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WP 2010-3
March 2010

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Universidad de Málaga
ISSN 1989-6908

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March 17, 2010

Abstract

We put forward a simple spatial competition model to study banks' strategic responses to the Spanish asymmetric geographic deregulation. We find that once geographic deregulation process finishes, inter-regional mergers between the savings banks are optimal. We claim that the public good nature of the merging activities together with the incentives provided by the deregulation process are the driving factors behind the equilibrium merger of the savings banks. It seems that the economic crisis will finally force regional politicians to allow inter-regional *caja* mergers, letting the consequences of the removal of geographic barriers in the 80's come to a fruition with a delay of thirty years.

JEL Codes: C72, G21, G28, L13, L41, L51

*The authors gratefully acknowledge financial support from the Spanish Ministry of Education (project ECO2008-04424) and from the Andalusian High Excellence Research Program (project SEJ 4794).

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1 Introduction

The current world economy crisis has lead to many debates all over the world. And the debates are necessarily louder for those countries who have experienced deeper (and long-lasting) recessions. This is the case of Spain. Even though the Spanish banking sector has not (yet) contemplated the bankrupcy and intervention of any major bank, many voices claim for structural reforms in the banking system.

There are two major types of institutions: savings and commercial banks, which differ in their legal nature and in their geographic presence. Savings banks are non-profitable institutions mostly located in the territory where they were founded several decades (in some cases, centuries) ago, whereas commercial banks are present nationwide and are profit seekers.

The required restructuring takes the form of a consolidation process, in which the weakest institutions are wiped out from the territory. And it happens that the weakest ones are mainly savings banks (due not only to large exposures to bad loans but also to a quick business switch from very traditional retail banking to more sophisticated product activities: securisation, mortages, etc). The governor of the Bank of Spain has actually quantified the magnitude of the reform: a third of the 45 unlisted regional savings banks are to be absorbed by stronger institutions (see Financial Times, Feb 24, 2010, "Bank of Spain chief in reform plea").

This view is actually shared by the main players of the banking sector. The president of the Spanish Association of Commercial Banks (AEB) has also warned about the "excess of capacity in the banking sector" (see El País, March 12, 2010) and the president of the Spanish Association of Savings Banks (CECA) has called for less political control and more legal reforms to ease the merger process (see El País, March 11, 2010). Also the socialist party (that currently rules the country) and the main opposition party share this view. However, despite this consensus, the progress of the consolidation process is slow (see Financial Times, Feb 23, 2010, "Bank of Spain hits at delays in caja mergers").

The main reason is that savings banks are controlled by regional politicians and they are reluctant to give them away (see The Economist, March 11, 2010 "All talk, no walk"). And it is actually this resistance from local politicians that explains the null presence of inter-regional mergers among savings banks since the end of the eighties, when geographic restrictions on the expansion of savings banks were lifted. Until those days, only commercial banks could operate nationwide.

Despite the fact that geographic expansion was limited in most countries, only a few papers have theoretically examined the interplay between geographic regulation and banking behaviour.[1] Economides et al (1996) rationalizes the setting of geographic restrictions as an effective mean to protect small banks from large banks in the US and Lozano et al (2010) analyses the consequences of the removal of these barriers in the US.

In this paper we claim that inter-regional savings banks mergers are the natural consequence not only of their weakest relative position to the current crisis (larger exposure to bad loans related to credits to property developers and builders) but also of the asymmetric branching regulation. We do so by setting forth a simple spatial competition model in which banks' strategic responses to geographic deregulation are studied.

Given that our focus is on inter-regional mergers, we consider the simplest case in which they can be studied: two (asymmetric) territories, i.e. regions (one richer than the other). In each territory, the local savings bank faces the competition of a unique universal commercial bank. Once deregulation is announced, incumbents in each territory have the opportunity to resize their branch network before entry decisions are taken. The crucial feature of the model is that the commercial bank is incumbent in both territories whereas each savings bank is incumbent in one territory and entrant in the other.

The equilibrium configuration of the deregulation game is taken as the initial condition for the merger game. There, two types of mergers (if any) can arise: homogeneous (involving institutions of the same type) and heteregenenous. We show that for any general merger protocol, a merger will take place and savings banks will find optimal to merge each other.

The intuition is that mergers in a model of banking competition have two crucial features: (i) they provide a benefit to the merging institutions but (ii) the non-merging institution benefits the most (public good nature). One can show that the merger target of any institution is the competitor with the smallest branch network. This together with the fact that the asymmetric

[1] The effects of the banking deregulation have been extensively analysed from an empirical point of view in most countries. For a review of the Spanish case, see for example Carbó (2004).

branching deregulation results in the commercial bank enjoying the largest branch network among all institutions in the banking system, imply that the commercial bank is not the target of any banking institution. Hence, savings banks merge each other.

Given that every cloud has a silver lining, it seems that the financial crisis will force regional politicians to finally allow inter-regional *caja* mergers, letting the consequences of the removal of geographic barriers in the 80's come to a fruition with a delay of thirty years. The rest of the paper is as follows. Next section presents the banking competition model. Sections 3 and 4 analyse the branching deregulation model and the incentives to merge. Section 5 concludes. All the proofs are in the Appendix.

2 Bank competition model

To model banking competition in a territory, we follow Economides et al. (1996) and Lozano et at (2010) and use Salop's spatial model (1979). We simplify matters by considering the limit case when transportation costs approach infinity. This way, we can dispense with interest rates decisions and focus on the number of branches settled by each banking institution. Assuming symmetric location of branches, profits to banking institution i in territory k amount to

$$\pi_{i,k} = n_{i,k}\left(\delta_k \widetilde{r}\frac{1}{N_k} - \Phi\right)$$

where δ_k represents territory k deposit density, $\widetilde{r}$ is market interest rate,[2] $n_{i,k}$ is the number of branches opened by banking institution i in territory k, N_k is the total number of branches (of all banking institutions) in territory k, and $\Phi > 0$ represents cost per branch opened.[3]

The profits to a banking institution that operates in several territories is the sum of her profits in each territory. Note that our formulation implicitly assumes that the functioning of a territory is isolated from the rest (we do not allow customers of a territory to fulfill their banking needs outside

[2] We assume that banks, regardless of their territory, can access the same market interest rate.

[3] This bank competition model is strategically equivalent to a particular case of the rent-seeking game proposed by Tullock (1980).

their territory and there are no economies of scale/scope in the banking technology).

3 The Spanish case

Until the mid seventies, both savings and commercial banks were only allowed to operate in their regional territories. In 1974 this restriction was lifted for commercial banks whereas it was not until 1988 that savings banks were also allowed to operate nationwide.[4]

In our model, we consider two territories and three players: two savings banks and one commercial bank. The two territories are asymmetric in that the deposits density of territory 1 is larger than that of territory 2. That is, we assume $\delta_1 = \delta > 0$ and $\delta_2 = \alpha\delta$ with $\alpha \in (0, 1]$.[5]

Our starting point is the asymmetric situation in which savings banks are restricted to operate in their own territory whereas the commercial bank is allowed to operate everywhere. Then, deregulation takes place as follows:

Definition 1 *Once the Regulatory Agency announces cross border activities at a given date,*

Stage 1. *Incumbents decide simultaneously the number of branches to open in their own territories;*

Stage 2. *Upon observing incumbents' decisions in Stage 1, the entrant decides how many branches to open in the new territory. Payoffs are given by the bank competition model in Section 2.*

3.1 The initial condition

In this paper each savings bank will be named by the territory in which she is initially constrained to operate. The subscript 3 is for the commercial bank.

[4] There were also regulations concerning the setting of loans and deposits interest rates, although we abstract from them in this paper. See Canals (1997) and Caminal et al (1989) for an account of the Spanish banking deregulation process.

[5] The parameter α measures how wealth (which will be deposited in the banking system) is distributed throughout the country. When $\alpha = 1$ the wealth is evenly distributed, whereas as $\alpha \to 0$ the percentage of wealth in territory 1 approaches 100%.

The asymmetric regulatory body precluded savings banks from operating outside their own territory. Hence, $n_{1,2} = n_{2,1} = 0$. The commercial bank is free to operate nationwide. We look for the the Nash equilibrium $\left\{n^*_{k,k}, n^*_{3,k}\right\}$ of the bank competition game in every territory k.

Proposition 1 *In each territory, the commercial and the local savings banks equally share the deposits market; each ones opens* $\frac{\delta_k}{4}\frac{\widetilde{r}}{\Phi}$ *branches.*

Note that the number of branches opened by each banking institution in each territory depends on the parameters of the model in an intuitive manner. The total number of branches in a given territory, $N_k = \frac{1}{2}\frac{\delta_k \widetilde{r}}{\Phi}$, depends positively on the territory deposit density δ_k and the interest rate $\widetilde{r}$. It depends negatively on the cost per branch Φ.

Next table displays the equilibrium outcome in this period period.

	MARKET		BRANCH NETWORK	
PLAYER	1	2	SIZE	RATIO
1	8ω	0	8ω	$\frac{1}{2(1+\alpha)}$
2	0	$8\alpha\omega$	$8\alpha\omega$	$\frac{\alpha}{2(1+\alpha)}$
3	8ω	$8\alpha\omega$	$8(1+\alpha)\,\omega$	$1/2$
TOTAL			$16(1+\alpha)\,\omega$	1

Table 1. Equilibrium outcome in the regulated period

where

$$\omega = \frac{\delta \widetilde{r}}{32\Phi}$$

As we see, each territory is equally shared by the commercial bank and the local savings bank. Given that territories differ in deposits densities, we obtain an asymmetric distribution of branch network sizes: half of the branches are owned by the commercial bank, whereas the remaining branches are distributed between the savings banks according to the relative weighs $\left(\frac{1}{1+\alpha}, \frac{\alpha}{1+\alpha}\right)$.

3.2 The Deregulation Game

Assume that the authority announces that branching restrictions will be lifted. A sequential game is defined as follows. First, the incumbents of each territory can (prior to the date in which the deregulation is effective) modify their number of branches. Then, in the second stage, the entrants decide how many branches to open in the new territory. We look for the subgame perfect equilibrium of this game.

3.2.1 The entrant stage

In this subgame, entrants decide on the number of branches to open in territory k, after observing the number of branches opened by the incumbents.

Lemma 1 *Let I_k be the number of branches opened by the incumbents in territory k. In the entry stage, the optimal number of branches opened by each entrant is*

$$e_k^*(I_k) = \begin{cases} \sqrt{2N_k\left(I_k + N_k\right)} - \left(N_k + I_k\right) & \text{if } I_k \leq N_k \\ 0 & \text{otherwise} \end{cases}$$

Note that entry can be prevented by doubling the number of existing branches.

3.2.2 The incumbents stage

We now analyze the behaviour of the two incumbents. They decide on their own optimal opening of branches anticipating the optimal behavior of the entrant described in Lemma 1.

Proposition 2 *The optimal number of branches opened by each incumbent of territory k is $\frac{N_k}{16}$.*

The optimal number of branches opened by the incumbents in territory k is $2i_k^* = \frac{1}{8}N_k$. Given that it is smaller than N_k it follows (Lemma 1) that entry prevention does not happen in equilibrium. In fact, we get that the optimal number of branches opened by the entrant in territory k is

$$e_k^*\left(2i_k^*\right) = 6i_k^*$$

3.2.3 Equilibrium outcome of the Deregulation game

We can now set forth the equilibrium outcome of the Deregulation Game. The number of branches of each banking institution in territory k is displayed in Table 2.

	Market 1			Market 2			Network
Player	Reg	Dereg	Total	Reg	Dereg	Total	Size
1	8ω	ω	9ω	0	$6\alpha\omega$	$6\alpha\omega$	$(9+6\alpha)\,\omega$
2	0	6ω	6ω	$8\alpha\omega$	$\alpha\omega$	$9\alpha\omega$	$(6+9\alpha)\,\omega$
3	8ω	ω	9ω	$8\alpha\omega$	$\alpha\omega$	$9\alpha\omega$	$9\,(1+\alpha)\,\omega$
Total			24ω			$24\alpha\omega$	$24\,(1+\alpha)\,\omega$

Table 2. Equilibrium outcome of the Deregulation game

Note that

$$\frac{9}{24} > \frac{9+6\alpha}{24\,(1+\alpha)} \geq \frac{6+9\alpha}{24\,(1+\alpha)}$$

which implies that the ranking of institutions by network size is 3, 1 and 2; i.e. the commercial bank enjoys the largest branch network (9/24), the savings bank originally settled in territory 1 comes in second place (it also scores 9/24 in the limit as $\alpha \to 0$) and the other savings bank appears at the bottom (there is a tie in case there are no asymmetries, i.e. $\alpha = 1$). Note that the relative network size of the commercial bank is 9/24 irrespective of the relative strenght of the two territories.

4 Mergers and Acquisitions

In the previous section we did not consider the possibility of mergers activities. We now contemplate this possibility in the post-deregulation era.[6] We consider a game with two stages. First there is a merger and acquisitions stage (M&A stage hereafter) with two possible outcomes: either a merger (involving two institutions) or no merger at all.[7] If there is no merger, then

[6] The initial conditions of the M&A game are not exogenous but are the equilibrium outcome of the deregulated game. This implies that we are assuming in the deregulated game players were not aware of the future merging possibilities.

[7] Full monopolisation is not allowed in the model.

the branch network remains unchanged and the payoff to each banking institution in a territory is simply the proportion of the total territory gains $(\delta_k \widetilde{r})$ given by the relative size of her branch network. This is so because branches have already been installed in the deregulation game. The payoffs in this case are the following

$$\begin{aligned} \bar{\pi}_1 &= \frac{9\omega}{24\omega}\delta_1\widetilde{r} + \frac{6\alpha\omega}{24\alpha\omega}\delta_2\widetilde{r} = (12+8\alpha)\,\widehat{w} \\ \bar{\pi}_2 &= \frac{6\omega}{24\omega}\delta_1\widetilde{r} + \frac{9\alpha\omega}{24\alpha\omega}\delta_2\widetilde{r} = (8+12\alpha)\,\widehat{w} \\ \bar{\pi}_3 &= \frac{9\omega}{24\omega}\delta_1\widetilde{r} + \frac{9\alpha\omega}{24\alpha\omega}\delta_2\widetilde{r} = (12+12\alpha)\,\widehat{w} \end{aligned} \tag{1}$$

where

$$\widehat{w} = \frac{\delta\widetilde{r}}{32}$$

If a merger comes out from the M&A stage, then there is a competition stage in which the (two) banking institutions decide on their number of branches in the two territories. We next solve for the subgame perfect equilibrium of the M&A game, starting out from the competition stage.

4.1 The competition stage

The merged institution decides on how many branches to operate in each territory: she can decide either the openning new branches (incurring in a cost Φ per new branch) or the closure of exiting ones (getting back Φ per branch closed) in each territory. Then, the competitor reacts and decides on her optimal number of branches. Next proposition characterizes the equilibrium outcome of the competition stage.

Proposition 3 *The merged institution and the competitor share each territory on an equal basis, reproducing the duopoly conditions of the regulated period.*

In equilibrium, the merged and the non-merged institutions share the two territories and reproduce the duopoly conditions. Two remarks here:

Remark 1 *The assumption of sequential movements is not crucial in this game. Simultaneous competition in the competition stage would result in the same equilibrium outcome.*

Remark 2 *The initial duopoly conditions arise as the equilibrium outcome of this stage because of the assumption of full cost recovery upon closing a branch. Partial recovery would not lead the merging institutions to the closure of all branches opened throughout the Deregulation Game.*[8]

Once the equilibrium outcome in the competition stage has been obtained, we step back and analyse the M&A stage.

4.2 The M&A stage

Mergers and acquisitions are decided at this stage. To this end, we need a merger protocol, i.e. a description of the rules governing who, when, to whom and at what price a merger is proposed. A merger protocol together with the equilibrium outcome of the competition stage defines a game and actually, a variety of games, i.e. protocols, can be found in the literature.

Before presenting our protocol, let us examine bank profits for every post-merger market structure. The analysis will reveal that the protocol choice is not that important in determining the outcome. Table 3 displays banks' profits for every post-merger market structure. Given that the outside option -no merge- gives participants their reservation values $\bar{\pi}$ given by (1), we find it convenient to also include in the table the surplus to each bank, defined as profits net of the reservation values.[9] For each $\{i, j\} \in \{1, 2, 3\}$ and $k \in \{1, 2, 3\} \setminus \{i, j\}$, we denote by $s_{i,j}$ the surplus of the merger $i + j$ and by s_k the competitor surplus when the merger $i + j$ occurs.

Market Structure	Profit		Surplus	
	Merged	Competitor	Merged	Competitor
1+2, 3	$23(1{+}\alpha)\,\widehat{w}$	$17(1{+}\alpha)\,\widehat{w}$	$3(1+\alpha)\,\widehat{w}$	$5(1+\alpha)\,\widehat{w}$
1+3, 2	$(26{+}23\alpha)\,\widehat{w}$	$(14{+}17\alpha)\,\widehat{w}$	$(2{+}3\alpha)\,\widehat{w}$	$(6{+}5\alpha)\,\widehat{w}$
2+3, 1	$(23{+}26\alpha)\,\widehat{w}$	$(17{+}14\alpha)\,\widehat{w}$	$(3{+}2\alpha)\,\widehat{w}$	$(5{+}6\alpha)\,\widehat{w}$

Table 3. Profit and Surplus for every post-merger market structure.

[8] Even with full recovery, it is not always the case that the non-merging institution finds optimal to close branches. See the Appendix for the analysis of a case in which the reaction to a merge between the commercial and one savings bank leads the other savings bank to open branches.

[9] Individual rationality precludes any player from getting less than her reservation value; hence the discussion needs to focus on surplus rather than profits.

Note that every merger generates a positive surplus both for the merging institutions and the non-merged bank. Note also that the competitor's surplus is larger than the merging's one. These two observations reveal the public good nature of the merging activities: banking institutions prefer a merger to take place and they prefer the most their two competitors merging each other.

Hence, the next step is the analysis of the target relationship among the banking institutions. We need a definition.

Definition 2 *The weakest competitor of an institution is her lowest branch network competitor.*[10]

Lemma 2 *The target of each institution is her weakest competitor.*

We already have all the ingredients for the merger game. Given that in our game there is only one merger at most, the dynamic aspects of the merger game are not so important; hence, we propose a simple simultaneous merger proposal protocol which tries to describe which mergers attempts will be under negotiation. In the case that more than one merger attempt is pursued, we assume that only one among those attempted will be successful.

We consider a three-player simultaneous game in which each institution has two pure strategies: "propose" (P) and "no propose" (NP). Once an acquirer is selected (randomly chosen among all proposers), the acquiree is the proposer's target and the acquirer gets all the surplus generated by the merger. The payoff matrix of this game is the following, where 1 is the row player, 2 is the column player and 3 is the matrix player.[11]

[10] Thus, given Table 2, the weakest competitor of 1 and 3 is 2 and the weakest competitor of 2 is 1.

[11] This payoff matrix can alternatively be considered the reduced form of a more general game in which each player i has three strategies: "No propose", "Propose a merger with j " and "propose a merger with k", with $j, k \neq i$. In this larger game, each player has a dominated strategy; i.e. the strategy "proposing a merger with my target" dominates the strategy "proposing a merger with a different competitor".

	P			NP		
P	$\bar{\pi}_1 + \frac{s_{1,2}+s_1}{3}$	$\bar{\pi}_2 + \frac{s_{1,2}}{3}$	$\bar{\pi}_3 + \frac{2s_3+s_{2,3}}{3}$	$\bar{\pi}_1 + \frac{s_{1,2}+s_1}{2}$	$\bar{\pi}_2$	$\bar{\pi}_3 + \frac{s_3+s_{2,3}}{2}$
NP	$\bar{\pi}_1 + \frac{s_1}{2}$	$\bar{\pi}_2 + \frac{s_{1,2}}{2}$	$\bar{\pi}_3 + \frac{s_3+s_{2,3}}{2}$	$\bar{\pi}_1 + s_1$	$\bar{\pi}_2$	$\bar{\pi}_3 + s_{2,3}$

P

	P			NP		
P	$\bar{\pi}_1 + \frac{s_{1,2}}{2}$	$\bar{\pi}_2 + \frac{s_{1,2}}{2}$	$\bar{\pi}_3 + s_3$	$\bar{\pi}_1 + s_{1,2}$	$\bar{\pi}_2$	$\bar{\pi}_3 + s_3$
NP	$\bar{\pi}_1$	$\bar{\pi}_2 + s_{1,2}$	$\bar{\pi}_3 + s_3$	$\bar{\pi}_1$	$\bar{\pi}_2$	$\bar{\pi}_3$

NP

Table 4. The payoff matrix of the merger game.

The main result of this paper is the following proposition.

Proposition 4 *The unique Nash equilibrium of the merger game is (P,P,NP).*

Corollary 1 *In equilibrium the two savings banks merge.*

This outcome is very intuitive: The commercial bank is not the target of any of the two savings banks, since it is the strongest institution (the one with the largest branch network). Given that each institution prefers acquiring rather than being acquired, the savings banks propose each other. This outcome should be robust to different merger protocols (games). And in fact this is the case.

Consider for example an alternative bargaining procedure between the merging institutions. As long as the bargaining procedure gives the proposer at least half of the surplus, it is easy to show that the same outcome is obtained. Implicit in our protocol is a dictator situation -the proposer gets the full surplus- but given that the final proposer is randomly selected, each savings bank gets half of the surplus in expected terms (ex-ante). The Nash bargaining solution (Nash, 1950) for example would result in the same equilibrium outcome but it will give each savings bank half of the surplus for sure (ex-post).[12]

We can also consider non-simultaneous proposal games. Think for example of the merger protocol proposed by Qiu and Zhou (2007). Players are sequentially selected either to make a take-it-or-leave-it merger proposal or

[12] A lower bound of half of the surplus is required as otherwise there would not be incentives to propose but to be proposed (targeted).

to pass. Rejected proposals precipitate the end of the merger stage and upon passing, another player is randomly selected. The take-it-or-leave-it nature makes the proposer to get the whole surplus of the merger. Hence, each savings bank when selected as proposer will propose the other savings bank and the commercial bank, if selected, would pass. As a consequence, the two savings bank will merge.

5 Conclusions

We present a simple spatial competition model in which banks' strategic responses to geographic deregulation can be studied. We apply it to the asymmetric regulatory body which Spanish savings and commercial banks were subjected to until the end of the 80's. Until the mid seventies, both savings and commercial banks were only allowed to operate in their regional territories. In 1974 this restriction was lifted for commercial banks whereas it was not until 1988 that savings banks were also allowed to operate nationwide.

We find that once full geographic deregulation effort finishes, savings bank should optimally merge. The public good nature of the merging activities together with the incentives structure provided by the deregulation game, turning the commercial bank into the strongest competitor, and therefore being the target of no merger proposal, are the driving factors behind the equilibrium merger of the savings banks. The particular bargaining procedure among the merging institutions and the simultaneous versus sequential ordering embedded in any model are not substantial.

It seems that the economic crisis will finally force regional politicians to allow inter-regional *caja* mergers, letting the consequences of the removal of geographic barriers in the 80's to come to a fruition with a delay of thirty years.

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APPENDIX

Proof of Proposition 1. For each $i \in \{k, 3\}$

$$n_{i,k}^{*} \in \arg \max_{\{n_{i,k}\}} n_{i,k} \left(\delta_k \widetilde{r} \frac{1}{n_{i,k} + n_{-i,k}^{*}} - \Phi \right)$$

The first order condition $\frac{\partial \pi_{i,k}}{\partial n_{i,k}} = 0$ yields

$$\frac{n_{-i,k}^{*}}{\left(n_{i,k} + n_{-i,k}^{*}\right)^2} = \frac{\Phi}{\delta_k \widetilde{r}}$$

Imposing symmetry, we get

$$n_{k,k}^{*} = n_{3,k}^{*} = \frac{\delta_k}{4} \frac{\widetilde{r}}{\Phi}$$

as requested. ■

Proof of Lemma 1. Let I_k be the total number of branches opened by the two incumbents in territory k and e_k the number of branches opened by the entrant. The problem of the entrant is

$$e_k^{*} \in \arg \max_{\{e_k\}} e_k \left(\delta_k \widetilde{r} \frac{1}{e_k + I_k + N_k} - \Phi \right)$$

The first order condition yields

$$2N_k(I_k + N_k) = (e_k^{*} + I_k + N_k)^2$$

Hence

$$e_k^{*} = \sqrt{2N_k \left(I_k + N_k\right)} - (N_k + I_k)$$

It is easy to show that the optimal number of branches opened by each entrant is decreasing in I_k and that it is null when the incumbent opens N_k branches. ■

Proof of Proposition 2. Let i_k and $\hat{\imath}_k$ be the number of new branches opened by each of the two incumbents of territory k. The problem of an incumbent is

$$i_k^{*} \in \arg \max_{\{i_k\}} i_k \left(\delta_k \widetilde{r} \frac{1}{i_k + \hat{\imath}_k^{*} + N_k + e_k^{*}\left(i_k + \hat{\imath}_k^{*}\right)} - \Phi \right)$$

The first order condition yields

$$\frac{\hat{i}_k^* + N_k + e_k^*\left(i_k + \hat{i}_k^*\right) - \frac{\partial e_k^*\left(i_k + \hat{i}_k^*\right)}{\partial i_k}}{\left(i_k + \hat{i}_k^* + N_k + e_k^*\left(i_k + \hat{i}_k^*\right)\right)^2} = \frac{1}{2N_k}$$

where

$$\frac{\partial e_k^*\left(i_k + \hat{i}_k^*\right)}{\partial i_k} = \frac{e_k^*\left(i_k + \hat{i}_k^*\right) - \left(i_k + \hat{i}_k^* + N_k\right)}{2\left(i_k + \hat{i}_k^* + N_k\right)}$$

Note that this derivative is linear in $e_k^*(\cdot)$. Imposing symmetry $(i_k^* = \hat{i}_k^*)$ we get

$$i_k^* = \frac{N_k}{16}$$

■

Proof of Proposition 3. Let m_k be the number of branches that the merged institution has decided to have in territory k once the merger takes place. Let p_k the number of branches the competitor had in territory k prior to the merger and a_k the number of new branches of the competitor in territory k after the merger (note that a_k can be negative). The problem of the competitor in territory k is the following

$$a_k^* \in \arg\max_{\{a_k\}} \delta_k \widetilde{r} \frac{p_k + a_k}{p_k + a_k + m_k} - a_k \Phi$$

The first order condition yields

$$\delta_k \widetilde{r} \frac{m_k}{\left(p_k + a_k + m_k\right)^2} - \Phi = 0$$

Solving we get

$$a_k^* = \sqrt{\frac{\delta_k \widetilde{r}}{\Phi} m_k} - m_k - p_k$$

Once the optimal reaction of the competitor has been computed, we step back to find the optimal number of new branches c_k^* of the merged institution in territory k (let C_k be the number of branches the two merging institutions had in territory k prior to the merger) The problem of the merged institution is the following

$$c_k^* \in \arg\max_{\{m_k\}} \delta_k \widetilde{r} \frac{c_k + C_k}{c_k + C_k + p_k + a_k^*\left(c_k + C_k\right)} - c_k \Phi$$

The first order condition yields

$$\frac{1}{2}\frac{\Phi}{\delta_k\widetilde{r}}\frac{-2C_k - 2c_k^* + \sqrt{\frac{\delta_k\widetilde{r}}{\Phi}\left(C_k + c_k^*\right)}}{C_k + c_k^*} = 0$$

Solving we get

$$c_k^* = \frac{\delta_k}{4}\frac{\widetilde{r}}{\Phi} - C_k \tag{2}$$

Note that $c_k^* + C_k = \frac{\delta_k}{4}\frac{\widetilde{r}}{\Phi}$. As regards the optimal response by the competitor we obtain

$$a_k^*\left(\frac{\delta_k}{4}\frac{\widetilde{r}}{\Phi}\right) = \sqrt{\frac{\delta_k\widetilde{r}}{\Phi}\left(c_k^* + C_k\right)} - \left(c_k^* + C_k\right) - p_k = \frac{\delta_k}{4}\frac{\widetilde{r}}{\Phi} - p_k \tag{3a}$$

Hence $a_k^* + p_k = \frac{\delta_k}{4}\frac{\widetilde{r}}{\Phi}$ as asserted. ■

Proof of Proposition 4. It follows from the characterisation of the best response function. The best response of 2 is to choose P. The best response of 3 is to choose P if $(a_1, a_2) = (NP, NP)$ and to choose NP otherwise. The best response of 1 is to choose NP if $(a_2, a_3) = (NP, P)$ and to choose P otherwise. ■

Analysis of the market structures.

- *Merger I: The two savings banks merge.*

In this case, the commercial bank closes ω branches and gets a share of 1/2 of the territory in territory $k = 1, 2$. Hence, its payoff is

$$\pi_3 = \frac{1}{2}\delta\widetilde{r} + \frac{1}{32}\delta\widetilde{r} + \frac{1}{2}\alpha\delta\widetilde{r} + \alpha\frac{\delta\widetilde{r}}{32} = 17\left(1+\alpha\right)\widehat{\omega}$$

The merged institution closes 7ω branches and gets a share of 1/2 of the territory in territory $k = 1, 2$. Hence, its payoff is

$$\pi_{12} = \frac{1}{2}\delta\widetilde{r} + 7\frac{\delta\widetilde{r}}{32} + \frac{1}{2}\alpha\delta\widetilde{r} + 7\alpha\frac{\delta\widetilde{r}}{32} = 23\left(1+\alpha\right)\widehat{\omega}$$

We now determine the individual benefit of each savings institution (1 and 2). We have two cases:

I.a) If the proposer is player 1, then the benefits are:

$$\begin{aligned} \pi_1 &= \pi_{12} - \bar{\pi}_2 = (15 + 11\alpha)\,\widehat{\omega} \\ \pi_2 &= \bar{\pi}_2 = (8 + 12\alpha)\,\widehat{\omega} \end{aligned}$$

I.b) If the proposer is player 2, then the benefits are:

$$\begin{aligned} \pi_1 &= \bar{\pi}_1 = (12 + 8\alpha)\,\widehat{\omega} \\ \pi_2 &= \pi_{12} - \bar{\pi}_1 = (11 + 15\alpha)\,\widehat{\omega} \end{aligned}$$

- *Merger II: The commercial bank and savings bank 1 merge.*

In this case, savings bank 2 opens 2ω branches in territory 1 and gets a share of 1/2 of the territory and closes $\alpha\omega$ in territory 2 and gets a share of 1/2 of the territory. Hence, its payoff is

$$\pi_2 = \frac{1}{2}\delta\widetilde{r} - 2\frac{\delta\widetilde{r}}{32} + \frac{1}{2}\alpha\delta\widetilde{r} + \alpha\frac{\delta\widetilde{r}}{32} = (14 + 17\alpha)\,\widehat{\omega}$$

The merged institution closes 10ω branches in territory 1 and gets a share of 1/2 of the territory and closes $7\alpha\omega$ in territory 2 and gets a share of 1/2 of the territory. Hence, its payoff is

$$\pi_{13} = \frac{1}{2}\delta\widetilde{r} + \frac{10}{32}\delta\widetilde{r} + \frac{1}{2}\alpha\delta\widetilde{r} + 7\alpha\frac{\delta\widetilde{r}}{32} = (26 + 23\alpha)\,\widehat{\omega}$$

We now determine the individual benefit of the commercial bank (player 3) and savings bank 1. We have two cases:
II.a) If the proposer is player 1, then the benefits are:

$$\begin{aligned} \pi_1 &= \pi_{13} - \bar{\pi}_3 = (14 + 11\alpha)\,\widehat{\omega} \\ \pi_3 &= \bar{\pi}_3 = 12\,(1 + \alpha)\,\widehat{\omega} \end{aligned}$$

II.b) If the proposer is player 3, then the benefits are:

$$\begin{aligned} \pi_1 &= \bar{\pi}_1 = (12 + 8\alpha)\,\widehat{\omega} \\ \pi_3 &= \pi_{13} - \bar{\pi}_1 = (14 + 15\alpha)\,\widehat{\omega} \end{aligned}$$

- *Merger III: The commercial bank and savings bank 2 merge.*

The analysis is analogous to the previous case. We get

$$\begin{aligned} \pi_1 &= (17 + 14\alpha)\,\widehat{\omega} \\ \pi_{23} &= (23 + 26\alpha)\,\widehat{\omega} \end{aligned}$$

III.a) If the proposer is player 2, then the benefits to 2 and 3 are:

$$\begin{aligned} \pi_2 &= \pi_{23} - \bar{\pi}_3 = (11 + 14\alpha)\,\widehat{\omega} \\ \pi_3 &= \bar{\pi}_3 = 12\,(1 + \alpha)\,\widehat{\omega} \end{aligned}$$

III.b) If the proposer is player 3, then the benefits to 2 and 3 are:

$$\begin{aligned} \pi_2 &= \bar{\pi}_2 = (8 + 12\alpha)\,\widehat{\omega} \\ \pi_3 &= \pi_{23} - \bar{\pi}_2 = (15 + 14\alpha)\,\widehat{\omega} \end{aligned}$$