Málaga Economic Theory
Research Center
Working Papers

A natural mechanism to choose the deserving winner when the jury is made up of all contestants

Pablo Amorós

WP 2010-7
July 2010

Departamento de Teoría e Historia Económica
Facultad de Ciencias Económicas y Empresariales
Universidad de Málaga
ISSN 1989-6908

A natural mechanism to choose the deserving winner when the jury is made up of all contestants*

Pablo Amorós†

July 1, 2010

Abstract

We analyze the problem of a jury choosing the winner from a set of agents when the identity of the deserving winner is common knowledge amongst the jurors but each juror is biased in favor of one different agent. We propose a simple and natural mechanism that implements the socially optimal rule (the winner is the deserving winner) in subgame perfect equilibria.

Key Words: mechanism design; contests; subgame perfect equilibrium.
J.E.L. Classification Numbers: C72, D71, D78.

*The author is grateful to L. Corchón and W. Thomson for their valuable suggestions. Financial assistance from Ministerio de Educación y Ciencia under project ECO2008-03674/ECON is gratefully acknowledged.

†Universidad de Málaga, Campus El Ejido, E-29013, Málaga, Spain; tel. +34 95 213 1245, fax: +34 95 213 1299; e-mail: pag@uma.es.

1 Introduction

In this paper we analyze the problem of a jury choosing one winner from a set of agents when the identity of the "deserving winner" is common knowledge amongst the jurors but each juror is biased towards one of the agents. More specifically, we study the case in which each juror can be identified with one different agent to whom he wants to favor. The typical example of this situation is that where the jury is made up of all agents, so that each juror wants to win the competition, but he is impartial with respect to the rest.

The socially optimal rule is that the deserving winner win. In a more general model, Amorós (2010) identifies restrictions on jurors partiality such that the socially optimal rule is implementable for different equilibrium concepts.[1] The necessary and sufficient condition for subgame perfect Nash implementation is that, for each pair of agents, the planner knows the identity of a juror who is impartial with respect to them.[2] This condition is trivially satisfied in the model of the present paper, and therefore we can affirm that the socially optimal rule is implementable in subgame perfect equilibria.[3]

To prove his result, Amorós (2010) proposes a mechanism à la Maskin (1999) that does the job. This type of mechanisms have received much criticism for being unnatural (see Jackson, 1992). Nevertheless, as argued by Serrano (2004), the main purpose of these mechanisms is the characterization of what can be implemented. They are necessarily quite abstract, as they are able to handle a large number of situations. More realistic mechanisms can be constructed when one deals with a specific application. This is precisely what we do in the present paper. In our setting, for each pair of agents, the planner not only knows the identity of a juror who is impartial with respect to them, but he has more information about their preferences. This information allows us to propose a "natural" extensive form mechanism that implements the socially optimal rule in subgame perfect equilibria. In

[1] Amorós (2010) analyzes a model in which a jury must choose $w \geq 1$ winners from the set of agents (in the present paper $w = 1$ and the jury is made up of all agents). Amorós et al. (2002) and Amorós (2009) also consider the problem of eliciting the "truth" from a group of partial jurors. In the model analyzed in these works, however, alternatives are rankings of all agents instead of sets of winners.

[2] Roughly speaking, a juror is impartial with respect to a group of agents if, when comparing any two sets of winners which only differ in agents in that group, the juror prefers the set including the deserving winners.

[3] In our model, each agent is a juror who is impartial with respect to all agents but himself.

this mechanism agents take turns announcing the winner. The announcement of the first agent is implemented only if he does not propose himself as the winner. Otherwise, the turn passes to the next agent and the process is repeated. If the turn comes to the last agent, his announcement is implemented even if he announces that he is the winner. This mechanism is such that truth-telling is a subgame perfect equilibrium, and any subgame perfect equilibrium results in the deserving winner.

The rest of the paper is organized as follows: Section 2 provides the model, Section 3 introduces the mechanism and proves that it implements the socially optimal rule in subgame prefect equilibria, and Section 4 provides the conclusions.

2 The model

Let $N = \{1, 2, 3, 4, ...\}$ be a set of $n \geq 4$ agents in a competition. The agents must choose one winner from among them. All agents know who the best agent is. We call this agent the **deserving winner**, $w_d \in N$. The socially optimal outcome is that the deserving winner win. Agents, however, are biased: Each agent always wants to win the competition, whoever the deserving winner is. At the same time, each agent is impartial with respect to the rest of agents in the sense that, if he does not win, he wants the deserving winner to win. Let us formalize this idea. Let $\Re$ be the class of preference relations defined over N. Each agent $i \in N$ has a **preference function** $R_i : N \longrightarrow \Re$ which associates with each deserving winner, $w_d \in N$, a preference relation $R_i(w_d) \in \Re$. Let $P_i(w_d)$ denote the strict part of $R_i(w_d)$.

Definition 1 *For each agent $i \in N$, the* ***preference function*** *$R_i : N \longrightarrow \Re$ is* ***admissible*** *if:*

(1) for each $w_d \in N$ and each $j \in N$ such that $j \neq i$, $i\ P_i(w_d)\ j$, and

(2) for each $w_d \in N$ and each $j \in N$ such that $j \neq w_d$ and $j \neq i$, $w_d\ P_i(w_d)\ j$.

Let $\mathcal{R}_i$ denote the class of all preference functions that are admissible for agent i. A **state of the world** is a pair $(R, w_d) \in \times_{i \in N} \mathcal{R}_i \times N$. Let $S \equiv \times_{i \in N} \mathcal{R}_i \times N$ be the set of admissible states of the world. The **socially optimal choice rule** is the function $\varphi : S \rightarrow N$ such that, for each $(R, w_d) \in S$, $\varphi(R, w_d) = w_d$ (*i.e.*, for each admissible state, φ selects the deserving winner).

Implementation of φ can be defined in the usual manner. A normal form mechanism is a pair (M, g), where $M = \times_{i \in N} M_i$, M_i is a message space for agent i, $g : M \to N$ is an outcome function, and agents send messages simultaneously. An **extensive form mechanism** is a dynamic mechanism in which agents make choices sequentially. An extensive form mechanism **implements φ in subgame perfect equilibria** if for each state $(R, w_d) \in S$, the only subgame perfect equilibrium outcome is w_d; *i.e.*, for each $(R, w_d) \in S$, (i) there exists a subgame perfect equilibrium of the mechanism that results in w_d, and (ii) there does not exist any subgame perfect equilibrium that results in some agent different from w_d.[4]

3 The mechanism

In this section, we propose a "natural" extensive form mechanism that implements φ in subgame perfect equilibria (Mechanism 1). It is not easy to define what a "natural" mechanism means. In our framework, however, the following can be seen as reasonable properties to be satisfied by a "natural" mechanism: (1) each agent only has to announce a winner, and (2) truth-telling is an equilibrium.[5] Mechanism 1 fulfills these properties. In this mechanism, agents take turns announcing the winner, starting with agent 1. If agent 1 announces a winner different from himself, his announcement is implemented and the mechanism stops. Otherwise agent 2's turn comes and the process is repeated. If the mechanism arrives to agent n, his announcement is implemented (even if he announces that he is the winner) and the mechanism stops.

Mechanism 1:

Stage 1: Juror 1 announces $m_1 \in N$. There are two possibilities:

1.1 If $m_1 \neq 1$, then m_1 is chosen as winner. STOP.

1.2 If $m_1 = 1$, then go to Stage 2.

Stage 2: Juror 2 announces $m_2 \in N$. There are two possibilities:

2.1 If $m_2 \neq 2$, then m_2 is chosen as winner. STOP.

[4] For each extensive form mechanism and each state of the world, a subgame perfect equilibrium induces a Nash equilibrium in every subgame (see Moore and Repullo, 1988; Abreu and Sen, 1990).

[5] Although we do not have a general proof of it, we have the conjecture that any mechanism satisfying these properties fails to implement φ in Nash equilibria.

2.2 If $m_2 = 2$, then go to Stage 3.

$\vdots$

Stage $n-1$: Juror $n-1$ announces $m_{n-1} \in N$. There are two possibilities:

(n-1).1 If $m_{n-1} \neq n-1$, then m_{n-1} is chosen as winner. STOP.

(n-1).2 If $m_{n-1} = n-1$, then go to Stage n.

Stage n: Juror n announces who the winner is, $m_n \in N$. Then m_n is chosen as winner. STOP.

Proposition 1 *Mechanism 1 implements φ in subgame perfect equilibria.*

Proof. Note that, for each agent $i \in N$ and each state $(R, w_d) \in S$, i and w_d are the most and second most preferred alternatives for i, respectively. Let $(R, w_d) \in S$.

Claim 1. At Stage n, agent n announces $m_n = n$, no matter who deserves to win.

The proof is trivial since n is the most preferred alternative for agent n.

Claim 2. At Stage $n-1$, if $w_d \neq n-1$, agent $n-1$ announces $m_{n-1} = w_d$.

Note that there is nothing that agent $n-1$ can do to be the winner. Therefore, his best option is to announce $m_{n-1} = w_d$ so that his second best alternative is selected (if $w_d = n$, agent $n-1$ would be indifferent between announcing $m_{n-1} = n$ or $\hat{m}_{n-1} = n-1$; in the latter case agent n would announce $m_n = n$ at Stage n and the deserving winner would be finally selected anyway).

Claim 3. At Stage $n-1$, if $w_d = n-1$, agent $n-1$ announces $m_{n-1} = i^*$ for some $i^* \in N \backslash \{n-1\}$.

If $w_d = n-1$, the second most preferred alternative for agent $n-1$ could be any i^* with $i^* \in N \backslash \{n-1\}$. Obviously in this case, agent $n-1$ will announce $m_{n-1} = i^*$ at Stage $n-1$ (as in the previous case, if $i^* = n$, agent $n-1$ would be indifferent between announcing $m_{n-1} = n$ or $\hat{m}_{n-1} = n-1$).

Claim 4. At Stage $n-2$, if $w_d \neq n-1$, agent $n-2$ announces $m_{n-2} = w_d$.

If $w_d = n-2$ and agent $n-2$ announces $m_{n-2} = n-2$, the mechanism goes to Stage $n-1$, agent $n-1$ announces $m_{n-1} = n-2$ and $n-2$ is chosen as winner (which is the best alternative for agent $n-2$). If $n-1 \neq w_d \neq n-2$, there is nothing that agent $n-2$ can do to be chosen as winner. Since w_d is his second most preferred alternative, agent $n-2$ announces $m_{n-2} = w_d$ and the deserving winner is chosen.

Claim 5. At Stage $n-2$, if $w_d = n-1$ and $i^* = n-2$, agent $n-2$ announces $m_{n-2} = n-2$.

If $w_d = n-1$ and agent $n-2$ announces $m_{n-2} = n-2$, the mechanism goes to Stage $n-1$, agent $n-1$ announces $m_{n-1} = i^* = n-2$ and $n-2$ is chosen as winner (which is the best alternative for agent $n-2$).

Claim 6. At Stage $n-2$, if $w_d = n-1$ and $i^* \neq n-2$, agent $n-2$ announces $m_{n-2} = w_d$.

In this case there is nothing that agent $n-2$ can do to be the winner (if he announces $m_{n-2} = n-2$, the mechanism goes to Stage $n-1$, agent $n-1$ announces $m_{n-1} = i^* \neq n-2$ and i^* is chosen as winner). Since w_d is his second best alternative, agent $n-2$ announces $m_{n-2} = w_d$ and the deserving winner is chosen.

Claim 7. At Stage $n-3$, agent $n-3$ announces $m_{n-3} = w_d$.

Suppose first that $w_d \neq n-1$. By Claim 4, if the mechanism goes to Stage $n-2$, agent $n-2$ will announce $m_{n-2} = w_d$ and the deserving winner will be chosen (not necessarily at Stage $n-2$ in case that $w_d = n-2$). Since there is nothing that agent $n-3$ can do in order to be chosen as winner at Stage $n-3$, his best option is to announce $m_{n-3} = w_d$ (if $w_d = n-3$, the mechanism will go to Stage $n-2$ and $n-3$ will be selected as winner; if $w_d \neq n-3$, the deserving winner will be chosen at Stage $n-3$).

Suppose now that $w_d = n-1$ and $i^* = n-2$. By Claim 5, if the mechanism goes to Stage $n-2$, agent $n-2$ will announce $m_{n-2} = n-2$ and the mechanism will go to Stage $n-1$. Then, by Claim 3, agent $n-1$ will announce $m_{n-1} = n-2$ and $n-2$ will be chosen as final winner. Since agent $n-3$ prefers $w_d = n-1$ rather than $n-2$, he will announce $m_{n-3} = n-1$.

Finally, suppose that $w_d = n-1$ and $i^* \neq n-2$. By Claim 6, if the mechanism goes to Stage $n-2$, agent $n-2$ will announce $m_{n-2} = w_d$. Then, by the same argument as in the case where $w_d \neq n-1$, the best option for agent $n-3$ is to announce $m_{n-3} = w_d$ (agent $n-3$ would be indifferent between announcing $m_{n-3} = w_d$ or $\hat{m}_{n-3} = n-3$; in the latter case the mechanism would go to the next stage and the deserving winner will be finally selected anyway).

Claim 8. If there are more than four agents, at Stage $n-k$, with $k \geq 4$, agent $n-k$ announces $m_{n-k} = w_d$.

To see this, note that agent $n-k$ cannot be selected as winner at Stage $n-k$. By the previous claims, if the mechanism goes to Stage $n-k-1$, then the deserving winner will be finally selected. Since $n-k$ and w_d are the best and second best alternatives for agent $n-k$, respectively, his best option is to announce $m_{n-k} = w_d$.

From Claims 1-8, for each $(R, w_d) \in S$, there exists a subgame perfect equilibrium of mechanism 1 such that agent 1 announces $m_1 = w_d$ and, in case that $w_d = 1$, agent 2 announces $m_2 = w_d$. Obviously, this equilibrium always results in w_d, whatever it is. Moreover, although in some situations there can exist more than one subgame perfect equilibrium, it is easy to see that all of them result in w_d.[6] ■

The next example makes it clear why Mechanism 1 needs at least four agents to work.

Example 1 *Suppose that* $N = \{1, 2, 3\}$*. Let* $\hat{R}_2 \in \times\mathcal{R}_2$ *be a preference function that is admissible for agent* 2 *such that* $2\ \hat{P}_2(w_d)\ 1\ \hat{P}_2(w_d)\ 3$ *when* $w_d = 2$*. It is easy to see that, at any state* $(R, w_d) \in \times_{i\in N}\mathcal{R}_i \times N$ *such that* $R_2 = \hat{R}_2$ *and* $w_d = 2$*, there is a subgame perfect equilibrium of Mechanism 1 where agent* 1 *announces* $m_1 = 1$ *and agent* 2 *announces* $m_2 = 1$*. Obviously, this equilibrium does not yield the deserving winner.*

4 Conclusion

We have analyzed the problem of choosing one winner from a set of $n \geq 4$ agents when the identity of the "deserving winner" is common knowledge amongst the jurors and the jury is made up of all agents, so that each juror wants to win the competition, but he is impartial with respect to the rest. The socially optimal rule is the rule that selects the deserving winner for each possible state. We have proposed a simple and natural mechanism that implements the socially optimal rule in subgame perfect equilibria. The mechanism is such each agent only has to announce a winner, truth-telling is a subgame perfect equilibrium, and any subgame perfect equilibrium results in the deserving winner.

[6] For example, if $N = \{1, 2, 3, 4\}$ and $w_d = 2$, there is another subgame perfect equilibrium where juror 1 announces $m_1 = 1$, juror 2 announces $m_2 = 2$ and juror 3 announces $m_3 = 2$. If we assume that the agents have intrinsic preferences for honesty in the sense that they dislike the idea of lying when it does not influence their welfare but instead goes against the intention of the planner, then these type of equilibria would not exist (see Matsushima, 2008).

References

[1] Abreu, D., and A. Sen (1990): Subgame perfect implementation: A necessary and sufficient condition, J. Econ. Theory, 50, 285-299.

[2] Amorós, P. (2010): Picking the winners, mimeo, Universidad de Málaga.

[3] Amorós, P. (2009): Eliciting socially optimal rankings from unfair jurors, J. Econ. Theory, 144, 1211-1226.

[4] Amorós, P., L. C. Corchón, and B. Moreno (2002): The scholarship assignment problem, Games Econ. Behav., 38, 1-18.

[5] Jackson, M. O. (1992): Implementation in undominated strategies: A look at bounded mechanisms, Rev. Econ. Stud., 59, 757-75.

[6] Maskin, E. (1999): Nash equilibrium and welfare optimality, Rev. Econ. Stud., 66, 23-38.

[7] Matsushima, H. (2008): Role of honesty in full implementation, J. Econ. Theory, 139, 353-359.

[8] Moore, J., and R. Repullo (1988): Subgame perfect implementation, Econometrica, 56, 1191-1220.

[9] Serrano, R. (2004): The theory of implementation of social choice rules, SIAM Review, 46, 377-414.