case, experts cannot unilaterally affect the receiver's decision. Then, a truthful equilibrium exist.

Let $\Lambda(m_1, m_2, ..., m_n)$ be the equilibrium posterior probability assigned to the state of the world being 0, upon observing the vector of messages $(m_1, m_2, ..., m_n)$. Additionally, let x (y) denote the out-of-equilibrium-path belief assigned to the state of the world being 0, upon observing all messages supporting policy 0 (1), but one.

Proposition 2. *In the game with one-sided incomplete information and competing experts:*

(i) There is no equilibrium in which one single type of expert is sincere.

(ii) There is a truthful equilibrium in which all experts are sincere if and only if $n \geq 3$, and out-of-equilibrium-path beliefs are $x > \frac{1}{2}$ and $y < \frac{1}{2}$.

Proof. We prove it in two steeps.

(i) Let us conjecture an equilibrium in which, for all $w \in \{0, 1\}$, $m(w) = w$ for one type of sender (either good or bad) and $m(w) = 0$ for the other type of sender (analogously, if $m(w) = 1$). By Bayes' rule, Λ("at least one message supports 1") $= 0$ and either $\Lambda(0, 0, ..., 0) = \frac{\theta}{\theta+(1-\theta)\alpha^n}$ or $\Lambda(0, 0, ..., 0) = \frac{\theta}{\theta+(1-\theta)(1-\alpha)^n}$, depending on whether the experts who pool at 0 are the good type or the bad one, respectively. According to these posteriors, the receiver optimally chooses a("at least one message supports 1") $= 1$. Additionally, he implements $a(0, 0, ..., 0) = 0$ if and only if θ is strictly greater than some threshold. This threshold is either $\frac{\alpha^n}{1+\alpha^n}$ or $\frac{(1-\alpha)^n}{1+(1-\alpha)^n}$, depending on whether the senders who pool at 0 are the good type or the bad one, respectively. Consider θ is above that threshold. If $w = 0$ and $m(0) = 0$, the payoff to a bad sender is $\frac{1-n}{n}$; whereas her payoff if she deviates to $m(0) = 1$ is 0. Consider now θ is below the threshold. If $w = 0$ and $m(0) = 0$, the payoff to a bad sender is -1; whereas her payoff if she deviates to $m(0) = 1$ is 0. Hence, there is no equilibrium of the type conjectured.

(ii) Let us now conjecture an equilibrium in which, for all $w \in \{0,1\}$, $m(w) = w$ for both types of sender. By Bayes' rule, $\Lambda(0,0,...,0) = 1$ and $\Lambda(1,1,...,1) = 0$. Let Λ("all messages support 0 but one") $= x \in [0,1]$ and Λ("all messages support 1 but one") $= y \in [0,1]$. According to these posteriors, the receiver optimally chooses $a(0,0,...,0) = 0$ and $a(1,1,...,1) = 1$. Additionally, he implements a("all messages support 0 but one") $= 0$ if and only if $x > \frac{1}{2}$ and a("all messages support 1 but one") $= 0$ if and only if $y > \frac{1}{2}$. Consider $y > \frac{1}{2}$ (with $x \in [0,1]$). If $w = 1$ and $m(1) = 1$, the payoff to a bad sender is $\frac{1-n}{n}$; whereas her payoff if she deviates to $m(1) = 0$ is 0. Consider now $x < \frac{1}{2}$ (with $y \in [0,1]$). If $w = 0$ and $m(0) = 0$, the payoff to a bad sender is $\frac{1-n}{n}$; whereas her payoff if she deviates to $m(0) = 1$ is 0. Note that, if $n = 2$, $x = y$. Hence, in this case, there is no equilibrium of the type conjectured. Last, consider $x > \frac{1}{2}$ and $y < \frac{1}{2}$. For all $w \in \{0,1\}$, if $m(w) = w$, the payoff to a bad sender is $\frac{1-n}{n}$; whereas her payoff if she deviates to $m(w) \neq w$ is -1. Analogously for the good sender. This completes the proof. □

From Proposition 2 we learn that, with competing experts, there is not necessarily an equilibrium in which information is revealed. This occurs even though senders know the receiver values information or, more interestingly, precisely because of it. The reason is that competition reduces the senders' payoffs, which increases the incentives of untruthful experts to look as if they were indeed being sincere. Moreover, as the prize is unique, they each want to appear as the only trustworthy expert in the economy. Hence, they have an incentive to separate their message from that of the truly sincere type and take advantage of the receiver's behavior when receiving conflicting reports. It is possible if $n = 2$, in which case, a sender always manages not to reveal information but to look as if she were indeed being truthful.[2] However, if $n \geq 3$, a single expert cannot always unilaterally change the receiver's behavior. This is the case if the receiver follows the herd, i.e., the advice of the majority. In this event, experts have no reason to deviate from truth-telling (despite the low equilibrium payoff), which allows information to be revealed in equilibrium. Hence, our first policy implication.

Corollary 1. *Low degrees of competition do not permit truthful information transmission whereas tougher competition, combined with the implementation of the most frequent advice, does.*

3 Uncertainty about the receiver's motives

We now expand the model slightly to consider that the receiver has some private information too. Our aim is to illustrate those situations in which experts doubt

[2] If $n = 2$ and a bad sender conjectures that if $m_1 \neq m_2$, the receiver will optimally choose to implement 1 (0), that sender benefits from deviating to 1 (0) when the state of the world is 0 (1); fooling the receiver, who believes state is 1 (0) in this case. Note that for a bad sender to believe this way, it has to be the case that, in equilibrium, there is either a type of sender who is pooling at 0 (1) or both types of sender are being truthful ($m_1 \neq m_2$ is out of the equilibrium path). In any case, if $w = 0$ ($w = 1$), experts are conjectured to send 0 (1) in equilibrium.

about the ultimate motives of a decision maker. In particular, we assume that senders think that the receiver may be either of two types: honest or biased. An honest receiver (the one considered so far) is assumed to have an utility that depends on the state of the world and his choice of action. For simplicity, her utility is given by $-(a-w)^2$. A biased receiver, however, is assumed to have a state independent preferred policy which, without loss of generality, we set equal to zero. His utility is $-a$. Senders are not sure about the preferences of the receiver but have a prior on the probability of the receiver being honest, which is $\beta \in (0,1)$. All other features of the model remain the same.

In this case, senders send their messages in the light of uncertainty about the motives of the receiver as well as of the other experts. We explore whether this uncertainty improves the ability of the decision maker of receiving information and obtain that this is the case. The *mechanism*, however, depends on the degree of competition between senders. Thus, for any number of experts, there is an equilibrium in which they specialize and tailor their messages to the type of receiver they want to please. In this equilibrium, good senders cater for the honest receiver (revealing) and bad senders cater for the biased one (sending 0). Additionally, there is an equilibrium in which both types of expert are truthful. However, for this equilibrium to hold, $n \geq 3$, as otherwise, the perverse effect of competition dominates.

One expert

We briefly comment the case of one single expert. In this case, we observe that the uncertainty about the receiver's motives reduces the incentives of the sender to be truthful, as it is now the case that the receiver may prefer biased information. In this sense, with one single expert, an outcome concerned receiver losses from any ambiguity on his motives. Hence, if that receiver were given the opportunity to speak about his preferences, he would clearly state them.

Let $\Lambda(m)$ be the equilibrium posterior probability assigned to the state of the world being 0, upon observing message m. We next characterize the informative equilibria in this case. We obtain that the bad sender is never truthful and that there is a unique partially-informative equilibrium in which the good sender reveals her information. This is the outcome except for the very particular case of $\beta = \frac{1}{2}$, where the bad expert could be the only truthful sender in the population.[3] Next proposition refers to non-degenerate cases.

Proposition 3. *In the game with two-sided incomplete information and one single expert, there is a unique equilibrium with information transmission. In this*

[3] If $\beta = \frac{1}{2}$, there is an equilibrium in which, for all $m \in \{0,1\}$, $a(m) = 0$ for the biased receiver and $a(m) = 1$ for the honest one. It requires one type of sender to be truthful (could be the bad type) and the other one to pool at 0 (otherwise, $a(m) = 1$ would not be a best response). In this equilibrium, the payoff to the bad sender is $-\frac{1}{2}$ and that of the good sender is $-\frac{1}{2}(1+\lambda)$, independently of their messages.

partially-truthful equilibrium, the good sender is sincere, the bad expert pools at 0 *and conditions* $\theta > \frac{1-\alpha}{2-\alpha}$ *and* $\beta > \frac{1}{1+\lambda}$ *must be satisfied.*

Proof. First, note that, for all $m \in \{0, 1\}$, $a(m) = 0$ for the biased receiver.

(1) Let us conjecture an equilibrium in which, for all $m \in \{0, 1\}$, $a(m) = 1$ for the honest receiver. If $m(\cdot) = 0$, the payoff to a bad sender is $\beta(-1)$; whereas if $m(\cdot) = 1$, her payoff is $(1 - \beta)(-1)$. Hence, if $\beta \neq \frac{1}{2}$, the bad sender is never truthful. Let us now conjecture an equilibrium in which, for all $m \in \{0, 1\}$, either $a(m) = 0$, $a(m) = m$ or $a(m) \neq m$ for the honest receiver. The bad sender always pools at 0, i.e., she is not informative.

(2) Thus, let us conjecture an equilibrium in which, for all $w \in \{0, 1\}$, $m(w) = w$ for the good sender. (2.i) Consider that, for all $w \in \{0, 1\}$, $m(w) = 1$ for the bad sender. By Bayes' rule, $\Lambda(0) = 1$. But then, a bad sender finds it optimal to deviate to 0. A contradiction. (2.ii) Consider that, for all $w \in \{0, 1\}$, $m(w) = 0$ for the bad sender. By Bayes' rule, $\Lambda(1) = 0$ and $\Lambda(0) = \frac{\theta}{\theta+(1-\theta)(1-\alpha)}$. According to these posteriors, for all $m \in \{0, 1\}$, $a(m) = m$ for the honest receiver if and only if $\theta > \frac{1-\alpha}{2-\alpha}$. If $\theta > \frac{1-\alpha}{2-\alpha}$, $m(\cdot) = 0$ for the bad sender; whereas the good sender is truthful if and only if $\beta > \frac{1}{1+\lambda}$. If $\theta < \frac{1-\alpha}{2-\alpha}$, the honest receiver pools at 1. Here, for all $w \in \{0, 1\}$, if $\beta > \frac{1}{2}$, $m(w) = 1$ for both types of sender; whereas if $\beta < \frac{1}{2}$, $m(w) = 0$ for both types. This completes the proof. □

From Proposition 3 we observe that the good sender is truthful when either the receiver is perceived as honest (high β) or/and the concern of the sender for the policy implemented is high enough (high λ). Otherwise, there is no sincere information transmission. Thus, introducing uncertainty with one single expert does not improve the conditions for information transmission but worsens them.

Competing experts

We now focus on the case of n competing experts. Here, senders send their reports in the light of uncertainty about the motives of the other experts as well as of the receiver. Each sender knows, however, the number of experts who compete for the contract and the fact that a biased receiver always implements 0, independently of the vector of messages $(m_1, m_2, .., m_n)$.

We first analyze whether truthful equilibria exist and obtain that it is the case if and only if $n \geq 3$. Introducing uncertainty in the case of two senders is thus not enough to guarantee sincere reporting by the two types of expert. It however serves when the number of competing experts is more than two. In this case, it sustains equilibria for those out-of-equilibrium-path beliefs for which an equilibrium existed in the case of certainty.

Proposition 4. *In the game with two-sided incomplete information and competing experts, there is a truthful equilibrium in which all experts are sincere if and only if* $n \geq 3$, $\beta > \frac{n}{n+1}$, *and out-of-equilibrium-path beliefs are* $x > \frac{1}{2}$ *and* $y < \frac{1}{2}$.

Proof. Let us conjecture an equilibrium in which, for all $w \in \{0,1\}$, $m(w) = w$ for both types of sender. By Bayes' rule, $\Lambda(0,0,...,0) = 1$ and $\Lambda(1,1,...,1) = 0$. Let Λ("all messages support 0 but one") $= x \in [0,1]$ and Λ("all messages support 1 but one") $= y \in [0,1]$. According to these posteriors, the honest receiver optimally chooses $a(0,0,...,0) = 0$ and $a(1,1,...,1) = 1$. Additionally, he implements a("all messages support 0 but one") $= 0$ if and only if $x > \frac{1}{2}$ and a("all messages support 1 but one") $= 0$ if and only if $y > \frac{1}{2}$.

(1) Consider $y > \frac{1}{2}$ (with $x \in [0,1]$). If $w = 1$ and $m(1) = 1$, the payoff to a bad sender is $\beta(\frac{1-n}{n}) + (1-\beta)(-1)$; whereas her payoff if she deviates to $m(1) = 0$ is 0. There is no equilibrium of the type conjectured.

(2) Consider now $y < \frac{1}{2}$ and $x < \frac{1}{2}$. Let us focus on the bad sender. If $w = 0$ and $m(0) = 0$, her payoff is $\frac{1-n}{n}$; whereas if she deviates to $m(0) = 1$, it is $(1-\beta)(-1)$. Hence, $m(0) = 0$ if and only if $\beta < \frac{1}{n}$. Now, if $w = 1$ and $m(1) = 1$, her payoff is $\beta(\frac{1-n}{n}) + (1-\beta)(-1)$; whereas if she deviates to $m(1) = 0$, it is $\beta(-1)$. Hence, $m(1) = 1$ if and only if $\beta > \frac{n}{n+1}$. As $\frac{n}{n+1} < \frac{1}{n}$ if and only if $n = 1$ and neither x nor y are defined in this case, there is no equilibrium of the type conjectured.

(3) Last, consider $y < \frac{1}{2}$ and $x > \frac{1}{2}$. If $w = 0$ and $m(0) = 0$, the payoff to a bad sender is $\frac{1-n}{n}$; whereas if she deviates to $m(0) = 1$, it is -1. Analogously for a good sender. Now, if $w = 1$ and $m(1) = 1$, the payoff to a bad sender is $\beta(\frac{1-n}{n}) + (1-\beta)(-1)$; whereas if she deviates to $m(1) = 0$, it is $\beta(-1)$. Similarly, if $m(1) = 1$, the payoff to a good sender is $\beta(\frac{1-n}{n}) + (1-\beta)(-1-\lambda)$; whereas if she deviates to $m(1) = 0$, it is $\beta(-1) + (1-\beta)(-\lambda)$. Hence, for all $w \in \{0,1\}$, $m(w) = w$ for both types of sender if and only if $\beta > \frac{n}{n+1}$. Note that, if $n = 2$, $x = y$; hence, in this case, this is not an equilibrium. This completes the proof. □

Similar to the case of one-sided incomplete information, we observe that for a truthful equilibrium to exist, the honest receiver must trust and implement the advice of the majority. Otherwise, cheating behavior would be encouraged.

On the other hand, Proposition 4 says that, if $n \geq 3$, condition $\beta > \frac{n}{n+1}$ is necessary for an informative equilibrium to exist. We observe that, as n increases, so as the values of β that sustain truthful information transmission. The reason is that, the tougher competition, the lower the equilibrium payoff associated to the event of the receiver being honest (whereas the out-of-equilibrium path payoff does not vary). Then, for the incentives to deviate not to increase too much as competition goes up, the probability of the decision maker being honest must be high enough.

Corollary 2. *For a truthful equilibrium to exist, the tougher competition, the higher the belief that the receiver is honest.*

So far we have focused on the scenario in which all experts are truthful. As in the case of one-sided incomplete information, we next analyze whether there is an equilibrium in which one single type of expert is sincere. In particular, we posit the most natural scenario: Good senders are sincere whereas bad senders are not (they manipulate their report and pool at 0). Contrary to the case of one-sided incomplete information, we obtain that this is an equilibrium with two-sided incomplete information. The intuition is straight forward: Uncertainty allows different types of experts to tailor their messages and cater for the preferences of different types of receiver.

Prior to the analysis of this set-up, note that in the conjectured equilibrium, senders are unsure about the number of experts sending each of the two possible messages. They thus have to compute their payoffs for each possible situation as well as the probability of those events occurring, which is given by the binomial distribution.

Proposition 5. *In the game with two-sided incomplete information and competing experts, there is an equilibrium in which one single type of expert is sincere. In this partially-truthful equilibrium, good senders are truthful, bad senders pool at* 0 *and conditions* $\theta > \frac{(1-\alpha)^n}{1+(1-\alpha)^n}$ *and* $\frac{\alpha(1-\alpha^n)}{1-\alpha^{n+1}+(1-\alpha)^n(n\alpha\lambda-1)} < \beta < \frac{1}{n}$ *must be satisfied.*

Proof. Let us conjecture an equilibrium in which, for all $w \in \{0,1\}$, $m(w) = w$ for good senders and $m(w) = 0$ for bad senders. By Bayes' rule, Λ("at least one message supports 1") $= 0$ and $\Lambda(0,0,...,0) = \frac{\theta}{\theta+(1-\theta)(1-\alpha)^n}$. According to these posteriors, the honest receiver optimally chooses a("at least one message supports 1") $= 1$. Additionally, he implements $a(0,0,...,0) = 0$ if and only if $\theta > \frac{(1-\alpha)^n}{1+(1-\alpha)^n}$.

(1) Consider $\theta < \frac{(1-\alpha)^n}{1+(1-\alpha)^n}$ and let us focus on the case of the good sender. If $w = 0$ and she chooses $m(0) = 0$, her payoff is $\beta(-1-\lambda) + (1-\beta)(1-\frac{1}{n})(-1)$; whereas if she deviates and sends $m(0) = 1$, it is $\beta(-\lambda) + (1-\beta)(-1)$. Hence, $m(0) = 0$ for the good sender if and only if $\beta < \frac{1}{n+1}$. Now, if $w = 1$ and the good sender chooses $m(1) = 1$, her payoff is $\sum_{j=0}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}[\beta(1-\frac{1}{j+1})(-1)+(1-\beta)(-1-\lambda)]$; whereas if she deviates and sends $m(1) = 0$, it is $\sum_{j=0}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}[\beta(-1)+(1-\beta)((1-\frac{1}{n-j})(-1)-\lambda)]$. As $\sum_{j=0}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1} = 1$, after some algebra we obtain that the payoff to a good sender that chooses $m(1) = 1$ is $(1-\beta)(-1-\lambda) - \beta + \beta\sum_{j=0}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}\frac{1}{j+1} = (1-\beta)(-1-\lambda) - \beta + \beta\frac{1-(1-\alpha)^n}{n\alpha}$. Similarly, the payoff to a good sender that chooses $m(1) = 0$ is $-\beta - (1-\beta)(1+\lambda) + (1-\beta)\sum_{j=0}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}\frac{1}{n-j} = -\beta - (1-\beta)(1+\lambda) + (1-\beta)\frac{1-\alpha^n}{n(1-\alpha)}$. Solving for β, we obtain that $m(1) = 1$ for the good sender if and only if $\beta > 1 + \frac{(1-\alpha)((1-\alpha)^n-1)}{1-(1-\alpha)^n+\alpha(1-\alpha)^n-\alpha^{n+1}}$. Lemma 1 in the Appendix shows that $1 + \frac{(1-\alpha)((1-\alpha)^n-1)}{1-(1-\alpha)^n+\alpha(1-\alpha)^n-\alpha^{n+1}} > \frac{1}{n+1}$. Hence, there is no equilibrium of the type conjectured.

(2) Consider now $\theta > \frac{(1-\alpha)^n}{1+(1-\alpha)^n}$. (i) Let us first focus on the bad sender. If $w = 0$ and $m(0) = 0$, her payoff is $\frac{1-n}{n}$; whereas if she deviates and sends $m(0) = 1$, it is $(1-\beta)(-1)$. Hence, $m(0) = 0$ for the bad sender if and only if $\beta < \frac{1}{n}$. Now, if $w = 1$ and $m(1) = 0$, her payoff is $(1-\alpha)^{n-1}\frac{1-n}{n} + \sum_{j=1}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}[\beta(-1)+(1-\beta)(1-\frac{1}{n-j})(-1)]$; whereas if she deviates and sends $m(1) =$

1, it is $\sum_{j=0}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}[\beta(1-\frac{1}{j+1})(-1)+(1-\beta)(-1)]$. Note that $\sum_{j=1}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}=1-(1-\alpha)^{n-1}$. Then, operating as before, we obtain that $m(1)=0$ for the bad sender if and only if $\beta<\frac{\alpha(1-\alpha^n)}{1-(1-\alpha)^n-\alpha^{n+1}}$. Lemma 2 in the Appendix shows that a sufficient condition for this inequality to hold is $\beta<\frac{1}{n}$. (ii) Now, let us consider the good sender. If $w=0$ and $m(0)=0$, her payoff is $\frac{1-n}{n}$; whereas if she deviates and sends $m(0)=1$, it is $\beta(-\lambda)+(1-\beta)(-1)$. Hence, $m(0)=0$ for the good sender if and only if $\frac{1}{n}>\beta(1-\lambda)$. Now, if $\lambda>1$, the aforementioned condition always hold; and if $\lambda\in(0,1)$, this condition can be rewritten as $\beta<\frac{1}{n(1-\lambda)}$ (a sufficient condition for this inequality to hold is $\beta<\frac{1}{n}$). Last, if $w=1$ and $m(1)=1$, the payoff to the good sender is $\sum_{j=0}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}[\beta(1-\frac{1}{j+1})(-1)+(1-\beta)(-1-\lambda)]$; whereas is she deviates to $m(1)=0$, her payoff is $(1-\alpha)^{n-1}(\frac{1-n}{n}-\lambda)+\sum_{j=1}^{n-1}\binom{n-1}{j}\alpha^j(1-\alpha)^{n-j-1}[\beta(-1)+(1-\beta)((1-\frac{1}{n-j})(-1)-\lambda)]$. Here again, $m(1)=1$ for the good sender if and only if $\beta>\frac{\alpha(1-\alpha^n)}{1-\alpha^{n+1}+(1-\alpha)^n(n\alpha\lambda-1)}$. This completes the proof. $\square$

Figures 1 and 2 below illustrate the region where, according to β, there may be an equilibrium in which good experts cater for the preferences of the honest receiver (by being truthful), whereas bad experts conform to the motives of the biased receiver (sending 0 for any state). The region is determined by $\frac{\alpha(1-\alpha^n)}{1-\alpha^{n+1}+(1-\alpha)^n(n\alpha\lambda-1)}<\beta<\frac{1}{n}$. We represent β in the vertical axis and depict the upper bound, $\frac{1}{n}$, in blue (black); and the lower bound, $\frac{\alpha(1-\alpha^n)}{1-\alpha^{n+1}+(1-\alpha)^n(n\alpha\lambda-1)}$, in green (grey). Thus, the area below the blue (black) surface and above the green (grey) one corresponds to the region where an equilibrium of the type conjectured may exist. Last, note that the equilibrium described in Proposition 3 is a subcase of that in Proposition 5, when $n=1$. Hence, the analysis that follows is done for $n\geq 1$.

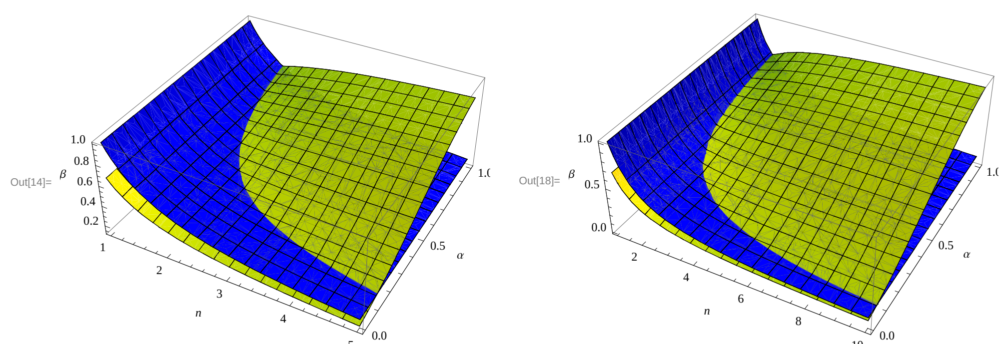

Figure 1: $\lambda=0.5$. Regions of existence of an equilibrium for $n=5$ and $n=10$

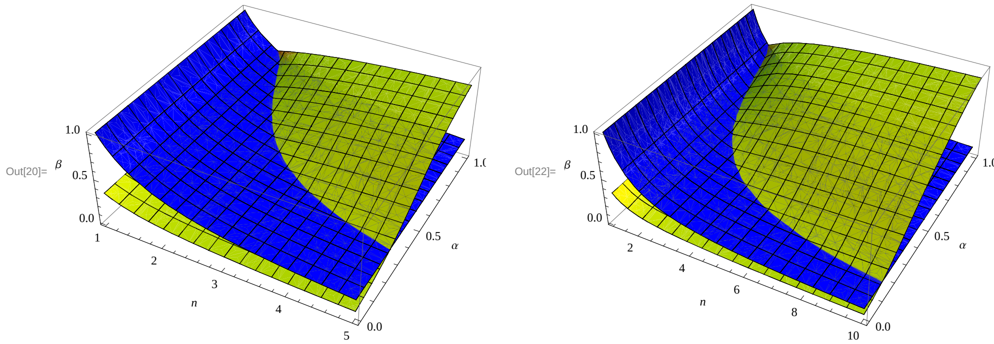

Figure 2: $\lambda = 2$. Regions of existence of an equilibrium for $n = 5$ and $n = 10$

The first two graphs correspond to a case in which good experts are relatively more concerned for approval than for the policy implemented ($\lambda = 0.5$); whereas in the last two graphs, it is the other way round ($\lambda = 2$). We observe that the greater the social concern of the sender, the lower $\frac{\alpha(1-\alpha^n)}{1-\alpha^{n+1}+(1-\alpha)^n(n\alpha\lambda-1)}$; then, the greater the region where an equilibrium of the type conjectured may exist. Additionally, we observe that as competition increases, the region where an equilibrium of this type may exist shrinks. More precisely, this region shrinks because, for all $\alpha \in (0, 1)$, β decreases as n increases. In words, as competition becomes fiercer, we need experts to doubt more about the honesty of the decision maker. Otherwise, the opportunistic senders (those with no concern for lying) would find it profitable to deviate. Note that this is the case in the truthful equilibrium previously analyzed, where we concluded that n and β must be positively related. In contrast, for a partially-truthful equilibrium to exist, the condition is reversed: n and β must be negatively related.

Corollary 3. *For a partially-truthful equilibrium to exist, the tougher competition, the lower the belief that the receiver is honest.*

To summarize, by introducing uncertainty in the case of competing experts, we learn that the truthful equilibrium, in which all experts are sincere, hold (when $n \geq 3$, which is also the condition with one-sided incomplete information); and that a partially-truthful equilibrium, in which experts specialize (tailoring their messages to the preferences of the receiver they want to please), emerges. Note that this partially-truthful equilibrium exists even though senders share the belief on the type of the decision maker (but do have different payoff structures). Additionally, note that the partially-truthful equilibrium exists for $n = 2$, in which case no information transmission can be sustained in equilibrium if there is only one-sided incomplete information. In this sense, uncertainty offsets to a certain extend the pernicious effect of competition and sustains information transmission by (at least) the good experts (in the hope that the receiver turns out to be honest). Hence, with multiple experts, if an outcome concerned receiver were given the opportunity to speak about his motives, he would choose not to clearly state them.

4 A case for ambiguity

From the previous analysis we learn that the incentives of a sender to be truthful may diminish with competition (case $n = 2$). However, as the number of competing experts increases, the probability of at least one sender being good raises. We next combine these two aspects and study whether competition affects positively or not both, the probability of an honest receiver obtaining a truthful message and that of him implementing the right policy. To this aim, we condition our analysis on an informative equilibrium being played.

Let us first consider the case in which senders know the preferences of the receiver. Here, with one single expert, there is an equilibrium in which both types of sender are truthful. In this equilibrium, the probability of receiving a truthful message is always one. Since, for all $m \in \{0, 1\}$, $a(m) = m$ in equilibrium; the implemented policy always corresponds to the state of the world. On the other hand, with competing experts, informative speeches are made only if $n \geq 3$, so the implemented policy is not necessarily optimal (case $n = 2$).

Remark 1. *In the game with one-sided incomplete information and conditioned (if possible) on playing a truthful equilibrium, competition between experts does not necessarily improve both: the quality of the information released and the optimality of the implemented policy.*

Let us now consider the case in which senders are uncertain about the motives of the receiver. In this case, there is a truthful equilibrium in which all senders are sincere only if $n \geq 3$. In this equilibrium, for all $i, j \in \{1, 2, ..., n\}$, $m_i = m_j$ and $a(\cdot) = m_i$. Hence, the implemented policy always corresponds to the state of the world. Additionally, there always exists a partially-truthful equilibrium (for any $n \geq 1$) in which good experts cater for the honest receiver, i.e., they are truthful; whereas bad experts cater for the biased receiver, i.e., they pool at 0. In the case of one single expert and, conditional on playing the partially-truthful equilibrium, the probability of the receiver obtaining a sincere message is $\theta + (1 - \theta)\alpha$. It coincides with the probability of him implementing the right policy as, in equilibrium, for all $m \in \{0, 1\}$, $a(m) = m$ for the honest receiver. On the other hand, in the case of competing experts, $a(0, 0, ..., 0) = 0$ for the honest receiver, otherwise he chooses 1. Hence, conditional on playing the partially-truthful equilibrium, the probability of the honest receiver implementing the right policy is equal to the probability of receiving (at least) one sincere message, which is $\theta + (1 - \theta)(1 - (1 - \alpha)^n)$. As $\alpha \in (0, 1)$, the next result follows.

Remark 2. *In the game with two-sided incomplete information and conditioned on playing a (partially) truthful equilibrium, competition between experts is beneficial to both: the quality of the information released and the optimality of the implemented policy.*

Hence, the benefit of competition depends on the informative structure of the game, i.e., whether the receiver's motive is his private information or not. This result has an interesting reading: If an outcome concerned local authority were given the opportunity to speak about his motives and he were to anticipate competition between experts, he would choose to be unclear and create confusion among firms. In contrast, if the local authority knew that one single expert would apply for the contract, he would rather prefer to be clear and precise in his objectives.

5 Conclusion

We consider a sender-receiver game in which the sender wants the approval of the receiver and the latter is outcome concerned. In this situation, the sender has an incentive to reveal her information. But if we introduce competition between experts and the receiver thinks that senders might also be outcome concerned, they each have an incentive to make the receiver believe that they actually are the unique trustworthy expert in the population. Then, information may not be transmitted in equilibrium. In this scenario, we show that it may be in the decision maker's interest to be ambiguous about his motives as, if succeeded, he could guarantee revelation of information by (at least) outcome concerned experts.

Our results have implications for public procurement processes, job promotions and other situations where more than one expert compete for a unique prize, i.e., the decision maker's approval. To any of these cases, our conclusion is that a bit of ambiguity may actually pay. There is, however, much to do to account for all the particularities of these decision making processes. For example, it could be interesting to consider experts of different quality (regarding their signals), to make the number of experts that compete for the contract award private information, or to consider more than two state spaces. The analysis of these variations is left for future research.

6 Appendix

Lemma 1. *For all $\alpha \in (0,1)$ and $n \geq 2$, $1 + \frac{(1-\alpha)((1-\alpha)^n-1)}{1-(1-\alpha)^n+\alpha(1-\alpha)^n-\alpha^{n+1}} > \frac{1}{n+1}$.*

Proof. Operating on the inequality we obtain that $1 + \frac{(1-\alpha)((1-\alpha)^n-1)}{1-(1-\alpha)^n+\alpha(1-\alpha)^n-\alpha^{n+1}} > \frac{1}{n+1}$ if and only if $\frac{(1-\alpha)(1-(1-\alpha)^n)}{1-(1-\alpha)^n-\alpha(\alpha^n-(1-\alpha)^n)} < \frac{n}{n+1}$. As $1-(1-\alpha)^n > \alpha^n - (1-\alpha)^n$ and $\alpha < 1$, the denominator is positive. Then, we can write the inequality as $\frac{(1-\alpha)(1-(1-\alpha)^n)}{\alpha(1-\alpha^n)} < n$. Let us denote $f(\alpha) = \frac{(1-\alpha)(1-(1-\alpha)^n)}{\alpha(1-\alpha^n)}$. Applying L'Hôpital's rule, we obtain $\lim_{\alpha\to 0} f(\alpha) = n$ and $\lim_{\alpha \to 1} f(\alpha) = \frac{1}{n}$. Then, to complete the proof, we have to show that $f'(\alpha) < 0$. We obtain $f'(\alpha) = \frac{\alpha(1-\alpha^n)[n(1-\alpha)^n-(1-(1-\alpha)^n)]}{\alpha^2(1-\alpha^n)^2} - \frac{(1-\alpha)(1-(1-\alpha)^n)((1-\alpha^n)-n\alpha^n)}{\alpha^2(1-\alpha^n)^2} < 0$ if and only if the numerator is negative. Rearranging, $f'(\alpha) < 0$ if and only if $g(\alpha) = \alpha(1-\alpha^n)[(1-\alpha)^n(n+1)-1] + (1-\alpha)(1-(1-\alpha)^n)(\alpha^n(n+1)-1) < 0$. Note that $g(0) = 0$ and $g(1) = 0$. Additionally, $g(\alpha) = g(1-\alpha)$, i.e., the function is symmetric around one half. Then, it is sufficient to prove that $g'(\alpha) < 0$ for all $\alpha \in (0, \frac{1}{2})$. We obtain $g'(\alpha) = \frac{-n(n+1)[(1-\alpha)^n\alpha^2-\alpha^n(1-\alpha)^2+\alpha^n(1-\alpha)^n(1-2\alpha)]}{n(1-\alpha)} < 0$ if and only if

$(1-\alpha)^n\alpha^2-\alpha^n(1-\alpha)^2+\alpha^n(1-\alpha)^n(1-2\alpha) > 0$. As $\alpha < \frac{1}{2}$, $\alpha^n(1-\alpha)^n(1-2\alpha) > 0$ and $(1-\alpha)^n\alpha^2 - \alpha^n(1-\alpha)^2 = (1-\alpha)^2\alpha^2[(1-\alpha)^{n-2} - \alpha^{n-2}] > 0$. This completes the proof. □

Lemma 2. *For all $\alpha \in (0,1)$ and $n \geq 2$, $\frac{1}{n} < \frac{\alpha(1-\alpha^n)}{1-(1-\alpha)^n-\alpha^{n+1}}$.*

Proof. Let us denote $h(\alpha) = \frac{\alpha(1-\alpha^n)}{1-(1-\alpha)^n-\alpha^{n+1}}$. Applying L'Hôpital's rule, we obtain $\lim_{\alpha\to 0} h(\alpha) = \frac{1}{n}$ and $\lim_{\alpha\to 1} h(\alpha) = \frac{n}{1+n}$. Then, we have to show that $h'(\alpha) > 0$. We obtain $h'(\alpha) = \frac{(1-(n+1)\alpha^n)(1-(1-\alpha)^n-\alpha^{n+1})-(n(1-\alpha)^{n-1}-(n+1)\alpha^n)(\alpha-\alpha^{n+1})}{(1-(1-\alpha)^n-\alpha^{n+1})^2} > 0$ if and only if the numerator is positive which, after some algebra, can be written as $w(\alpha,n) = n\alpha[\alpha^n - \alpha^{n-1} - (1-\alpha)^{n-1} + \alpha^{n-1}(1-\alpha)^{n-1}] + [1-\alpha^n-(1-\alpha)^n + \alpha^n(1-\alpha)^n] > 0$. Note that $w(\alpha,2) = (-1+\alpha)^2\alpha^2 > 0$ and $\frac{\partial w(\alpha,n)}{\partial n} > 0$. This completes the proof.

□

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