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The Closed Primaries versus the Top-two Primary*

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Abstract

The top-two primary is the new primary system passed in several states of the US that creates a single ballot in which the top two vote getters pass to the general election. Primary elections induce a sequential game with three stages: the candidate-entry stage, the primary election stage, and the general election. We analyze the electoral winner in equilibrium of the top-two primaries versus the traditional closed party primaries in terms of the Condorcet Consistency criterion, when voters and candidates are strategic. We show that up to four potential candidates (with no more than two democrats and no more than two republicans), the top-two system generally elects the median voter's most preferred candidate. On the contrary, with the closed party primaries, extreme candidates can be elected even when the median voter prefers the moderated counterpart. When there are more potential candidates, the closed primaries system does not show, in general, any other different deviation. The top-two system then shows every type of deviation from the Condorcet Consistency criterion: it can elect an extreme candidate when the median voter prefers the moderated counterpart, or it can elect a democratic candidate when the median voter's most preferred candidate is republican (or the other way around).

Keywords: Closed primaries; Open primaries; Top-two primary; Citizen-candidate; Strategic Voting; Sequential voting. Condorcet consistency.
JEL Classification Numbers: C72; D72.

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1 Introduction

The primary elections describe the process by which the electorate chooses its nominees for general election.[1] On the one hand, empirical evidence has shown that primaries have fostered competition, especially when there is a lack of two-party competition (Key, 1958; Grau, 1981; Jewell and Olson, 1978). On the other hand, more than a century of primaries in U.S. politics has shown some of their faults. In this line, Ansolabehere et al. (2010) and Hirano et al. (2010) highlight the decline of competition in U.S. primary elections. Among other reasons, their evidence shows that the rise in the value of incumbency has contributed to less competition.

Primary elections are of key interest and there is a growing number of political parties in Western democratic countries as well as in Latin American countries, with interests in incorporating primaries to their governing constitutions (Kenig, 2009; Carey and Polga-Hecimovich, 2006; Hazan, 1997; Wauters, 2010). Primary elections can be classified as lying somewhere on a scale from open primaries to closed primaries. In an open primary, registered voters can vote in any party's primary regardless of their party affiliation (these are also called blanket primaries). In the closed primaries, only those voters that are officially registered members of the party are eligible to vote in the primary. In a semi-closed primary, unaffiliated voters can participate as well.

Recently, several states in the U.S. have pass an alternative open primary: the *top-two primary election*. This is the primary ratified by voters in 2004 for Washington State, in 2010 for California, and in 2011 for Alaska. Depending on the state, the top-two primary applies to the State Senate, House of Representatives, State Legislature, and Governor among others. Louisiana has been using a similar system since 1975 and other states, such as Arizona, Colorado, New York, Oregon or Wisconsin, keep a lively debate on the convenience of modifying their primaries by incorporating a similar top-two system.[2]

The top-two primary election eliminates the closed party primaries from the electoral process and creates a system where all voters (partisan or not) equally participate at every stage. In the top-two primary, all the candidates, whatever their affiliation (if any), are placed on the same ballot, and only the first and second vote getters advance to the general election. Candidates have the option to add their party affiliation to their name on the ballot. Among other cases, two members of the same party can move forward to the general election.

The top-two primary system has been surrounded by strong controversy and

[1] The Progressive Movement represented by Robert La Follette, governor of Wisconsin from 1901 until 1906, established direct primary elections in which voters, instead of party officials, had the right to select their candidates. Prior to this, candidates had been selected by private caucuses and conventions rather than by a direct vote by electors (Hofstadter, 1955; Lovejoy, 1941; Merriam, 1909; Merriam and Overacker 1928; Ranney, 1975).

[2] The difference of the Louisiana primaries with respect to the top-two is that if a candidate wins a simple majority in the first round there is no second round. Other states such as Alabama, Arkansas, Georgia, Mississippi, South Carolina and Texas have closed party primaries in which a runoff between the top two is required when the candidates do not reach certain threshold (Bullock and Johnson, 1992; Engstrom and Engstrom, 2008).

it has opened a hot debate. Supporters of the top-two system argue that this will result in more moderated politicians.[3] As recently argued by Senator Charles Schumer, Democrat of New York:

"Polarization and partisanship are a plague on American politics. [...] The partisan primary system, has contributed to the election of more extreme office-holders and increased political polarization. [...] While there are no guarantees, it seems likely that a top-two primary system would encourage more participation in primaries and undo tendencies toward default extremism." New York Times, July 22, 2014.

The purpose of this paper is to provide, in a clear theoretical model, a comparison between the two different primary procedures: the closed party primaries vesus the top-two primary. We want an answer to the following puzzle: Do the closed-primaries and the top-two primaries elect different candidates? In solving this question we compare two parallel models, one in which parties select nominees according to closed party-primaries (the traditional election system) and another in which nominees are selected according to a single ballot in the top-two primary election.

We present a new stylized model in which political partisanship is divided into two groups, democrats and republicans. Four potential candidates labeled as extreme and moderate partisans, and six different types of voters labeled as strong, weak and lean partisans, participate in the electoral process to select a representative. We analyze the sequential decisions of candidates and voters by which first, the candidates strategically decide whether to run or not, second, voters cast their ballots at the primary election, and third, votes cast their ballot at the general election. Two relevant features of our analysis are the endogenous entry of candidates and the strategic voting decisions of the electorate.[4]

We solve the proposed sequential games according to the subgame perfect Nash equilibrium concept in which, at each stage of the game, players' strategies are weakly undominated given the equilibrium continuation strategies of the game (see Bag et al. 2009). We analyze the equilibrium prediction at each of the subgames. We characterize the equilibrium set of candidates running in the primaries, the nominated candidates, and the candidate winning the general election in terms of the median voter's ideology. We compare the equilibria of the two election system. Finally, we analyze two extensions of the model, one in which candidates face an entry cost and another in which there are more than four potential candidates.

With the closed party primaries, we find that an extreme candidate can win the general election even when the electorate median voter prefers the moderated candidate over the extreme one. Intuitively, if partisan voters know that both of their candidates can win the general election, they will opt for an extremist candidate when their partisan median voter is strong. We also find that with the

[3] Proponents of Proposition 14 on California's June 2010 Ballot.

[4] The endogenous entry is the key assumption in the citizen-candidate model (see Osborne and Slivinsky, 1996; Besley and Coate, 1997). In contrast to this model, we introduce an intermediate stage with the primary election (see also Cadigan and Janeba, 2002).

closed party primaries, this is rarely the case in which a republican candidate wins the general election when the electorate median voter is a democratic candidate (or the other way round).

With the top-two election system, the median voter's most preferred candidate (among the potential candidates), almost always wins the general election. Intuitively, in the top-two primary, the strategic exit of candidates can transform a four-candidate race into a three candidate race in which, by strategic voting, the most preferred candidate for the median voter is elected. However, when there are more than four potential candidates, not only the most preferred candidate for the median voter can be elected but also every other candidate. For example, in a strong democratic district, democratic partisan voters can split their vote among several democratic candidates and, eventually, a republican candidate can win the general election.

There are several contributions that analyze the benefits or costs associated to adopting primary elections. Adams and Merrill (2008) show that although primaries draw candidates away from the center, they also identify high-quality candidates. In contrast to our model, voters are assumed to vote sincerely at the primary stage. In the framework of a citizen-candidate model with primary elections and a continuous of potential candidates, Cadigan and Janeba (2002) show that the party closed primaries mitigates the pressure for convergent platforms. Some of their results are embedded in ours since we find that an extremist candidate can win the general election in a closed party primary system. The focus of these authors is different since the party interval is a key variable which generates different electoral results. Serra (2011) and Hortala-Vallve and Mueller (2012) analyze the party elites' decision concerning the convenience of holding primary elections. The former author shows that primaries increase the valence of the nominee at the expenses of an extra cost of moving policy position. The later authors highlight that primaries can act as a mechanism that prevents political parties from splitting into more homogenous groups. Hirano et al. (2010) show that the primary election systems do not appear to generate polarization of the political parties, in contrast to widespread arguments defending the opposite. Snyder and Ting (2011) show, from a combined empirical and theoretical perspective, that primaries raise the expected quality of party's candidates but, at the same time, primaries hurt the ex-ante preferred party in a competitive electorate. In a theoretical framework, Hummel (2013) shows that higher quality candidates choose more moderated policies.

Closely related to our motivation, we know of three other contributions that compare different candidate selection procedures in terms of the induced electoral outcome. Gerber and Morton (1998) show, according to evidence based on U.S. primary elections, that representatives from closed primaries take policy positions that are furthest from their district's estimated median voters, whereas semi-closed primaries select even more moderate representatives than open primaries (see also Cain and Gerber, 2002). Jackson et al. (2007) develop a two-stage model with a first nomination stage and a second general election stage and show that more open selection induces more centrist candidates (in contrast to our analysis, they do not propose a concrete primary election procedure and their equilibrium concept does not account for endogenous entry of candidates). Finally, in an statewide experiment in California, Ahler et al. (2014) compare the closed primaries with the top-two primaries. They find that voters fail to discern ideological differences between extreme and moderate candidates of the same party. As a consequence, they find that moderate candidates cannot do better in the top-two primaries (see also Snyder and Ting, 2002).

The rest of the paper is organized as follows. First, we present the model describing a common setting for the analysis of both election systems. We then analyzes the equilibria according to the traditional election system and the top-two election system. We extend our analysis to the cases in which there is a cost of running and to the case in which there are more than four ex-ante candidates. All proofs are in the Appendix.

2 The model

We consider an electoral district that has to elect a representative to serve in the legislature. There are two party labels, democrats and republicans. We analyze two different electoral systems, one characterized by the closed party primaries, and another characterized by the top-two primary.

Consider a group of ex-ante candidates $\mathcal{C} = \{D^+, D^-, R^+, R^-\}$ where the letters D and R refer to the **democratic** and **republican** candidates and the superscripts $+$ and $-$ mean **extremist** and **moderate**. For example, the candidate D^+ refers to an extremist democratic candidate and R^- does to a moderated republican candidate. The four fixed policy positions is a simplifying assumption which captures that this is hard for voters to distinguish among more than two different policy positions within democratic partisan candidates or within republican partisan candidates (see Ahler et al., 2014; Snyder and Ting, 2002).

General elements of the set of candidates $\mathcal{C}$ are denoted by x, y, etc. Each $x \in \mathcal{C}$ is identified with a fixed policy position in the interval $[0, 1]$ as in Figure 1, so that $\mathcal{C}$ is an ordered set with $D^+ < D^- < R^- < R^+$.

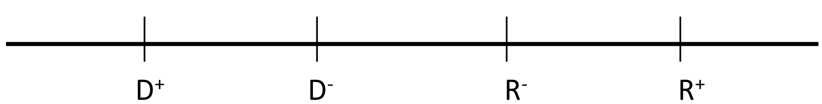

Figure 1 Position of the ex-ante candidates.

Let $\mathcal{V} = \{1, \ldots, v\}$ be the set of voters in the electoral district. General elements of $\mathcal{V}$ are denoted by i, j, etc. Each voter $i \in \mathcal{V}$ has a (strict) single-peaked preference relation over the set of candidates, $\succ_i$. There is one candidate, called peak and denoted by $p(\succ_i)$. The peak represents the most preferred candidate for the voter, and the closer a candidate is to the peak, the more preferred the candidate is for the voter. Formally, for all x, $y \in \mathcal{C}$, if $y < x <$

$p(\succ_i)$ or $p(\succ_i) < x < y$, then $x \succ_i y$. There is no measure of the distance between two adjacent candidates and therefore, if the peak of a voter is for example D^-, then either D^+ or R^- can be the second best preferred candidate for this voter.

We call **democratic partisans** to the voters whose peaks are a democratic candidate (either D^+ or D^-) and **republican partisans** to those voters whose peaks are a republican candidate (either R^+ or R^-). We suppose that democratic partisans always prefer the extreme democratic candidate over the extreme republican candidate, and republican partisans always prefer the extreme republican candidate over the extreme democratic candidate, we however admit every other single-peaked preference order.[5] Then, the admissible preferences for each voter i over the set of ex-ante candidates $\mathcal{C}$ are those represented in Table 1 (where higher candidates in the table are preferred to lower candidates).

Strong D $\succ_{D^+}$	Weak D $\succ^1_{D^-}$	Lean D $\succ^2_{D^-}$	Lean R $\succ^2_{R^-}$	Weak R $\succ^1_{R^-}$	Strong R $\succ_{R^+}$
D^+	D^-	D^-	R^-	R^-	R^+
D^-	D^+	R^-	D^-	R^+	R^-
R^-	R^-	D^+	R^+	D^-	D^-
R^+	R^+	R^+	D^+	D^+	D^+

Table 1 Admissible preferences for the voters.

Let $\mathbb{P} = \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$ be the set of admissible preference relations and let $\succ = (\succ_i)_{i\in\mathcal{V}} \in \mathbb{P}^v$ be a preference profile for voters in $\mathcal{V}$.

We define the set of democratic and republican partisan voters as $\mathcal{V}_D$ and $\mathcal{V}_R$ where

$$\begin{aligned}\mathcal{V}_D &= \{i \in \mathcal{V} : \succ_i \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}\}\} \\ \mathcal{V}_R &= \{i \in \mathcal{V} : \succ_i \in \{\succ_{R^+}, \succ^1_{R^-}, \succ^2_{R^-}\}\}.\end{aligned}$$

Within each group of partisan voters, each type of voter is labeled as **strong** (when their peak is an extremist candidate, which implies that their preferences are $\succ_{D^+}$or $\succ_{R^+}$), **weak** (when their peak is a moderated candidate and their second best preferred candidate has the same party-affiliation, which implies that their preferences are $\succ^1_{D^-}$ or $\succ^1_{R^-}$), and **lean** (when their peak is a moderated candidate and their second best preferred candidate is the moderated candidate of the other political party, this implies that preferences are $\succ^2_{D^-}$or $\succ^2_{R^-}$). For example, the preferences of a lean democratic voter are $\succ^2_{D^-}$.

Let $\succ_{D^+} < \succ^1_{D^-} < \succ^2_{D^-} < \succ^2_{R^-} < \succ^1_{R^-} < \succ_{R^+}$ be the order for the elements of $\mathbb{P}$. Given this order, and for each $\succ \in \mathbb{P}^v$, let $\succ^m$ be the median of the elements

[5] Thus, the single-peaked preference relations $\succ^*_{D^-}$ and $\succ^*_{R^-}$ such that $D^- \succ^*_{D^-} R^- \succ^*_{D^-} R^+ \succ^*_{D^-} D^+$ and $R^- \succ^*_{R^-} D^- \succ^*_{R^-} D^+ \succ^*_{R^-} R^+$ are not admissible. This is a simplifying assumption that can be interpreted as a consistency requirement over the preferences. Bouton (2013) analyzes the runoff system when there are three candidates and were there is no natural order of the candidates that guarantees a single-peaked domain of preferences.

of $\mathbb{P}$ at $\succ$; *i.e.*, $\succ^m \in \mathbb{P}$ is such that $\#\{i \in \mathcal{V} : \succ_i \leq \succ^m\} \geq \frac{v}{2}$ and $\#\{i \in \mathcal{V} : \succ_i \geq \succ^m\} \geq \frac{v}{2}$. Suppose, for the sake of simplicity, that $\succ^m$ is unique. We call $\succ^m$ the **median voter's preferences**. Notice that, for all $x, y \in \mathcal{C}$ such that $x \succ^m y$, either (1) $x \succ_i y$ for all $i \in \mathcal{V}$ such that $\succ_i \leq \succ^m$, or (2) $x \succ_i y$ for all $i \in \mathcal{V}$ such that $\succ_i \geq \succ^m$. Hence, when comparing any two candidates x and y, if the median voter prefers x to y, then a majority of voters also prefer x to y.[6]

We also refer to the median voter within each political party, democrat and republican. We call $\succ_D^m$ to the **median democratic partisan's preferences**, which for each $\succ \in \mathbb{P}^v$, $\succ_D^m$ is the median of the elements of the set $\{\succ_{D^+}, \succ_{D^-}^1, \succ_{D^-}^2\}$ and, as a consequence, $\succ_D^m \in \{\succ_{D^+}, \succ_{D^-}^1, \succ_{D^-}^2\}$. More precisely, the median democratic partisans's preferences $\succ_D^m$ satisfy that $\#\{i \in \mathcal{V}_D : \succ_i \leq \succ_D^m\} \geq \frac{v_D}{2}$ and $\#\{i \in \mathcal{V}_D : \succ_i \geq \succ_D^m\} \geq \frac{v_D}{2}$. The **median republican partisan's preferences**, $\succ_R^m \in \{\succ_{R^-}^2, \succ_{R^-}^1, \succ_{R^+}\}$, are defined in a similar way. Suppose, for simplicity, that $\succ_D^m$ and $\succ_R^m$ are unique. Abusing notation, we write $\succ_D^m = \succ_{D^-}$ and $\succ_R^m = \succ_{R^-}$ to denote $\succ_D^m \in \{\succ_{D^-}^1, \succ_{D^-}^2\}$ and $\succ_R^m \in \{\succ_{R^-}^1, \succ_{R^-}^2\}$, respectively.

Note that there exists a relationship between the median voter and the median partisans. For example, if the median voter is a weak democratic partisan $\succ_{D^-}^1$, then the median democratic partisan can only be a strong or weak voter $\succ_{D^+}$, $\succ_D^1$ (and therefore, the possibility of lean partisan $\succ_D^2$ is excluded).

Each ex-ante candidate $x \in \mathcal{C}$ also has a (strict) single-peaked preference relation over $\mathcal{C}$, $\succ_x \in \mathbb{P}$, such that $p(\succ_x) = x$ (i.e., the peak of each candidate is his/her self). Thus, the preference relations of candidates D^+ and R^+ are the preferences $\succ_{D^+}$ and $\succ_{R^+}$ defined in Table 1, respectively. Similarly, $\succ_{D^-}^1$ and $\succ_{D^-}^2$ are admissible preference relations for candidate D^-, while $\succ_R^1$ and $\succ_R^2$ are admissible preference relations for candidate R^-.

In the election systems described below, there is a primary procedure and candidates decide whether to run or not. We denote by $\emptyset$ the situation where no candidate is running and assume that in terms of the preferences of voters, the peak of the voter is always strictly preferred to a situation with no candidate (i.e., for each $i \in \mathcal{V}$, $p(\succ_i) \succ_i \emptyset$) and for each candidate, his/her self is preferred to a situation with no candidate (i.e., for each $x \in \mathcal{C}$, $x \succ_x \emptyset$).

Description of the traditional election system

In this section, we analyze the election procedure when each political party, democrat and republican, run a closed primary procedure to pick their nominee for general election. According to this system, there are two separated primaries, the democratic primary and the republican primary. In each of these primaries, there are at most two candidates over which, the corresponding partisans vote. We consider a preliminary stage in which ex-ante candidates decide whether or

[6]For this result to be true, it is crucial that the median of the elements of $\mathbb{P}$ is defined with respect to the order $\succ_{D^+} < \succ_{D^-}^1 < \succ_{D^-}^2 < \succ_{R^-}^2 < \succ_{R^-}^1 < \succ_{R^+}$. Note also that, if the preference relations $\succ_{D^-}^*$ and $\succ_{R^-}^*$ defined in Footnote 7 were admissible, there would not be any order for the elements of $\mathbb{P}$ for which the median voter predicts the winner of a majoritarian election.

not to run for the primary election. The traditional election system induces a sequential game form with three stages.

Stage 1: In this stage, the four candidates simultaneously decide whether to run or not for their party primary. Each candidate $x \in \mathcal{C}$ chooses between running (Y) or not (N). Let $S_x^1 = \{Y, N\}$ denote the strategy space of candidate x. We call $s_x^1 \in S_x^1$ a strategy of candidate x and $s^1 \in S^1 = \times_{x \in \mathcal{C}} S_x^1$ a strategy profile played by the four candidates.[7]

Let $2^{\mathcal{C}}$ be the set of all subsets of $\mathcal{C}$. Let $\mathcal{C}^r \in 2^{\mathcal{C}}$ be the set of candidates who are running and let $\mathcal{C}_D^r$ and $\mathcal{C}_R^r$ be the set of candidates that are running in the democratic and republican primaries respectively. Formally, $\mathcal{C}_D^r = \{D^+, D^-\} \cap \mathcal{C}^r$ and $\mathcal{C}_R^r = \{R^+, R^-\} \cap \mathcal{C}^r$.

Stage 2: In the second stage, the republican and the democratic parties hold their conventions. In a republican (democratic) party convention, only republican (democratic) partisans vote over the candidates that presented their candidacy.

Each voter i knows the set of candidates who are running in the primaries $\mathcal{C}^r$, the type of the median voter within each party, and the type of the median voter of the overall population. Depending on the set of candidates, the voters have the option of voting for one of the candidates. Besides, if there is no candidate running in the primary, the strategy of the voter is the empty set $\emptyset$.

In the democratic convention, the strategy of each partisan voter $i \in \mathcal{V}_D$, is denoted by s_i^2 and it indicates, for each possible set of candidates $\mathcal{C}^r$, the voting decision of agent i. Thus, $s_i^2 : 2^{\mathcal{C}} \longrightarrow \{D^+, D^-, \emptyset\}$ is a mapping such that, for each $\mathcal{C}^r \in 2^{\mathcal{C}}$, $s_i^2(\mathcal{C}^r) \in \mathcal{C}_D^r$ is the candidate for whom i will vote in the primary of the democratic party when $\mathcal{C}^r$ is the set of candidates. Let S_D^2 denote the set of all these mappings for the democratic partisan voters. For each republican partisan voter $i \in \mathcal{V}_R$, we define in a similar way the mapping $s_i^2 : 2^{\mathcal{C}} \longrightarrow \{R^+, R^-, \emptyset\}$ and the set S_R^2. Let $S^2 = \times_{i \in \mathcal{V}} S_i^2$ (where $S_i^2 = S_D^2$ if $i \in \mathcal{V}_D$ and $S_i^2 = S_R^2$ if $i \in \mathcal{V}_R$), and let $s^2 = (s_i^2)_{i \in \mathcal{V}} \in S^2$.

Once each party has celebrated its convention, the candidates who get more votes in the democratic and republican primaries become nominees. In each of the closed party primary, if there is a tie, any of the two candidates is equally likely to be the nominee. We denote the democratic and the republican nominees by $x_D^n \in \mathcal{C}_R^r$ and $x_R^n \in \mathcal{C}_R^r$ respectively.

Stage 3: In the third stage, all the voters cast their ballot at the general election for one of the nominees. Each voter knows the democratic and the republican nominees (x_D^n and x_R^n) if any. For each voter $i \in \mathcal{V}$, the strategy at Stage 3 is denoted by s_i^3 and it indicates for each possible pair of nominees, the voting decision of voter i. Thus, $s_i^3 : \{D^+, D^-, \emptyset\} \times \{R^+, R^-, \emptyset\} \longrightarrow \{D^+, D^-, R^+, R^-, \emptyset\}$ is a mapping such that, for each pair of nominees x_D^n and x_R^n, $s_i^3(x_D^n, x_R^n) \in \{x_D^n, x_R^n\}$ is the candidate for whom i will vote in the general election.

The candidate who collects the most votes at the third stage is the winner of the general election. Let S_i^3 denote the set of all these mappings, $S^3 =$

[7] Throughout the paper, only pure strategies are considered.

$\times_{i\in\mathcal{V}} S_i^3$, and $s^3 = (s_i^3)_{i\in\mathcal{V}} \in S^3$. Then, for each $s^1 \in S^1$, $s^2 \in S^2$ and $s^3 \in S^3$, the candidate who wins the general election is denoted by $x(s^1, s^2, s^3) \in \{D^+, D^-, R^+, R^-, \emptyset\}$. If there is a tie in the general election, the two candidates are equally likely to win.

Description of the top-two election system

In this system, there is no party primary and instead, all the candidates run in a single non-partisan primary. Voters know the candidates' ideological label (democrat versus republican, strong versus moderate). In the top-two primary, the two candidates that get the most votes pass to the general election.

Analogously to the traditional election system, the top-two election system induces a sequential game with three stages.

Stage 1: The four candidates simultaneously decide whether to run or not in the unique primary. For each $x \in \mathcal{C}$, $T_x^1 = \{Y, N\}$ denotes the strategy space of candidate x, $t_x^1 \in T_x^1$ is the strategy of candidate x, and $t^1 \in T^1 = \times_{x\in\mathcal{C}} T_x^1$ is a candidates' strategy profile. Thus, at most four candidates are running in the non-partisan primary.

Stage 2: All the voters, whatever their affiliation, vote for one of the self-declared candidates.

Each voter knows the set of candidates that are running in the top-two primary $\mathcal{C}^r$, the type of the median voter within each party, and the type of the median voter of the overall population. A strategy for a voter at this stage indicates, for each possible set of self-declared candidates, the candidate for whom the voter cast his/her ballot. Formally, for every $i \in \mathcal{V}$, a strategy is a mapping $t_i^2 : 2^{\mathcal{C}} \longrightarrow \{D^+, D^-, R^+, R^-, \emptyset\}$ where, for each $\mathcal{C}^r \in 2^{\mathcal{C}}$, $t_i^2(\mathcal{C}^r) \in \mathcal{C}^r$ is the candidate for whom i will vote. The strategy space for i at the second stage, T_i^2, is the set of all these mappings and $t^2 \in T^2 = \times_{i\in\mathcal{V}} T_i^2$ is a profile of strategies for the voters at this second stage.

The two candidates with more votes become nominees and they pass to the general election. The nominees are denoted by x_1^n, x_2^n where $x_1^n, x_2^n \in \{D^+, D^-, R^+, R^-, \emptyset\}$. We assume that, if there is a tie, any potential pair of candidates is equally likely to pass to the third stage.[8]

Stage 3: In the third stage, all the voters cast their ballot at the general election for one of the nominees. Each voter $i \in \mathcal{V}$ knows who the nominees are. A strategy at the third stage for i is a mapping $t_i^3 : \{D^+, D^-, R^+, R^-, \emptyset\} \times \{D^+, D^-, R^+, R^-, \emptyset\} \longrightarrow \{D^+, D^-, R^+, R^-, \emptyset\}$ such that, for each pair $x_1^n, x_2^n \in \{D^+, D^-, R^+, R^-, \emptyset\}$, $t_i^3(x_1^n, x_2^n) \in \{x_1^n, x_2^n\}$ is the candidate for whom i votes in the general election. Let T_i^3 be the set of all these mappings, $T^3 = \times_{i\in\mathcal{V}} T_i^3$, and $t^3 = (t_i^3)_{i\in\mathcal{V}} \in T^3$. For each $t^1 \in T^1$, $t^2 \in T^2$, and $t^3 \in T^3$, let $x(t^1, t^2, t^3) \in \{D^+, D^-, R^+, R^-, \emptyset\}$ be the candidate who gets the most votes at the third stage. If there is a tie, the two candidates are equally likely to win.

[8] For instance, if R^+ is the candidate who gets the most votes and D^+ and D^- are tied for second place, then the confrontations R^+ versus D^+ and R^+ versus D^- are equally likely in the third stage. Similarly, if R^+, D^+, and D^- are tied for first place then the confrontations R^+ versus D^+, R^+ versus D^-, and D^+ versus D^- are equally likely in the third stage.

Equilibrium concept

Since the proposed electoral games have a dynamic structure, we will consider the subgame perfect Nash equilibrium concept. As is common in the literature on voting, we need to eliminate choices that are weakly dominated. Otherwise, there is a large number of trivial equilibria in which each voter's choice is immaterial.

Following Bag et al. (2009), we require that, at each stage of the game, the strategies of each player are not weakly dominated given the equilibrium continuation strategies in future stages. Note that this equilibrium notion is stronger than the undominated subgame perfect equilibrium (a weakly undominated strategy may be weakly dominated if we consider that in the continuation game the players play equilibrium strategies).[9]

Consider the traditional election system. For any $s^1 \in S^1$ and $x \in \mathcal{C}$, let $s^1_{-x} \equiv (s^1_y)_{y \in \mathcal{C} \setminus \{x\}}$ be the list of strategies of the profile s^1 for all candidates except x. Denote the set of such s^1_{-x} by S^1_{-x}. Similarly, for any $s^k \in S^k$ ($k \in \{2,3\}$) and $i \in \mathcal{V}$, let s^k_{-i} be the list $(s^k_j)_{j \in \mathcal{V} \setminus \{i\}}$ and let S^k_{-i} denote the set of such s^k_{-i}. Any equilibrium profile of strategies $s^* = (s^{*1}, s^{*2}, s^{*3}) \in S^1 \times S^2 \times S^3$ should satisfy the following properties. In any subgame at the third stage, s^{*3} should be a weakly undominated Nash equilibrium in the subgame. In any subgame starting at the second stage, the voters' strategies s^{*2} should be an undominated Nash equilibrium in the subgame given that the voters play according to s^{*3} in the continuation game. At the first stage, the candidates' strategies s^{*1} should be an undominated Nash equilibrium given that the voters play according to s^{*2} and s^{*3} in the continuation game.

Definition: *A profile of strategies $s^* = (s^{*1}, s^{*2}, s^{*3}) \in S^1 \times S^2 \times S^3$ is an equilibrium of the traditional election system if:*

(a) Subgame perfection: in any subgame, s^* is a Nash equilibrium.

(b) Non weak domination in the continuation strategy in future stages:

(b.1) for each $x \in \mathcal{C}$, there is no $s^1_x \in S^1_x$ such that:

$$x((s^1_x, s^1_{-x}), s^{*2}, s^{*3}) \succeq_x x((s^{*1}_x, s^1_{-x}), s^{*2}, s^{*3}) \text{ for all } s^1_{-x} \in S^1_{-x}, \text{ and}$$
$$x((s^1_x, s^1_{-x}), s^{*2}, s^{*3}) \succ_x x((s^{*1}_x, s^1_{-x}), s^{*2}, s^{*3}) \text{ for some } s^1_{-x} \in S^1_{-x}.$$

(b.2) for each $s^1 \in S^1$ and $i \in \mathcal{V}$, there is no $s^2_i \in S^2_i$ such that:

$$x(s^1, (s^2_i, s^2_{-i}), s^{*3}) \succeq_i x(s^1, (s^{*2}_i, s^2_{-i}), s^{*3}) \text{ for all } s^2_{-i} \in S^2_{-i}, \text{ and}$$
$$x(s^1, (s^2_i, s^2_{-i}), s^{*3}) \succ_i x(s^1, (s^{*2}_i, s^2_{-i}), s^{*3}) \text{ for some } s^2_{-i} \in S^2_{-i}.$$

(b.3) for each $s^1 \in S^1$, $s^2 \in S^2$, and $i \in \mathcal{V}$, there is no $s^3_i \in S^3_i$ such that:

$$x(s^1, s^2, (s^3_i, s^3_{-i})) \succeq_i x(s^1, s^2, (s^{*3}_i, s^3_{-i})) \text{ for all } s^3_{-i} \in S^3_{-i}, \text{ and}$$
$$x(s^1, s^2, (s^3_i, s^3_{-i})) \succ_i x(s^1, s^2, (s^{*3}_i, s^3_{-i})) \text{ for some } s^3_{-i} \in S^3_{-i}.$$

The definition of an equilibrium of the top-two election system, $t^* = (t^{*1}, t^{*2}, t^{*3}) \in T^1 \times T^2 \times T^3$, is analogous and we omit it in the interest of space.

[9] If we simply impose undominated subgame perfection, any candidate might win the election in equilibrium.

3 Distortionary effects of the primary systems

There is a debate in U.S. politics by which different primary procedures are receiving different critics. In this section, we identify some of the drawbacks of the primary election procedures, namely the *extreme candidate effect* (EC, for short) and the *switching party effect* (SP, for short).

The Extreme Candidate effect occurs when the median voter's most preferred candidate is moderated, whereas the winner of the general election is the corresponding extreme candidate of the same party. More specifically,

Definition: *An electoral system generates the Extreme Candidate effect (EC) when the median voter is a weak or lean democrat (republican), but the winner in the general election is an extreme democrat D^+ (with respect to an extreme republican R^+).*

There are several reasons why we may observe the EC effect. One of the reason is that the moderated candidate does not present his/her candidacy. Another possible reason is that the moderated candidate may not get the nomination in his party primaries.

The Switching Party effect emerges when the median voter and the winner of the election belong to different parties. That is, when the median voter is democratic partisan but the winner of the general election is republican partisan, or the other way round, when the median voter is republican partisan but the winner of the general election is democratic partisan.

Definition: *An electoral system generates the Switching Party effect (SP) when the median voter and the winner of the general election have different party-affiliation.*

We do neither specify if the median voter is strong, weak or lean partisan, nor if the electoral winner is extreme or moderate. For example, the median voter can be lean partisan and the electoral winner can be moderate with different party-affiliation. There are several reasons why we can observe the SP effect. Consider the case in which the median voter is lean democrat then, he/she prefers a moderated republican to an extremist democrat. In this case, if the nominees are the moderated republican and the extremist democrat, the republican nominee will win the general election.

Interestingly, the two proposed effects are violations of the Condorcet Consistency criteria. To clarify this point, we provide the following definitions. A candidate is a *Condorcet winner* if he is preferred (by the voters) to any other candidate in pairwise comparisons. In the domain of single-peaked preferences we consider, a Condorcet winner always exists and it is the peak of the median voter. Then, we say that a voting rule satisfies *Condorcet Consistency*, when this always selects the Condorcet winner (whenever it exists).

Notice that when the EC effect occurs, the Condorcet winner is D^- (or R^-), but the winner is D^+ (with respect to R^+). In a similar way, when the SP effect occurs, the Condorcet winner is either D^- or D^+ (R^- or R^+), but the winner is R^- or R^+ (with respect to D^- or D^+).

We analyze whether the two proposed effects, the EC and the SP, occur in one or both of the proposed electoral systems. This is a key question which provides objective arguments to evaluate the two alternative primary procedures, the closed party primaries and the top-two primary.

4 The traditional election system

In this section, we describe the equilibria of the sequential game induced by the traditional system. We are particularly interested in figuring out who will win the general election in equilibrium depending on the ideology of the median voter. We consider the cases in which the median voter is weak or lean (democrat or republican).[10] We describe the electoral outcome associated to the equilibrium strategies for each of the subgames of the game.

Third stage of the traditional election system

At this stage, the democratic (x_D^n) and republican (x_R^n) nominees compete in the general election. We analyze all the possibilities, from the trivial ones in which none or just one candidate is at the general election, to all the other cases in which two candidates, each one from a different party, compete in the general election. There are up to nine different types of subgames beginning at the third stage depending on who the nominees are.

Any profile of equilibrium strategies is such that, in each of these subgames, the median voter's favorite candidate between x_D^n and x_R^n wins the election. For each voter, casting his/her ballot for his/her most preferred candidate is a weakly dominant strategy. Thus, the candidate winning the general election in the subgames beginning at the third stage are as described in Table 2.

Nominees		**Median voter**			
x_D^n	x_R^n	Weak D	Lean D	Lean R	Weak R
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
D^+	$\emptyset$	D^+	D^+	D^+	D^+
D^-	$\emptyset$	D^-	D^-	D^-	D^-
$\emptyset$	R^+	R^+	R^+	R^+	R^+
$\emptyset$	R^-	R^-	R^-	R^-	R^-
D^+	R^+	D^+ *	D^+ *	R^+ *	R^+ *
D^+	R^-	D^+ *	R^- **	R^-	R^-
D^-	R^+	D^-	D^-	D^- **	R^+ *
D^-	R^-	D^-	D^-	R^-	R^-

Table 2 Solving the third stage of the game.

[10] Empirical evidence shows that median voters are moderate: Kousser et al., 2013; Ahler et al., 2014). The case in which the median voter is a strong democrat or republican is included in the working paper version (see Amorós et al., 2013).

The first column indicates every possible pair of nominees. In the remaining columns we indicate the electoral winner for each ideology of the median voter. In those cases in which there are two nominees, the symbol ($_*$) indicates that the EC effect occurs and the symbol ($_{**}$) indicates that the SP effect occurs.

We observe that when D^+ or R^+ are the only candidates at the general election, they win. In this case, we find that the EC and the SP effects occur in a trivial way.

The are other two reasons why the EC effect occurs (symbol ($_*$) in Table 2): i) there is a confrontation between two extreme candidates, and therefore, the winner is an extreme candidate, and ii) there is a confrontation between an extreme and a moderated candidate, but the preferences of the median voter are weak which means that a majority of voters prefers an extremist candidate to the moderated candidate of the opposite party.

The other reason why the SP effect occurs (symbol ($_{**}$) in Table 2) is when the nominees are D^+ and R^- and the median voter is lean democrat (or symmetrically, when the nominees are R^+ and D^- and the median voter is lean republican). Then, a majority of voters prefers R^- over D^+ and R^- is eventually elected.

Second stage of the traditional election system

At the second stage, the parties simultaneously hold their conventions to elect their nominee for general election. In the republican (democratic) party convention, each republican (democratic) partisan votes for one of the republican (democratic) candidates if any.

Table 3 indicates, for each possible set of candidates presenting their candidacy at the primary election, the candidate that wins the general election. When necessary, we describe the median democratic partisan's preferences, and the median republican partisan's preferences. There are up to sixteen different types of subgames beginning at the second stage depending on who the running candidates are. Observe that in the last row of the table, we find two cases in which all candidates are running and where the multiplicity of equilibria can result in two different candidates winning the general election.[11]

Lemma 1: *Any profile of equilibrium strategies of the traditional election system is such that the candidates winning the general election in the subgames beginning at the second stage are as described in Table 3.*

According to Table 3, if there is at most one candidate from each party then, there is no decision to be made at the second stage and the favorite between them for the median voter wins the general election. If only two democratic (republican) candidates are running, then the favorite between them for the median democratic (republican) partisan voter is nominated and wins the general election.

[11] E.g., if all candidates are running, the median voter is $\succ^2_{D^-}$, and the median democratic partisan is $\succ_{D^+}$, there exist equilibria resulting in R^- and equilibria resulting in D^- winning the general election.

Candidates C_D^r	C_R^r	Median partisan	Median voter: Weak D	Lean D	Lean R	Weak R
$\emptyset$	$\emptyset$		$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
D^+	$\emptyset$		D^+	D^+	D^+	D^+
D^-	$\emptyset$		D^-	D^-	D^-	D^-
$\emptyset$	R^+		R^+	R^+	R^+	R^+
$\emptyset$	R^-		R^-	R^-	R^-	R^-
D^+	R^+		D^+	D^+	R^+	R^+
D^+	R^-		D^+	R^-	R^-	R^-
D^-	R^+		D^-	D^-	D^-	R^+
D^-	R^-		D^-	D^-	R^-	R^-
$\emptyset$	R^+R^-	$\succ_R^m=\succ_{R^+}$ $\succ_R^m=\succ_{R^-}$	R^+ R^-	R^+ R^-	R^+ R^-	R^+ R^-
D^+D^-	$\emptyset$	$\succ_D^m=\succ_{D^+}$ $\succ_D^m=\succ_{D^-}$	D^+ D^-	D^+ D^-	D^+ D^-	D^+ D^-
D^+	R^+R^-	$\succ_R^m=\succ_{R^+}$ $\succ_R^m=\succ_{R^-}$	$D^+{}_*$ $D^+{}_*$	$R^-{}_{**}$ $R^-{}_{**}$	$R^+{}_*$ R^-	$R^+{}_*$ R^-
D^-	R^+R^-	$\succ_R^m=\succ_{R^+}$ $\succ_R^m=\succ_{R^-}$	D^- D^-	D^- D^-	R^- R^-	$R^+{}_*$ R^-
D^+D^-	R^+	$\succ_D^m=\succ_{D^+}$ $\succ_D^m=\succ_{D^-}$	$D^+{}_*$ D^-	$D^+{}_*$ D^-	$D^-{}_{**}$ $D^-{}_{**}$	$R^+{}_*$ $R^+{}_*$
D^+D^-	R^-	$\succ_D^m=\succ_{D^+}$ $\succ_D^m=\succ_{D^-}$	$D^+{}_*$ D^-	D^- D^-	R^- R^-	R^- R^-
D^+D^-	R^+R^-	$\succ_D^m=\succ_{D^+}$ $\succ_D^m=\succ_{D^-}$ $\succ_R^m=\succ_{R^+}$ $\succ_R^m=\succ_{R^-}$	$D^+{}_*$ D^-	$R^-{}_{**}$ or D^- D^-	$D^-{}_{**}$ or R^- R^-	$R^+{}_*$ R^-

Table 3 Results of Lemma 1.

Regarding the remaining cases, the symbols ($_*$) and ($_{**}$) indicate when the EC effect or the SP effect occur respectively. We just explain those cases in which the median is democratic (the case of a republican median is analogous):

- There are two reasons why the EC effect occurs (symbol ($_*$) in Table 3): (1) There are three candidates and the extreme democratic candidate is the only candidate in the democratic primary. In this case, if the median voter is weak democrat, half of the population prefers D^+ over R^- or R^+. (2) There are three candidates, two democrats and a republican, or there are four candidates. The median voter is a weak democrat and the *median partisan democrat is a strong democrat*. In this case, candidate D^+ defeats R^- or R^+ in the general election and therefore, the median partisan democrat elects D^+.[12] Besides, if the only republican candidate in

[12] Voting D^+ is a weakly dominant strategy in the democratic primary for a majority of democrats, given the equlibrium strategies in the continuation game.

the primaries is R^+, then a democratic median voter is enough for D^+ to defeat R^+.

- The SP effect (symbol ($_{**}$) in Table 3) can only occur when the *median voter is lean democrat.* There are two reasons why a republican candidate may win the general election when the median of the population is democrat:
(1) There are three candidates and the extreme democratic candidate is the only candidate in the democratic primary. In this case, republican voters strategically elect a moderated republican nominee R^- since candidate R^- can defeat D^+ in the general election.
(2) There are four candidates and democratic voters *split their vote* at the primary election between candidates D^+ and D^-. In this case, D^+ can become the democratic nominee and is defeated by R^- in the general election. When this nominee faces R^- at the general election and the median voter is lean democrat, then a majority of voters prefers R^- over D^+.

As noted in the proof of Lemma 1, the strategies by which democratic voters split their vote between D^+ and D^- does not survive the refinement in which weakly dominated strategies are iteratively eliminated given the equilibrium strategies in the continuation game. For concreteness, voting for R^- is a weakly dominant strategy for each republican partisan at the second stage of the game and then, once eliminated this strategy for republican voters, voting for D^+ is weakly dominated, for a majority of democrats, by voting for D^-.

First stage of the traditional election system

From the previous analysis, we know who wins the general election depending on who is running. We use this information to calculate which candidates run and which of them win the general election in equilibrium. Notice that at the first stage, candidates strategically decide whether or not to present their candidacy, accounting for the equilibrium strategies in the continuation game. As we mentioned earlier, the preferences of candidates are such that their most preferred option is his/her self. Thus, the preferences of an extreme democrat can only be those of a strong democratic voter, but the preferences of a moderated democrat can be either those of a weak democrat or those of a lean democrat (a similar argument applies for republican candidates).

Lemma 2: *If the voting system is the traditional election system, then equilibrium always exists. The candidates running and the candidate winning the general election in any equilibrium are as described in Table 4.*

Table 4 shows that an extreme candidate can win the general election even when the median of the population is moderated (the EC effect). This effect occurs when there are four candidates, the preferences of the median partisan voter are strong, and the preferences of the median voter are weak (but not lean). In this case, both candidates D^+ and D^- can defeat R^- and R^+ at the general election and so, voting for D^+ in the democratic primary is a weakly dominant strategy for a majority of democratic partisan voters (who strictly prefer D^+ over D^-).

Median voter	Candidates running in equilibrium	Winner in equilibrium
Weak D	D^+, D^-, R^-, R^+	If $\succ_D^m = \succ_{D^+}$: $D^+{}_*$ If $\succ_D^m = \succ_{D^-}$: D^-
Lean D	D^+, D^-, R^-, R^+ D^-, R^-, R^+ (if $\succ_D^m = \succ_{D^+}$)	D^- D^-
Lean R	D^+, D^-, R^-, R^+ D^+, D^-, R^- (if $\succ_R^m = \succ_{R^+}$)	R^- R^-
Weak R	D^+, D^-, R^-, R^+	If $\succ_R^m = \succ_{R^-}$: R^- If $\succ_R^m = \succ_{R^+}$: $R^+{}_*$

Table 4 Equilibrium outcomes in the traditional election system

We also find that the strategic entry of candidates eliminates some of the drawbacks of the traditional primary system since in no case, a republican nominee can win the general election when the median voter is democrat (and no democrat can win when the median voter is republican). When the median voter is democrat and candidate R^- has a chance of winning the general election, candidate D^+ has incentives to withdraw from the primary contest since by doing so, candidate D^- becomes the democratic nominee and he/she defeats the republican nominee (R^- or R^+).

Interestingly, the entry stage generates, for each profile of voters' and candidates' preferences, a unique equilibrium outcome in terms of the winning candidate. A direct consequence of Lemma 2 is the following result.

Proposition 1: *The traditional election system:*
i) only generates the EC effect when the median voter is weak democrat (weak republican) and the median democratic partisan is strong democrat (with respect to strong republican),
ii) does not generate the SP effect.

By ii), the traditional election system always elects a candidate which party affiliation coincides with that of the median's voter. By i), the traditional election system can generate certain violation of the Condorcet Consistency criterion in the form of an EC effect. Consequently, in every equilibrium in which an extreme candidate is the eventual winner of the general election, a majority of voters prefers the moderated candidate to the elected extreme candidate. However, the winner is the most preferred candidate for the *strong median partisan voter.*

We finally describe the equilibrium pairs of nominees according to our results in Lemma 1 and Lemma 2.

Median voter	Nominees in equilibrium	Winner in equilibrium
Weak D	$\{D^+, R^+\}$ or $\{D^+, R^-\}$ if $\succ_D^m = \succ_{D^+}$	$D^+{}_*$
	$\{D^-, R^+\}$ or $\{D^-, R^-\}$ if $\succ_D^m = \succ_{D^-}$	D^-
Lean D	$\{D^-, R^-\}$	D^-
Lean R	$\{D^-, R^-\}$	R^-
Weak R	$\{D^+, R^-\}$ or $\{D^-, R^-\}$ if $\succ_R^m = \succ_{R^-}$	R^-
	$\{D^+, R^+\}$ or $\{D^-, R^+\}$ if $\succ_R^m = \succ_{R^+}$	$R^+{}_*$

Table 5 Equilibrium nominees in the traditional election system

Those districts with a weak median voter and an strong median partisan voter (both democrats or both republicans), can have two extreme candidates as nominees for the general election. Notice that when the median voter is weak democrat (or republican), republicans (with respect to democrats) are indifferent between nominating an extreme or a moderated candidate since none of them can defeat the democratic nominee. In all the other cases, two moderated candidates run against each other.

5 The top-two election system

In this section we follow the same steps than in the previous section to analyze the top-two election system.

Third stage of the top-two election system

The top two vote getters pass to the general election. There are up to eleven different types of subgames beginning at the third stage depending on who the nominees are. There are two additional subgames with respect to the traditional election system in which either two democratic candidates (D^+ and D^-) or two republican candidates (R^+ and R^-) face each other at the general election.

In each of the subgames, the median voter's favorite candidate between the two contenders wins the election. This third stage of the game is similar to the one of the traditional election system given that for each voter, casting his/her ballot for his/her most preferred candidate is a weakly dominant strategy. Thus, all the results of Table 2 hold and besides, there are two additional confrontations between two democratic or two republican candidates represented in the following table.

		Median voter			
Nominees		Weak D	Lean D	Lean R	Weak R
D^+	D^-	D^-	D^-	$D^-{}_{**}$	$D^-{}_{**}$
R^+	R^-	$R^-{}_{**}$	$R^-{}_{**}$	R^-	R^-

Table 6 Two additional confrontations at the third stage of the game

We find that the two additional confrontations generate new cases in which the SP effect occurs (symbol ($_{**}$) in Table 6). If the median voter is democrat (or republican) but the two candidates that pass to the general election are republican (with respect to democrats) then, the winner and the median voter have different party-affiliation. We observe that there in no new case in which the EC effect occurs.

Second stage of the top-two election system

At the second stage, the top-two primary is held. All the self-declared candidates are place on the same ballot and only the two candidates who get the most votes advance to the general election.

There are sixteen different types of subgames beginning at the second stage depending on who the running candidates are. Table 7 indicates, for each subgame and each ideology of the median voter, who wins the general election.

Lemma 3: *Any profile of equilibrium strategies of the top-two election system is such that the candidates winning the general election in the subgames beginning at the second stage are as described in Table 7.*

	Median voter			
Candidates	Weak D	Lean D	Lean R	Weak R
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
D^+	D^+	D^+	D^+	D^+
D^-	D^-	D^-	D^-	D^-
R^+	R^+	R^+	R^+	R^+
R^-	R^-	R^-	R^-	R^-
D^+R^+	D^+	D^+	R^+	R^+
D^+R^-	D^+	R^-	R^-	R^-
D^-R^+	D^-	D^-	D^-	R^+
D^-R^-	D^-	D^-	R^-	R^-
D^+D^-	D^-	D^-	D^-	D^-
R^+R^-	R^-	R^-	R^-	R^-
$D^+D^-R^-$	D^-	D^-	R^-	R^-
$D^+D^-R^+$	D^-	D^-	$D^-{}_{**}$	$R^+{}_{*}$
$D^+R^-R^+$	$D^+{}_{*}$	$R^-{}_{**}$	R^-	R^-
$D^-R^-R^+$	D^-	D^-	R^-	R^-
$D^+D^-R^-R^+$	D^- or $D^+{}_{*}$ or $R^{-(a)}{}_{**}$	D^- or $R^-{}_{**}$ or $D^{+(b)}{}_{*}$	$R^-{}_{**}$ or $D^-{}_{**}$ or $R^{+(c)}{}_{*}$	R^- or $R^+{}_{*}$ or $D^{-(d)}{}_{**}$

$^{(a)}$ Only if $\#\{i \in V :\succ_i = \succ^1_{D^-}\} < v/2$; $^{(b)}$ Only if $\#\{i \in V :\succ_i = \succ^2_{D^-}\} < v/2$
$^{(c)}$ Only if $\#\{i \in V :\succ_i = \succ^2_{R^-}\} < v/2$; $^{(d)}$ Only if $\#\{i \in V :\succ_i = \succ^1_{R^-}\} < v/2$

Table 7 Results of Lemma 3

If there are at most two candidates running, the favorite between them for the median voter wins the general election.

If there are three candidates, due to strategic voting, the most preferred candidate among them for the median voter wins the general election. Notice that in this case, only two candidates can pass to the general election. Consequently, only the two most preferred candidates (among the three) for the median voter have a chance of winning the general election. This is a weakly dominant strategy to vote in the primary for the most preferred candidate between the two that have a chance of winning the general election. Therefore, the most preferred candidate for the median voter is nominated and eventually elected. This implies that when the median's ideal candidate runs in the primary election, this candidate (which is the Condorcet winner) is elected. The EC effect and the SP effect arise in the case of three candidates when the Condorcet winner does not present his/her candidacy. In such case, the second best preferred option for the median voter wins the general election. We then find that if the preferences of the median voter are weak then, the EC effect occurs, and if the preferences of the median voter are lean, the SP effect occurs.

The last row in Table 7 describes the top-two primary when there are four candidates in the race.[13] We explain the case in which the median voter is democrat (the case in which the median voter is republican is analogous).

If the median is weak democrat, for those voters with preferences of type $\succ^1_{D^-}$, voting for R^+ and R^- in the top-two primary are two strategies that are weakly dominated by voting for D^-. No other strategy is weakly dominated for any other voter and therefore, all the candidates can become nominees, and all but R^+ can win the general election. For example, if D^+ is expected to be nominated and there is a tie for the second nomination between D^- and R^+, then voting for R^+ is an optimal strategy for strong democratic voters. Thus, one possibility is that candidates R^- and R^+ be nominated, and candidate R^- wins the general election (for this to be the case, less than half of the population can be of type $\succ^1_{D^-}$).[14] We deduce that the SP effect can occur and besides, the EC effect can occur in the same way than in the traditional primaries.

If the median is lean democrat, for those voters with preferences of type $\succ^2_{D^-}$, voting for D^+ and R^+ in the top-two primary are two strategies that are weakly dominated by voting for D^-. No other strategy is weakly dominated for any other voter and therefore, all the candidates can become nominees, and all but R^+ can win the general election. Note that for D^+ to win the general election, candidates D^+ and R^+ have to be nominated and thus, less than half of the population can be of type $\succ^2_{D^-}$. We deduce that the SP effect can occur in the same way than in the traditional primaries and besides, the EC effect can

[13] In this case, equilibrium refinements consisting of iterative elimination of weakly dominated strategies given the continuation strategies of the game do not eliminate any of the described equilibrium outcomes.

[14] The described equilibrium strategies for strong democratic voters would not satisfy the rational expectation condition (Cox 1997; Palfrey 1989) given that the described optimal strategies are based on some expected voting strategies (with which D^+ and R^+ are nominated) that differ from the equilibrium strategies of the other voters (with which R^+ and R^- are nominated). Notice that our proposed equilibrium concept is weak in this sense.

also occur even when the median is lean democrat.

For the case in which the median is democrat, we summarize the cases with three or more candidates and where either the EC effect or the SP effect occur (the case of a republican median is analogous):

- There are two reasons for the EC effect (symbol ($_*$) in Table 7):
  (1) There are three candidates and only the extreme democratic candidate presents his/her candidacy at the primary election. In such case, if the median is a weak democrat then, half of the population prefers D^+ over R^- or R^+ (similar to the traditional primaries).
  (2) There are four candidates competing in the primary. If the median is weak or lean (and, in the last case, less than half of the population is lean democrat), then D^+ can be elected. The more candidates there are in the primary election, the more tie-breaking circumstances are there by which, voting for an extreme candidate is not a weakly dominated strategy at the second stage of the game for any voter. In contrast to the case of the traditional election system, the EC effect is not related to the preferences of the median partisan voter.

- The SP effect can occur not only when the median voter is lean democrat, but also when the median is weak democrat (in contrast to the traditional primaries in which the median has to be lean democrat). There are two reasons why a republican candidate wins when the median of the population is democrat (symbol ($_{**}$) in Table 7):
  (1) There are three candidates and only the extreme democratic candidate presents his/her candidacy. Then, a lean democratic median prefers R^- over D^+ and D^+ over R^+. Thus, only R^- and D^+ have a chance of winning the general election, and given that half of the electorate prefers R^- to D^+, then R^- wins (this argument is similar, but not equal, to the one of the traditional primaries).
  (2) There are four candidates. Consequently, there are more tie-breaking circumstances and few voting strategy are weakly dominated. Voters can *split their vote* at the top-two primary among the four candidates D^+, D^-, R^- and R^+. If R^- and R^+ pass to the general election and the median is weak democrat, R^- is elected. If either R^- and D^+ or R^- and R^+ pass to the general election and the median is lean democrat, R^- is elected. This is particularly striking that two republicans can eventually run against each other in the general election when the median voter is democrat (when R^- runs against D^+, this is also a plausible case in the traditional election system).

Notice that with the top-two election system, there are additional cases in which the SP effect occurs. The fact that more candidates compete in the top-two primaries implies that there are more tie-breaking circumstances and more strategies survive the refinement of weak dominance. Consequently, voters can split their votes at the top-two primary among several candidates which can generate the SP effect.

First stage of the top-two election system

From our previous analysis we know which candidates win the general election depending on who is running. We use this information to calculate which candidates run and which of them win the general election in equilibrium.

Lemma 4: *If the voting system is the top-two election system, then equilibrium always exists. The candidates running and the candidate winning the general election in any equilibrium are as described in Tables 8a and 8b.*

Equilibrium descriptions in the Tables 8a and 8b depend on the identity of the candidate winning the general election when the four candidates are in the race (column 2).[15] Column 3 describes the candidates running in the top-two primary and column 4 indicates the winning candidate. For example, when the median voter is a weak democrat, there are seven equilibrium configurations and only in one of them, the extreme candidate is elected.

Median voter	Winner if all candidates run	Candidates running in equilibrium	Winner in equilibrium
Weak D	D^+	D^+, D^-, R^-	D^-
		D^+, D^-, R^+	D^-
		D^+, D^-	D^-
	D^-	D^+, D^-, R^-, R^+	D^-
	$R^{-(a)}$	If D^- is type $\succ^1_{D^-}$: D^-, R^-, R^+	D^-
		If D^- is type $\succ^1_{D^-}$: D^+, R^-, R^+	$D^+{}_*$
		If D^- is type $\succ^2_{D^-}$: D^-, R^-, R^+	D^-
Lean D	$D^{+(b)}$	If D^- is type $\succ^1_{D^-}$: D^+, D^-, R^-	D^-
		If D^- is type $\succ^1_{D^-}$: D^+, D^-, R^+	D^-
		If D^- is type $\succ^1_{D^-}$: D^+, D^-	D^-
		If D^- is type $\succ^2_{D^-}$: D^+, D^-, R^-	D^-
		If D^- is type $\succ^2_{D^-}$: D^+, D^-, R^+	D^-
		If D^- is type $\succ^2_{D^-}$: D^+, D^-	D^-
		If D^- is type $\succ^2_{D^-}$: D^+, R^-, R^+	$R^-{}_{**}$
	D^-	D^+, D^-, R^-, R^+	D^-
	R^-	D^-, R^-, R^+	D^-

$^{(a)}$ Only if $\#\{i \in V : \succ_i = \succ^1_{D^-}\} < v/2$ $^{(b)}$ Only if $\#\{i \in V : \succ_i = \succ^2_{D^-}\} < v/2$

Table 8a Equilibrium outcomes in the top-two election system.

[15] Which, in all but one of the equilibria, this describes the out of equilibrium path strategies of voters.

Median voter	Winner if all candidates run	Candidates running in equilibrium		Winner in equilibrium
Lean R	$R^{+(b)}$	If R^- is type $\succ^1_{R^-}$:	D^-, R^-, R^+	R^-
			D^+, R^-, R^+	R^-
			R^-, R^+	R^-
		If R^- is type $\succ^2_{R^-}$:	D^-, R^-, R^+	R^-
			D^+, R^-, R^+	R^-
			R^-, R^+	R^-
			D^+, D^-, R^+	$D^-{}_{**}$
	R^-	D^+, D^-, R^-, R^+		R^-
	D^-	D^+, D^-, R^-		R^-
Weak R	R^+	D^-, R^-, R^+		R^-
		D^+, R^-, R^+		R^-
		R^-, R^+		R^-
	R^-	D^+, D^-, R^-, R^+		R^-
	$D^{-(a)}$	If R^- is type $\succ^1_{R^-}$:	D^+, D^-, R^-	R^-
			D^+, D^-, R^+	$R^+{}_{*}$
		If R^- is type $\succ^2_{R^-}$:	D^+, D^-, R^-	R^-

$^{(a)}$ Only if $\#\{i \in V : \succ_i = \succ^1_{R^-}\} < v/2$ $^{(b)}$ Only if $\#\{i \in V : \succ_i = \succ^2_{R^-}\} < v/2$

Table 8b Equilibrium outcomes in the top-two election system.

We interpret the results in Table 8a (those of Table 8b are analogous). In all but one of the equilibria in which the median is weak democrat, the median voter's favorite candidate D^- wins the general election. In the only exception, candidate D^+ wins (and therefore the EC effect occurs). For this to be the case, voters must split their vote, out of the equilibrium path, since a moderated republican R^- must be the winner when the four candidates run in the primary election.[16] In such a case, if candidate D^- is weak democrat, he/she opts out to guarantee the victory of D^+ over R^-. Thus, the threat of the SP effect provides incentives for candidate D^- to withdraw from the contest and, as a consequence, there is an equilibrium of the entire game in which the EC effect occurs.

All the other equilibrium strategies of the subgames in the second stage of the game in which there is a EC effect, are not in the equilibrium path of the entire game given that a candidate have incentives to opt in or to opt out of the contest. The top-two election system displays multiple equilibrium predictions and one (out of six) induces the EC effect. Besides, the equilibrium showing

[16] As we have explained, voting for R^- is not weakly dominated for a majority of voters given the equilibrium continuation strategies.

the EC effect is based in one of the "unexpected" results of the top-two election system, namely that off-the-equilibrium-path the nominees are R^+ and R^- when the median voter is democrat and the four candidates are in the race.

In all but one of the equilibria in which the median is lean democrat, the median voter's favorite candidate D^- wins the general election. In the only exception, candidate R^- wins (and therefore the SP effect occurs). For this to be the case, out of the equilibrium path, a strong democrat D^+ must be the winner when the four candidates run in the primary election.[17] In such a case, if candidate D^- is lean democrat, he/she opts out to guarantee the victory of R^- over D^+. Thus, we find that the threat of the EC effect provides incentives for candidate D^- to withdraw from the contest and, as a consequence, there is an equilibrium of the entire game in which the SP effect occurs. Once again, the top-two election system generates multiple equilibria when the median is lean democrat and it is in one of them (characterized by the EC effect out of the equilibrium path and D^+ running against R^+ in the general election), that the SP effect occurs. The later equilibrium is based in one of the "unexpected" results of the top-two primary election by which two extreme candidates are nominated when the four candidates are in the unique primary and the median voter is a moderated democrat.

A direct consequence of Lemma 4 is the following result.

Proposition 2: *The top-two election system:*
i) only generates an EC effect when:
- less than half of the electorate is weak democrat (weak republican)
- the subgame in which the four candidates are running displays a SP effect in which R^- and R^+ are nominated (D^- and D^+ are nominated),
- the moderated democratic candidate D^- is weak democrat (R^- is *weak republican*).
ii) only generates a SP effect when:
- less than half of the electorate is lean democrat (lean republican)
- the subgame in which the four candidates are running displays an EC effect in which D^+ and R^+ are nominated,
- the moderated democratic candidate D^- is lean democrat (R^- is *lean republican*).

Therefore, the top-two election system can generate not only the EC effect, but also the SP effect. However, those conditions for these two effects about the strategies out of equilibrium path are certainly stringent. Besides, the top-two election system displays multiple equilibria and this is only in one of them (out of six), that the top-two election system induces the EC effect or the SP effect. Thus, roughly speaking, the candidates' entry stage provides a refinement against the multiple cases in which the EC and the SP effect occur. This is a rare case to have either the EC effect or the SP effect when following the top-two election system.

[17] Voting for D^+ is not weakly dominated in the second stage of the game for a majority of voters given the equilibrium continuation strategies.

Finally, we describe the ideology of the nominees for the general election according to our results in Lemma 3 and Lemma 4.

Median voter	Nominees in equilibrium	Winner in equilibrium
Weak D	$\{D^+, R^+\}$ or $\{D^+, R^-\}$ only if D^- is type $\succ^1_{D^-}$ and less than half of the electorate is type $\succ^1_{D^-}$.	$D^+{}_*$
	$\{D^+, D^-\}$ or $\{D^-, R^-\}$ or $\{D^-, R^+\}$.	D^-
Lean D	$\{D^+, D^-\}$ or $\{D^-, R^-\}$ or $\{D^-, R^+\}$.	D^-
	$\{D^+, R^-\}$ or $\{R^-, R^+\}$ only if D^- is type $\succ^2_{D^-}$ and less than half of the electorate is of type $\succ^2_{D^-}$.	$R^-{}_{**}$
Lean R	$\{D^+, R^-\}$ or $\{D^-, R^-\}$ or $\{R^-, R^+\}$.	R^-
	$\{D^+, D^-\}$ or $\{D^-, R^+\}$ only if R^- is type $\succ^2_{R^-}$ and less than half of the electorate is of type $\succ^2_{R^-}$.	$D^-{}_{**}$
Weak R	$\{D^+, R^-\}$ or $\{D^-, R^-\}$ or $\{R^-, R^+\}$.	R^-
	$\{D^+, R^+\}$ or $\{D^-, R^+\}$ only if R^- is type $\succ^1_{R^-}$ and less than half of the electorate is type $\succ^1_{R^-}$.	$R^+{}_*$

Table 9 Equilibrium nominees in the traditional election system

We can compare this table with the corresponding table for the traditional election system (Table 5). If we skip those equilibria in which the EC and the SP effect occurs, we find that in no other case the two most extreme candidates are simultaneously nominees for the general election. However, we find that there are many other confrontations among two democrats (when the median voter is democrat), or among a moderated democrat and an extreme republican (when the median is democrat) that can be sustained in equilibrium. Thus, only in very rare circumstances, the two nominees in the top-two primary are two extremist candidates (in contrast to the traditional election system). However, almost always we find that a moderated nominee with the same party-affiliation than the median voter runs, in the general election, against his/her extreme party counterpart or against candidates of the other party.

6 Other modeling assumptions

In this section we study how robust are the obtained results to the case in which candidates face a cost for running in the primary election, and to the case in which there are more than four potential candidates.

6.1 Candidates' entry cost

The cost of the candidates is formulated in terms of the following assumption.[18]

Assumption A. *Each candidate $x \in C$ prefers to run if by doing so he/she alters the result of the election and the winner is more preferred for him/her. If the election result is the same whether x is running or not, then x prefers not to run.*

The analysis for the third and second stage of the traditional and the top-two election systems is still valid under Assumption A. Next, we analyze the first stage of both election systems when there is a cost of running.

We denote by $s^1 = (s^1_{D^+}, s^1_{D^-}, s^1_{R^-}, s^1_{R^+}) \in S^1$ a strategy profile played by the four candidates (for example, $s^1 = (Y, N, N, Y)$ denotes the situation where D^+ and R^+ are running while D^- and R^- are not). Abusing notation, for any $x \in \mathcal{C}$ and $s^1, \hat{s}^1 \in S^1$, we write $s^1 \succ_x \hat{s}^1$ if one of the two following cases occurs: (i) x prefers any possible equilibrium result in equilibrium after candidates played s^1 in the first stage to any possible equilibrium result after they played $\hat{s}^1$, or (ii) $s^1_x = N$, $\hat{s}^1_x = Y$, and the only possible equilibrium result after candidates played s^1 in the first stage coincides with the only possible equilibrium result after they played $\hat{s}^1$.

Lemma 5 shows who runs and who wins in equilibrium in the traditional election system when there is a cost of running.

Lemma 5: *Suppose that Assumption A holds and the voting system is the traditional election system. Then, if (i) $\succ^m = \succ^2_{D^-}$ and $\succ^m_D = \succ_{D^+}$, or (ii) $\succ^m = \succ^2_{R^-}$ and $\succ^m_R = \succ_{R^+}$, there is no profile of equilibrium strategies. Otherwise, equilibrium exists. The candidates winning the general election in equilibrium are as described in Table 10 and any equilibrium is such that the winning candidate is the only one running.*

Median voter	Winner in equilibrium
Weak D	If $\succ^m_D = \succ_{D^+}$: D^+ If $\succ^m_D = \succ_{D^-}$: D^-
Lean D	If $\succ^m_D = \succ_{D^+}$: $\nexists$ Equilibrium If $\succ^m_D = \succ_{D^-}$: D^-
Lean R	If $\succ^m_R = \succ_{R^+}$: $\nexists$ Equilibrium If $\succ^m_R = \succ_{R^-}$: R^-
Weak R	If $\succ^m_R = \succ_{D^+}$: R^+ If $\succ^m_R = \succ_{R^-}$: R^-

Table 10 Results of Lemma 5

Note that these results are very similar to those obtained in the case in which there is no cost of running (Lemma 2). There are just two differences in Table

[18]This extension was proposed by James Snyder, to whom we gratefully acknowledge his interest for our results.

10 with respect to the case in which there is a cost of running. The first one is that now, all equilibria are such that only one candidate is running. The second difference is that, for some profiles of voter's preferences, there is no equilibrium. Recall that, if there is no cost of running and $\succ^m = \succ^2_{D^-}$ and $\succ^m_D = \succ_{D^+}$, there are equilibria in which D^- wins the general election. If there is a cost of running, however, a situation where only D^- is running is not an equilibrium because D^+ would prefer to run and win the general election, and a situation where only D^+ and D^- are running is not an equilibrium either, because D^- would prefer not to run, since D^+ wins anyway. Given the symmetry of our model, if $\succ^m = \succ^2_{R^-}$ and $\succ^m_R = \succ_{R^+}$, there is no equilibrium either. According to Lemma 5, we deduce the following result.

Proposition 3: *If Assumption A holds, and an equilibrium of the traditional election system exists, then the traditional election system:*
i) generates an EC effect when the median voter is weak democrat (weak republican) and the median democratic partisan is strong democrat (with respect to strong republican),
ii) does not generate a SP effect.

Thus, we find that the main result concerning the impact of the median partisan voter in the traditional election system, namely the EC effect, survives even when there is an entry cost.

The following result shows the equilibrium winners of the top-two election system when candidates face an entry cost.

Lemma 6: *Suppose that Assumption A holds and the voting system is the top-two election system. Then, equilibrium always exists. The candidates winning the general election in equilibrium are as described in Table 11 and any equilibrium is such that the winning candidate is the only one running.*

Median voter	Winner in equilibrium
Weak D	D^-
Lean D	D^-
Lean R	R^-
Weak R	R^-

Table 11 Results of Lemma 6

In this case, equilibrium always exists. Moreover, every equilibrium outcome is such that the median voters' favorite candidate is the only one running and therefore winning the general election. The two types of equilibria in which the EC effect and the SP effect occur, now disappear. Here are the main intuitions for this.

- In the equilibrium in which the EC effect occurs, candidate D^+ may wins the general election when the following four conditions hold: there is no cost of running, $\succ^m = \succ^1_{D^-}$, the preferences of candidate D^- are of type

$\succ^1_{D^-}$, and less than half of the voters are of type $\succ^1_{D^-}$. In such case, all candidates except D^- were running. Then, R^+ and R^- would prefer not to run either, since D^+ will win anyway. However, a situation where only D^+ is running is not an equilibrium because D^- would prefer to run and win the general election.

- In the equilibrium in which the SP effect occurs, candidate R^- wins the general election when the following conditions hold: there is no cost of running, $\succ^m = \succ^2_{D^-}$, the preferences of candidate D^- are of type $\succ^2_{D^-}$, and less than half of the voters are of type $\succ^2_{D^-}$. In such case, all candidates except D^- were running. Then, D^+ and R^+ would prefer not to run, since R^- would win anyway. However, a situation where only R^- is running is not an equilibrium because D^- would prefer to run and win the general election.

According to Lemma 6, we deduce the following result.

Proposition 4: *If Assumption A holds, the top-two election system always elects the most preferred candidate for the median voter and, in no case, this electoral system generates neither the EC effect nor the SP effect.*

The entry cost eliminates all the uncommon cases in which the EC and the SP effect occur when the election system follows the top-two primary. However, we have shown that the entry cost can not eliminate the EC effect when the election system follows the closed party primaries.

6.2 More than four potential candidates

Suppose that in each of the proposed policy positions D^+, D^-, R^-, R^+ there can be more than one candidate. We basically assume that voters can not distinguish among more than two different policy positions within democratic or republican partisans (Ahler et al., 2014; Snyder and Ting, 2002). For example, the set of potential candidates $\mathcal{C} = \left\{D_1^+, D_2^+, D_1^-, D_2^-, R^-, R^+\right\}$ indicates that there are two extreme democratic candidates, two moderated democratic candidates, a moderated republican and an extreme republican. We assume that every voter is indifferent between two candidates with the same ideological position.[19]

The proposed simplified assumption gives us sufficient insights to analyze the robustness of our results in Proposition 1 and Proposition 2. Our first results refers to the traditional election system.

Proposition 5: *If there are more than four potential candidates, the traditional election system is such that:*
i) the EC effect can occur when the median voter is weak democrat (weak republican), but this can not occur when the median voter is lean democrat (with

[19] When there are more potential candidates, all the analyzed equilibrium outcomes can be sustained as equilibrium outcomes of the new game. Just consider that the entry of an additional candidate may not modify the voting strategies at the primary stage (and this is still a weakly undominated strategy for every voter). Besides, if there are additional equilibrium outcomes, more than four candidates may be in the primary race.

respect to lean republican),
ii) the SP effect can occur when the median voter is lean democrat (lean republican) but this can not occur when the median voter is weak democrat (weak republican).

We show that a strong partisan median voter is not any more, a necessary condition to deduce the EC effect when there are three or more democratic or republican candidates. Consider that the median voter is weak democrat. Then, if there are several moderated democratic candidates, democratic voters (which are a majority), can split their vote among them in the primary election. Consequently, the extreme democratic candidate is nominated and wins the general election. Besides, this is easy to show that the proposed electoral outcome, can be immune to the candidates' strategic entry decision.

We also find that there is no EC effect when the median is lean democrat. This follows from the fact that, in the general election, an extreme candidate cannot defeat a moderated candidate of the opposite party. When the median voter is lean democrat, for every republican partisan, voting for an extreme republican is weakly dominated. Therefore, a moderated republican is always nominated and he/she defeats the extreme democratic candidate.

We also find that the SP effect reappears when the median is lean democrat. We deduce that those equilibrium strategies in which the SP effect occur, does not survive the iterative elimination of weakly dominated strategies given the equilibrium strategies in the continuation game. We interpret this fact as a weakness of statement ii) in Proposition 5, since it indicates that the SP effect may rarely occur.

Our second result is about the top-two election system.

Proposition 6: *If there are more than four potential candidates, the top-two election system is such that:*
i) the EC effect can occur when the median voter is weak or lean democrat (weak or lean republican),
ii) the SP effect can occur when the median voter is weak or lean democrat (weak or lean republican).

Thus, the SP and the EC effect can occur both, when the median voter is weak or when the median voter lean. We find that five candidates are enough to build an equilibrium example where the EC effect occurs, however, we need six candidates to build an equilibrium example where the SP effect occurs.

Interestingly, we find that every equilibrium outcome of the subgame starting at the second stage of the game when there are four candidates, can be sustained as an equilibrium outcome when we include additional candidates. Table 7 shows the equilibrium outcomes starting in the second stage of the game. Intuitively, when there are additional candidates in the race, the strategic exit of a candidate may have no effect since there can be an equivalent candidate located in the same ideological position that receives a transfer of votes (without modifying the elected candidate). Besides, the strategic entry of candidates may not modify the electoral outcome since it does not generate additional weakly dominated strategies.

Notice that with the top-two election system, every candidate can be elected for a particular configuration of the set of potential candidates. The more candidates are there in the race, the *more divided can be the votes* among the candidates. Besides, with the top-two election system, few voting strategies are weakly dominated. Therefore, even two equivalent candidates, whose ideology is the least preferred ideology for the median voter, can pass to the general election and one of them can be elected.

7 Conclusion and final remarks

The comparison between the traditional election system and the top-two system is made in terms of the Condorcet Consistency criterion which, in our setting, measures whether the electoral system selects the most preferred candidate for the median voter.

We identify two types of violation of the Condorcet Consistency criterion that we call the *Extreme Candidate effect*, and the *Switching Party effect*. The first effect refers to those equilibrium situations in which an extreme candidate wins the general election, whereas the median voter's most preferred candidate is the moderated counterpart . The second effect refers to those equilibrium situations in which the winner candidate and the median voter's most preferred candidate have different party-affiliation.

We have solved each of the proposed sequential games according to the subgame perfect Nash equilibrium concept in which, at each stage of the game, players' strategies are weakly undominated given the equilibrium continuation strategies of the game. This is a very weak equilibrium concept which, in the case of the top-two system generates multiple equilibria. We analyze every equilibrium outcome induced by each of the electoral systems.

We show that the traditional election system can generate the Extreme Candidate effect, but rarely it generates the Switching Party effect. The Extreme Candidate effect rests on a key condition, namely, on a *median partisan strong voter*. Interestingly, this Extreme Candidate effect persists even when we consider that candidates face an entry cost, and when there are more than four candidates in which case, the median partisan needs not be a strong voter. Intuitively, when there are more than four candidates, democrats and republicans can split their vote among several candidates resulting in an extreme candidate winning the election. However, only when the median voter is weak democrat or weak republican (but this is not lean) there are equilibria in which the Extreme Candidate effect can occur.

We show that the top-two election system rarely generates the Extreme Candidate effect or the Switching Party effect when there are no more than four potential candidates. Intuitively, in the top-two primary, the strategic exit of candidates can transform a four-candidate race into a three candidate race in which, by strategic voting, the most preferred candidate for the median voter is elected. We find that the top-two election system generates multiple equilibria and only in two of them (out of twelve), the Extreme Candidate and the Switching Party effects can occur. However, these two divergent equilibria rest on very particular strategies out of the equilibrium path. We also find that when candidates face an entry cost, neither the Extreme Candidate effect nor the Switching Party effect can occur. This is when we account for more than four potential candidates, that we find that the top-two primary can elect every single candidate. In particular, we do not need stringent conditions to show that then, the top-two election system can generate both effects, the Extreme Candidate and the Switching Party.

Roughly speaking, up to four candidates (with no more than two democrats and no more than two republicans), the top-two system is generally Condorcet Consistent and, in this respect, this system outperforms the closed party primaries. When more than four candidate are in the race, the top-two can deviate in any direction from the Condorcet Consistency criterion, whereas the only drawnback of the closed party primaries is the Extreme Candidate effect.

As an illustration, the State of California follows the top-two primary system since 2010. According to the 2014 US House Elections in the 53's California districts, no more than two republican candidates and two democratic candidates were in the race in 35 out of the 53 districts, which represents 66 percent of the districts.[20] There were 14 races with two candidates, 14 races with 3 candidates and 7 races with four candidates. In these cases, our model predicts that the top-two either selects the same moderate candidate than the closed-party primaries, or it outperforms the closed-party primaries by selecting a moderate candidate when the median voter prefers a moderate over the extreme counterpart. In the remaining districts, there were either three or more democrats, or three or more republicans. According to our analysis, any candidate can be the winner in these districts. One of the most striking cases is District 33, in which 18 candidates were running in the primary race (among them, 10 democrats and 3 republicans) and where one democrat was elected.

The top-two election system shares some similarities with the runoff voting system used in many countries to elect president (France, Poland, Argentina, Brazil, and Colombia, among others). In a runoff system, each candidate either has the support of a political party or is independent, and there cannot be two candidates from the same party. Moreover, if a candidate receives an absolute majority, there is no need for a second round. In contrast, the top-two election system does not restrict the candidates to be members of different political parties, and there is always a second voting round (the general election) in which two candidates with the same party-affiliation can run against each other. These differences between the two systems generate different strategic considerations. The analysis of the runoff-system has focused on information aggregation (Martinelli, 2002) and its implications in terms of Duverger's Law (Cox, 1997; Bouton, 2013).

We have introduced a simplified framework to compare the electoral consequences of the top-two primaries with those of the closed party primaries. Our framework can be useful in the comparison of additional party primaries:

[20] Interestingly, in 7 races out of 53, the top-two vote getters had the same party affiliation.

closed, semi-closed and open. Our framework can also be extended to incorporate additional key aspects in primary elections such as uncertainty about the decision of other voters (Palfrey, 1989; Cox 1997), candidates' uncertainty about the location of the median voter, incumbency advantage, or candidates' valence. The detailed analysis of these questions is left for future research.

Appendix

PROOF OF LEMMA 1: We distinguish four cases.

Case 1. The subgame is such that $C_D^r \neq \{D^+, D^-\}$ and $C_R^r \neq \{R^+, R^-\}$.

This case is trivial, since there is no decision to be made at the second stage.

Case 2. The subgame is such that (i) $C_D^r = \emptyset$ and $C_R^r = \{R^+, R^-\}$, or (ii) $C_D^r = \{D^+, D^-\}$ and $C_R^r = \emptyset$.

In any subgame beginning at the second stage with $C_D^r = \emptyset$ and $C_R^r = \{R^+, R^-\}$, the candidate who wins the republican primaries eventually wins the general election at the third stage. Therefore, voting for his/her most preferred candidate, R^+ or R^-, is a weakly dominant strategy for each republican partisan $i \in V_R$. Hence, the candidate winning the election in equilibrium will be R^+ if $\succ_R^m = \succ_{R^+}$ and R^- if $\succ_R^m = \succ_{R^-}$. The case in which $C_D^r = \{D^+, D^-\}$ and $C_R^r = \emptyset$ is symmetric.

Case 3. The subgame is such that (i) $C_D^r = D^+$ and $C_R^r = \{R^+, R^-\}$, or (ii) $C_D^r = D^-$ and $C_R^r = \{R^+, R^-\}$, or (iii) $C_D^r = \{D^+, D^-\}$ and $C_R^r = R^+$, or (iv) $C_D^r = \{D^+, D^-\}$ and $C_R^r = R^-$.

Suppose that $C_D^r = D^+$ and $C_R^r = \{R^+, R^-\}$. In this case the candidate who wins the republican primaries will end up running against D^+. Let $x_{D^+R^+}$ be the candidate who wins the general election in equilibrium at the third stage if $(x_D^n, x_R^n) = (D^+, R^+)$ (from Table 2 we know who this candidate is). Let candidate $x_{D^+R^-}$ be defined in an analogous manner. Note that, for each $i \in V_R$, if $x_{D^+R^+} \succ_i x_{D^+R^-}$ ($x_{D^+R^-} \succ_i x_{D^+R^+}$, respectively), then voting for R^+ (R^-, respectively) in the republican primaries is a weakly dominant strategy at the second stage given the continuation equilibrium strategies at the third stage. Therefore, the favorite candidate between $x_{D^+R^+}$ and $x_{D^+R^-}$ for the median republican partisan will win the election in equilibrium.[21] The cases (ii) $C_D^r = D^-$ and $C_R^r = \{R^+, R^-\}$, (iii) $C_D^r = \{D^+, D^-\}$ and $C_R^r = R^+$, and (iv) $C_D^r = \{D^+, D^-\}$ and $C_R^r = R^-$, are analogous.

Case 4. The subgame is such that $C_D^r = \{D^+, D^-\}$ and $C_R^r = \{R^+, R^-\}$.

Suppose first that $\succ^m = \succ_{D^-}^1$. From Table 2 we have that the democratic nominee eventually wins the general election, no matter who the republican nominee is. Therefore, voting for his/her most preferred candidate, D^+ or D^-, is a weakly dominant strategy for each democratic partisan at the second stage given the continuation equilibrium strategies at the third stage of the game.

Suppose now that $\succ^m = \succ_{D^-}^2$. From Table 2 we have that (i) if $x_D^n = D^-$, then D^- will win the general election no matter who x_R^n is, (ii) if $x_D^n = D^+$ and $x_R^n = R^+$, then D^+ will win the election, and (iii) if $x_D^n = D^+$ and $x_R^n = R^-$, then R^- will win the election. On the one hand, since for all $i \in V_R$, $R^- \succ_i D^+$, then voting for R^- is a weakly dominant strategy for each republican partisan at the second stage given the continuation equilibrium strategies at the third stage. On the other hand, since for all democratic partisan $i \in V_D$ such that $\succ_i \in \{\succ_{D^-}^1, \succ_{D^-}^2\}$, $D^- \succ_i D^+$ and $D^- \succ_i R^-$, then voting for D^- is a weakly dominant strategy for those voters at the second stage given the continuation equilibrium strategies at the third stage. Then, if

[21] For instance, if $\succ^m = \succ_{R^-}^2$ then $x_{D^+R^+} = R^+$ and $x_{D^+R^-} = R^-$. Therefore, if $\succ_R^m = \succ_{R^-}$, R^- will win the election in equilibrium-

$\succ^m_D = \succ_{D^-}$, D^- will win the election in equilibrium, and if $\succ^m_D = \succ_{D^+}$, both D^- and R^- can be sustained as an equilibrium.[22] The case in which $\succ^m = \succ^2_{R^-}$ is analogous. ■

PROOF OF LEMMA 2: We distinguish two cases.

Case 1. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose that $\succ^m = \succ^1_{D^-}$. From Lemma 1, we know who wins the general election depending on who is running. Table 12 summarizes this information. It can be observed that Y is a weakly dominant strategy for each candidate at the first stage given the continuation equilibrium strategies. Then, any profile of equilibrium strategies is such that all candidates are running, D^+ wins the general election if $\succ^m_D = \succ_{D^+}$ and D^- wins the general election if $\succ^m_D = \succ_{D^-}$. The case in which $\succ^m = \succ^1_{R^-}$ is analogous.

Case 2. $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}\}$.

Suppose that $\succ^m = \succ^2_{D^-}$. From Lemma 1, we know who wins the general election depending on who is running. Table 13 shows this information (note that if all candidates are running and $\succ^m_D = \succ_{D^+}$, then both R^- and D^- can win the general election in equilibrium).

It can be observed that Y is a weakly dominant strategy for candidates D^-, R^-, and R^+ at the first stage given any possible continuation equilibrium strategies. Moreover, if $\succ^m_D = \succ_{D^-}$ then Y is also a weakly dominant strategy for candidate D^+ at the first stage given any possible continuation equilibrium strategies. In this case, any profile of equilibrium strategies in the traditional election system is such that all candidates are running and D^- wins the general election. If $\succ^m_D = \succ_{D^+}$, however, Y is not a weakly dominant strategy for D^+ at the first stage given any possible continuation equilibrium strategies.[23] In this case, there are two possible types of equilibrium strategies: one in which D^+ is not running while D^-, R^-, and R^+ are running and D^- wins the general election, and another one in which all candidates are running and D^- wins the general election.[24] Then, if $\succ^m = \succ^2_{D^-}$, any profile of equilibrium strategies in the traditional election system is such that D^-, R^-, and

[22] If $\succ^m_D = \succ_{D^+}$, there are Nash equilibria in the game that begins at the second stage where all democratic partisans with preferences $\succ_{D^+}$ vote for D^- and all republican partisans vote for R^- (and then D^- will win the election), since a single voter cannot benefit from unilateral deviating. Moreover, in these equilibria, the strategies of all voters are undominated given the equilibrium continuation strategies in the third stage. Similarly, there are equilibria where all democratic partisans with preferences $\succ_{D^+}$ vote for D^+ and all republican partisans vote for R^- (and then R^- will win the election). Note that, once we have eliminated the strategy of voting for R^+ in the republican primaries, then the strategy of voting for D^+ is weakly dominated (given the continuation equilibrium strategies at the third stage) for each democratic partisan. In this paper, however, we only consider one round of deletion of weakly dominated strategies.

[23] In particular, if (i) D^-, R^-, and R^+ decide to run and (ii) the continuation equilibrium strategies are such that when all candidates are running R^- wins the general election, then candidate D^+ is strictly better off not running than running (since $D^- \succ_{D^+} R^-$).

[24] Note that there is no equilibrium where all candidates are running and R^- wins the general election since, in that case, candidate D^+ would prefer to deviate and not run (because in this case, given any possible continuation equilibrium strategies, D^- would win the general election).

R^+ are running and D^- wins the general election. The case in which $\succ^m = \succ^2_{R^-}$ is analogous. ■

		R^-: Y		R^-: N	
R^+	D^+	D^-: Y	D^-: N	D^-: Y	D^-: N
Y	Y	$D^{+(1)}$ / $D^{-(2)}$	D^+	$D^{+(1)}$ / $D^{-(2)}$	D^+
Y	N	D^-	$R^{+(3)}$ / $R^{-(4)}$	D^-	R^+
N	Y	$D^{+(1)}$ / $D^{-(2)}$	D^+	$D^{+(1)}$ / $D^{-(2)}$	D^+
N	N	D^-	R^-	D^-	$\emptyset$

$^{(1)}$If $\succ^m_D = \succ_{D^+}$ $^{(2)}$If $\succ^m_D = \succ_{D^-}$ $^{(3)}$If $\succ^m_R = \succ_{R^+}$ $^{(4)}$If $\succ^m_R = \succ_{R^-}$

Table 12 Closed party primaries: Candidates' entry stage when $\succ^m = \succ^1_{D^-}$.

		R^-: Y		R^-: N	
R^+	D^+	D^-: Y	D^-: N	D^-: Y	D^-: N
Y	Y	R^- or $D^{-(1)}$ / $D^{-(2)}$	R^-	$D^{+(1)}$ / $D^{-(2)}$	D^+
Y	N	D^-	$R^{+(3)}$ / $R^{-(4)}$	D^-	R^+
N	Y	D^-	R^-	$D^{+(1)}$ / $D^{-(2)}$	D^+
N	N	D^-	R^-	D^-	$\emptyset$

$^{(1)}$If $\succ^m_D = \succ_{D^+}$ $^{(2)}$If $\succ^m_D = \succ_{D^-}$ $^{(3)}$If $\succ^m_R = \succ_{R^+}$ $^{(4)}$If $\succ^m_R = \succ_{R^-}$

Table 13 Closed party-primaries: Candidates' entry stage when $\succ^m = \succ^2_{D^-}$.

PROOF OF LEMMA 3: To prove Lemma 3 we need two previous results, Claim 1 and Claim 2.

Claim 1: If $\succ^m = \succ^1_{D^-}$, any subgame beginning at the second stage of the top-two election system where all candidates are running is such that:

(1) voting for R^+ and voting for R^- in the to-two primary are weakly dominated strategies (given the equilibrium continuation strategies) for any voter i such that $\succ_i = \succ^1_{D^-}$,

(2) voting for R^+ and voting for R^- in the top-two primary are not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ_{D^+}, \succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$, and

(3) voting for D^+ and voting for D^- in the top-two primary are not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ_{D^+}, \succ^1_{D^-}\}$.

Proof. First note that, for any voter i such that $\succ_i = \succ^1_{D^-}$, voting for D^- in the top-two primary weakly dominates to voting for R^+ and to voting for R^- (given the equilibrium continuation strategies). We omit the proof of this point. It follows from the fact that, since $\succ^m = \succ^1_{D^-}$, if D^- is one of the candidates passing to the next round, then D^- (the most preferred candidate for any voter with preferences type $\succ^1_{D^-}$) will win the general election (see Table 2 and Table 6).

Suppose, without loss of generality that $v = 100$. Now we prove point (2). Let i be such that $\succ_i = \succ_{D^+}$. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (40, 20, 20, 19)$ then i will be better off voting for R^+ than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 29, 12, 28)$ then i will be better off voting for R^+ than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 20, 10, 19)$ then i will be better off voting for R^+ than voting for R^-. To see that voting for R^- is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (70, 10, 9, 10)$ then i will be better off voting for R^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 29, 28, 12)$ then i will be better off voting for R^- than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 20, 19, 10)$ then i will be better off voting for R^- than voting for R^+.

Let i be such that $\succ_i = \succ^2_{D^-}$. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 10, 50, 19)$ then i will be better off voting for R^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 9, 31, 29)$ then i will be better off voting for R^+ than voting for D^+, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (10, 30, 29, 30)$ then i will be better off voting for R^+ than voting for R^-. To see that voting for R^- is not weakly dominated for i note that: (i) if t^2_{-i} such that $(D^+, D^-, R^-, R^+) = (30, 9, 30, 30)$ then i will be better off voting for R^- than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 8, 31, 30)$, then i will be better off voting for R^- than voting for D^+, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (10, 30, 30, 29)$, then i will be better off voting for R^- than voting for R^+.

Let i be such that $\succ_i = \succ^2_{R^-}$. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20,\ 10,\ 50,\ 19)$ then i will be better off voting for R^+ than voting for D^-, (ii) if t^2_{-i} is such that

$(D^+, D^-, R^-, R^+) = (30, 9, 31, 29)$ then i will be better off voting for R^+ than voting for D^+, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (10, 20, 50, 19)$ then i will be better off voting for R^+ than voting for R^-. To see that voting for R^- is not weakly dominated for i note that: (i) if t^2_{-i} such that $(D^+, D^-, R^-, R^+) = (30, 9, 30, 30)$ then i will be better off voting for R^- than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 8, 31, 30)$ then i will be better off voting for R^- than voting for D^+, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (10, 20, 19, 50)$ then i will be better off voting for R^- than voting for R^+.

The proof that voting for R^+ and voting for R^- are not weakly dominated for any voter i such that $\succ_i \in \{\succ^1_{R^-}, \succ_{R^+}\}$ is identical to the proof for the case that $\succ_i = \succ^2_{R^-}$.

Finally, we prove point (3). Let i be such that $\succ_i = \succ^1_{D^-}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 8, 31, 30)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 50, 20)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for D^- is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 20, 50, 9)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+.

Let i be such that $\succ_i = \succ_{D^+}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 20, 9, 50)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 50, 20)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for D^- is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (11, 29, 29, 30)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+. ■

Claim 2: If $\succ^m = \succ^2_{D^-}$, any subgame beginning at the second stage of the top-two election system where all candidates are running is such that:
(1) voting for D^+ and voting for R^+ in the top-two primary are weakly dominated strategies (given the equilibrium continuation strategies) for any voter i such that $\succ_i = \succ^2_{D^-}$,
(2) voting for D^+ and voting for R^+ in the top-two primary are not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$,
(3) voting for D^- in the top-two primary is not weakly dominated by any other strategy (given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ_{D^+}, \succ^1_{D^-}, \succ^2_{D^-}\}$, and
(4) voting for R^- in the top-two primary is not weakly dominated by any other strategy

(given the equilibrium continuation strategies) for any voter i such that $\succ_i \in \{\succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$.

Proof. First, note that voting for D^- in the top-two primary weakly dominates to voting for D^+ and to voting for R^+ (given the equilibrium continuation strategies) for any voter i such that $\succ_i = \succ^2_{D^-}$. We omit the proof of this point. It follows from the fact that, since $\succ^m = \succ^2_{D^-}$, if D^- is one of the candidates passing to the next round, then D^- will win the general election (see Table 2 and Table 6).

Suppose without loss of generality that $v = 100$. Now we prove point (2). Let i be such that $\succ_i = \succ_{D^+}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 20, 9, 50)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (0, 0, 0, 99)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (40, 20, 20, 19)$ then i will be better off voting for R^+ than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (30, 29, 12, 28)$ then i will be better off voting for R^+ than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 20, 10, 19)$ then i will be better off voting for R^+ than voting for R^-.

Let i be such that $\succ_i = \succ^1_{D^-}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (19, 10, 20, 50)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (0, 0, 0, 99)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 9, 20, 20)$ then i will be better off voting for R^+ than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 9, 20, 20)$ then i will be better off voting for R^+ than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 9, 20, 20)$ then i will be better off voting for R^+ than voting for R^-.

Let i be such that $\succ_i = \succ^2_{R^-}$. To see that voting for D^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (8, 20, 50, 21)$ then i will be better off voting for D^+ than voting for D^-, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (0, 1, 98, 0)$ then i will be better off voting for D^+ than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 10, 20, 19)$ then i will be better off voting for D^+ than voting for R^+. To see that voting for R^+ is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 21, 8, 50)$ then i will be better off voting for R^+ than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 20, 21, 8)$ then i will be better off voting for R^+ than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (8, 21, 50, 20)$ then i will be better off voting for R^+ than voting for R^-.

Let i be such that $\succ_i \in \{\succ^1_{R^-}, \succ_{R^+}\}$. The proof that voting for D^+ and voting for R^+ are not weakly dominated for i is identical to the proof in the case that $\succ_i = \succ^2_{R^-}$.

Next, we prove point (3). Let i be such that $\succ_i = \succ_{D^+}$. To see that voting for D^- is not weakly dominated for any i such that $\succ_i = \succ_{D^+}$ note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (11, 29, 29, 30)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+. To see that voting for D^- is not weakly dominated for i such that $\succ_i = \succ^1_{D^-}$ note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (11, 29, 29, 30)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+. To see that voting for D^- is not weakly dominated for i such that $\succ_i = \succ^2_{D^-}$ note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 20, 50, 9)$ then i will be better off voting for D^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 20, 50)$ then i will be better off voting for D^- than voting for R^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (9, 20, 50, 20)$ then i will be better off voting for D^- than voting for R^+.

Finally, we prove point (4). Let i be such that $\succ_i = \succ^2_{D^-}$. To see that voting for R^- is not weakly dominated for i note that: (i) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (20, 9, 20, 50)$ then i will be better off voting for R^- than voting for D^+, (ii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (31, 8, 30, 30)$ then i will be better off voting for R^- than voting for D^-, and (iii) if t^2_{-i} is such that $(D^+, D^-, R^-, R^+) = (50, 9, 20, 20)$ then i will be better off voting for R^- than voting for R^+. The proof that voting for R^- is not weakly dominated for any voter i such that $\succ_i \in \{\succ^2_{R^-}, \succ^1_{R^-}, \succ_{R^+}\}$ is identical to the proof for the case that $\succ_i = \succ^2_{D^-}$. ■

Now, we can prove Lemma 3. We distinguish four cases.

Case 1. The subgame is such that, at most, there are two candidates running.

This case is trivial since the voters do not have to take any decision at the second stage.

Case 2. The subgame is such that only three candidates are running.

Suppose first that $C^r = D^+D^-R^-$. Then, there are three potential pairs of candidates that may pass to the next round: D^+D^-, D^+R^-, and D^-R^-. From Table 2 and Table 6 it can be observed that, in this case, if $\succ^m = \succ^1_{D^-}$, only two candidates may win the general election: D^+ (if D^+ passes to the third stage) and D^- (if D^+ does not pass to the third stage). Then, voting for D^- in the top-two primary is a weakly dominant strategy for any voter who prefers D^- to D^+, and D^- will win the election in equilibrium. Using a similar argument it can be shown that, (i) if $\succ^m = \succ^2_{D^-}$, voting for D^- in the top-two primary is a weakly dominant strategy for any voter who prefers D^- to R^-, and D^- will win the election in equilibrium; (ii) if $\succ^m = \succ^2_{R^-}$, voting for R^- in the top-two primary is a weakly dominant strategy for any voter who prefers R^- to D^-, and therefore R^- will win the election in equilibrium; (iv) if $\succ^m = \succ^1_{R^-}$, voting for R^- in the top-two primary is a weakly dominant strategy for any voter who prefers R^- to D^-, and therefore R^- will win the election in equilibrium.

Suppose now that $C^r = D^+D^-R^+$. In this case, only the pairs D^+D^-, D^+R^+, and D^-R^+ may pass to the next round. From Table 2 and Table 6 and using a similar argument to previous case it can be shown that: (i) if $\succ^m \in \{\succ^1_{D^-}, \succ^2_{D^-}, \succ^2_{R^-}\}$, a majority of voters will vote for D^- in the top-two primary and D^- will win the election in equilibrium; (ii) if $\succ^m = \succ^1_{R^-}$, a majority of voters will vote for R^+ in the top-two primary and R^+ will win the election in equilibrium.

If $C^r = D^+R^-R^+$, only the pairs D^+R^-, D^+R^+, and R^-R^+ may pass to the next round. From Table 2 and Table 6 and using a similar argument to previous cases it can be shown that: (i) if $\succ^m = \succ^1_{D^-}$, a majority of voters will vote for D^+ in the top-two primary and D^+ will win the election in equilibrium; (ii) if $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}, \succ^1_{R^-}\}$, a majority of voters will vote for R^- in the top-two primary and R^- will win the election in equilibrium.

Finally, if $C^r = D^-R^-R^+$, only the pairs D^-R^-, D^-R^+, and R^-R^+ may pass to the next round. From Table 2 and Table 6 and using a similar argument to previous cases it can be shown that: (i) if $\succ^m \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$, a majority of voters will vote for D^- in the top-two primary and D^- will win the election in equilibrium; (ii) if $\succ^m \in \{\succ^2_{R^-}, \succ^1_{R^-}\}$, a majority of voters will vote for R^- in the top-two primary and R^- will win the election in equilibrium.

Case 3. The subgame is such that the four candidates are running.

Subcase 3.1. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose that $\succ^m = \succ^1_{D^-}$. From points (1) and (3) of Claim 1 we have that, if more than half of the voters were of type $\succ^1_{D^-}$, then more than $\frac{v}{4}$ of the voters would vote for D^+ and/or D^- in the top-two primary. In this case, from Table 2 and Table 6, D^+ and D^- would be the only two candidates who might win in equilibrium. The fact that $\succ^m = \succ^1_{D^-}$, however, does not imply that more than $\frac{v}{2}$ voters are of type $\succ^1_{D^-}$. Then, given points (2) and (3) of Claim 1, we cannot rule out the possibility that any pair of candidates can pass to the next round in equilibrium. In particular, it is possible that the candidates who pass to the next round are R^- and R^+ in which case the winning candidate would be R^-. The only equilibrium result that we can rule out is that R^+ wins the general election (from Table 2, since $\succ^m = \succ^1_{D^-}$, the only chance for R^+ to win would be that all voters were voting for R^+ in the top-two primary; this situation, however, would never be an equilibrium since any democratic partisan would prefer to vote for D^+ or D^-). Therefore, D^+, D^-, or R^- may win the general election in equilibrium.[25] The case in which $\succ^m = \succ^1_{R^-}$ is symmetric to the case in which $\succ^m = \succ^1_{D^-}$ (there is also a symmetric version of Claim 1 for the case in which $\succ^m = \succ^1_{R^-}$).

Subcase 3.2. $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}\}$.

Suppose that $\succ^m = \succ^2_{D^-}$. From points (1), (3), and (4) of Claim 2, if more than half of the voters were of type $\succ^2_{D^-}$, then more than $\frac{v}{4}$ of the voters would vote

[25] For example, a situation where all voters type $\succ_{D^+}$ and $\succ^1_{D^-}$ vote for D^- while the rest vote for R^- would be an equilibrium in the second stage resulting in D^- (since $\succ^m = \succ^1_{D^-}$, more than a half of the voters are of type $\succ_{D^+}$ or $\succ^1_{D^-}$). Similarly, a situation where all voters type $\succ_{D^+}$ and $\succ^1_{D^-}$ vote for D^+, while the rest vote for R^- would be an equilibrium in the second stage resulting in D^+. Finally, if less than a half of the voters are of type $\succ^1_{D^-}$, a situation where half of the voters type $\succ^1_{D^-}$ vote for D^-, the other half of the voters type $\succ^1_{D^-}$ vote for D^+, half of the rest of voters vote for R^-, while the other half vote for R^+, would be an equilibrium in the second stage resulting in R^-.

for R^- and/or D^- in the top-two primary. In this case, from Table 2 and Table 6, R^- and D^- would be the only two candidates who might win in equilibrium. The fact that $\succ^m = \succ^2_{D^-}$, however, does not imply that more than $\frac{v}{2}$ voters are of type $\succ^2_{D^-}$. Given point (2) of Claim 2, it is possible that the candidates who pass to the next round are D^+ and R^+ in which case the winning candidate would be D^+. As in the case that $\succ^m = \succ^1_{D^-}$, the only equilibrium result that we can rule out is that R^+ wins the general election. Therefore, D^-, R^-, or D^+ may win the general election in equilibrium.[26] The case in which $\succ^m = \succ^2_{R^-}$ is symmetric to the case in which $\succ^m = \succ^2_{D^-}$ (there is also a symmetric version of Claim 2 for the case in which $\succ^m = \succ^2_{R^-}$). ■

PROOF OF LEMMA 4: We distinguish two cases.

Case 1. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose first that $\succ^m = \succ^1_{D^-}$. From Lemma 3, we know who wins the general election depending on who is running in this case. Table 14 summarizes this information.

		R^-			
		Y		N	
		D^-		D^-	
R^+	D^+	Y	N	Y	N
Y	Y	D^+ or D^- or $R^{-(*)}$	D^+	D^-	D^+
Y	N	D^-	R^-	D^-	R^+
N	Y	D^-	D^+	D^-	D^+
N	N	D^-	R^-	D^-	$\emptyset$

$^{(*)}$ Only if $\#\{i \in V : \succ_i = \succ^1_{D^-}\} < v/2$

Table 14 Top-two primaries: Candidates' entry stage when $\succ^m = \succ^1_{D^-}$.

We distinguish three subcases:

[26] For example, a situation where all voters type $\succ_{D^+}$, $\succ^1_{D^-}$, and $\succ^2_{D^-}$ vote for D^- while the rest vote for D^+ would be an equilibrium in the second stage resulting in D^- (since $\succ^m = \succ^2_{D^-}$, more than a half of the voters are of type $\succ_{D^+}$, $\succ^1_{D^-}$, or $\succ^2_{D^-}$). Similarly, a situation where all voters type $\succ^2_{D^-}$, $\succ^2_{R^-}$, $\succ^1_{R^-}$, and $\succ_{R^+}$ vote for R^-, while the rest vote for D^+ would be an equilibrium in the second stage resulting in R^- (since $\succ^m = \succ^2_{D^-}$, more than a half of the voters are of type $\succ^2_{D^-}$, $\succ^2_{R^-}$, $\succ^1_{R^-}$, or $\succ_{R^+}$) Finally, if less than a half of the voters are of type $\succ^2_{D^-}$, a situation where half of the voters type $\succ^2_{D^-}$ vote for R^-, the other half of the voters type $\succ^2_{D^-}$ vote for D^-, half of the rest of voters vote for R^+, while the other half vote for D^+, would be an equilibrium in the second stage resulting in D^+.

Subcase 2.1. The equilibrium strategies in the second and third stages are such that, if all candidates are running, D^+ wins the general election (i.e., D^+ is in the upper left cell in Table 14).

In this case, Y is a weakly dominant strategy for candidates D^+ and D^- at the first stage given the continuation equilibrium strategies. Given this, in Table 14 can be seen that there are three types of equilibria: one in which all candidates except R^+ are running, one in which all candidates except R^- are running, and one in which D^+ and D^- are running and R^+ and R^- are not running (all candidates running is not an equilibrium because in that case both, R^+ and R^-, have incentives to unilaterally deviate). The three types of equilibrium yields the same result: D^- wins the general election.

Subcase 2.2. The equilibrium strategies in the second and third stages are such that, if all candidates are running, D^- wins the general election (i.e., D^- is in the upper left cell in Table 14).

In this case, Y is a weakly dominant strategy for all candidates at the first stage given the continuation equilibrium strategies, and then D^- wins the general election.

Subcase 2.3. The equilibrium strategies in the second and third stages are such that, if all candidates are running, R^- wins the general election (i.e., R^- is in the upper left cell in Table 14).

In this case, Y is a weakly dominant strategy for candidates R^+ and R^- at the first stage given the continuation equilibrium strategies. Moreover, Y is a weakly dominant strategy for candidate D^- if his/her preferences are of type $\succ^2_{D^-}$, but not if his/her preferences are of type $\succ^1_{D^-}$ (in the latter case, if the other three candidates are running, D^- prefers not to run, since he/she prefers D^+ to R^-). Then, if the preferences of candidate D^- are of type $\succ^2_{D^-}$, candidates R^+, R^-, and D^- will run and, given this, D^+ prefers not to run. This equilibrium results in D^- winning the general election. If the preferences of candidate D^- are of type $\succ^1_{D^-}$, then there are two types of equilibria: one in which all candidates except D^+ are running (which result in D^- winning the general election), and one in which all candidates except D^- are running (which result in D^+ winning the general election).

The case in which $\succ^m = \succ^1_{R^-}$ is analogous.

Case 3. $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}\}$.

From Lemma 3, we know who wins the general election depending on who is running in this case. Table 15 summarizes this information.

We distinguish three subcases:

Subcase 3.1. The equilibrium strategies in the second and third stages are such that, if all candidates are running, D^+ wins the general election (i.e., D^+ is in the upper left cell in Table 15).

In this case, Y is a weakly dominant strategy for candidate D^+ at the first stage given the continuation equilibrium strategies. If the preferences of candidate D^- are of type $\succ^1_{D^-}$, then Y is also a weakly dominant strategy for him/her given the continuation equilibrium strategies, and there are three types of equilibria: one in which all candidates except R^+ are running, one in which all candidates except R^- are running, and one in which D^+ and D^- are running and R^+ and R^- are not running (all candidates running is not an equilibrium because in that case both, R^+ and R^-, have incentives to unilaterally deviate). The three types of equilibrium yields the same result: D^- wins the general election. If the preferences of candidate D^- are of type $\succ^2_{D^-}$, then Y is not a weakly dominant strategy for him/her given the continuation equilibrium strategies (if the other three candidates are running, D^- prefers not to run, since he/she prefers R^- to D^+). In this case, there are four types of equilibria: one in which all candidates except D^- are running, one in which all candidates except R^+ are running, one in which all candidates except R^- are running, and one in which D^+ and D^- are running and R^+ and R^- are not running. In the first type of equilibrium R^- wins the general election, while in the other three types of equilibria D^- wins the general election.

		R^-: Y		R^-: N	
		D^-: Y	D^-: N	D^-: Y	D^-: N
R^+: Y	D^+: Y	$D^{+(*)}$ or D^- or R^-	R^-	D^-	D^+
	D^+: N	D^-	R^-	D^-	R^+
R^+: N	D^+: Y	D^-	R^-	D^-	D^+
	D^+: N	D^-	R^-	D^-	$\emptyset$

$^{(*)}$ Only if $\#\{i \in V : \succ_i = \succ^2_{D^-}\} < v/2$

Table 15 Top-two primaries: Candidates' entry stage when $\succ^m = \succ^2_{D^-}$.

Subcase 3.2. The equilibrium strategies in the second and third stages are such that, if all candidates are running, D^- wins the general election (i.e., D^- is in the upper left cell in Table 15).

In this case, Y is a weakly dominant strategy for all candidates at the first stage given the continuation equilibrium strategies, and then D^- wins the general election.

Subcase 3.3. The equilibrium strategies in the second and third stages are such that, if all candidates are running, R^- wins the general election (i.e., R^- is in the upper left cell in Table 15).

In this case, Y is a weakly dominant strategy for candidates D^-, R^-, and R^+ at the first stage given the continuation equilibrium strategies. Then, candidates D^-, R^-, and R^+ will run and, given this, D^+ prefers not to run. This equilibrium results in D^- winning the general election.

The case in which $\succ^m = \succ^2_{R^-}$ is analogous. ■

PROOF OF LEMMA 5: We distinguish two cases:

Case 1. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose that $\succ^m = \succ^1_{D^-}$. In this case, Table 12 shows who wins the general election depending on who is running when the election system is the traditional election system. Note that $(Y,Y,Y,N) \succ_{R^+} (Y,Y,Y,Y)$, $(Y,N,N,Y) \succ_{R^-} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(Y,Y,N,N) \succ_{R^-} (Y,Y,Y,N)$, $(Y,N,N,N) \succ_{R^-} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(Y,N,N,N) \succ_{D^+} (N,N,N,N)$, and $(N,Y,N,N) \succ_{D^-} (N,N,N,N)$. Moreover, if $\succ^m_D = \succ_{D^+}$, then $(Y,N,N,N) \succ_{D^-} (Y,Y,N,N)$ and $(Y,Y,N,N) \succ_{D^+} (N,Y,N,N)$. Similarly, if $\succ^m_D \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$, then $(N,Y,N,N) \succ_{D^+} (Y,Y,N,N)$ and $(Y,Y,N,N) \succ_{D^-} (Y,N,N,N)$. Therefore: (i) if $\succ^m_D = \succ_{D^+}$, any profile of equilibrium strategies is such that $s^1 = (Y,N,N,N)$ and D^+ wins the general election, and (ii) if $\succ^m_D \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$, any profile of equilibrium strategies is such that $s^1 = (N,Y,N,N)$ and D^- wins the general election.[27] The case in which $\succ^m = \succ^1_{R^-}$ is analogous.

<u>Case 2</u>. $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}\}$.

Suppose that $\succ^m = \succ^2_{D^-}$. In this case, Table 13 shows who wins the general election depending on who is running when the election system is the traditional election system. Note that $(N,Y,Y,Y) \succ_{D^+} (Y,Y,Y,Y)$, $(Y,N,Y,N) \succ_{R^+} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(N,Y,Y,N) \succ_{D^+} (Y,Y,Y,N)$, $(N,N,Y,N) \succ_{D^+} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(Y,N,Y,N) \succ_{R^-} (Y,N,N,N)$, and $(N,Y,N,N) \succ_{D^-} (N,N,N,N)$. Moreover, if $\succ^m_D = \succ_{D^+}$, then $(Y,N,N,N) \succ_{D^-} (Y,Y,N,N)$ and $(Y,Y,N,N) \succ_{D^+} (N,Y,N,N)$, and hence there is no profile of equilibrium strategies. If $\succ^m_D \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$, however, then $(N,Y,N,N) \succ_{D^+} (Y,Y,N,N)$, and therefore any profile of equilibrium strategies is such that $s^1 = (N,Y,N,N)$ and D^- wins the general election. The case in which $\succ^m = \succ^2_{R^-}$ is analogous.[28] ■

PROOF OF LEMMA 6: We distinguish two cases:

<u>Case 1</u>. $\succ^m \in \{\succ^1_{D^-}, \succ^1_{R^-}\}$.

Suppose that $\succ^m = \succ^1_{D^-}$. In this case, Table 14 shows who wins the general election depending on who is running when the election system is the top-two system. Note that $(Y,N,N,Y) \succ_{R^-} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(Y,Y,N,N) \succ_{R^-} (Y,Y,Y,N)$, $(Y,N,N,N) \succ_{R^-} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(N,Y,N,N) \succ_{D^+} (Y,Y,N,N)$, $(Y,Y,N,N) \succ_{D^-} (Y,N,N,N)$, and $(N,Y,N,N) \succ_{D^-} (N,N,N,N)$. Moreover,

[27] Note that, in this case, Y is not a weakly dominant strategy for R^- and R^+ given the continuation equilibrium strategies. Moreover, if $\succ^m_D = \succ_{D^+}$, then Y is not a weakly dominant strategy for D^-, and if $\succ^m_D \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$, then Y is not a weakly dominant strategy for D^+ (given the continuation equilibrium strategies).

[28] Note that, in this case, (i) Y is not a weakly dominant strategy for R^- and R^+, (ii) if $\succ^m_D = \succ_{D^+}$, then Y is not a weakly dominant strategy for D^-, and (iii) if $\succ^m_D \in \{\succ^1_{D^-}, \succ^2_{D^-}\}$, then Y is not a weakly dominant strategy for D^+ (given the continuation equilibrium strategies).

if the equilibrium strategies in the second and third stages are such that D^+ (respectively D^- or R^-) wins the general election if all candidates are running, then $(Y,N,Y,Y) \succ_{D^-} (Y,Y,Y,Y)$ (respectively $(N,Y,Y,Y) \succ_{D^+} (Y,Y,Y,Y)$). Therefore, any profile of equilibrium strategies is such that $s^1 = (N,Y,N,N)$ and D^- wins the general election.[29] The case in which $\succ^m = \succ^1_{R^-}$ is analogous.

Case 2. $\succ^m \in \{\succ^2_{D^-}, \succ^2_{R^-}\}$.

Suppose that $\succ^m = \succ^2_{D^-}$. Table 15 shows who wins the general election depending on who is running when the election system is the top-two system. Note that $(N,N,Y,Y) \succ_{D^+} (Y,N,Y,Y)$, $(N,Y,Y,N) \succ_{R^+} (N,Y,Y,Y)$, $(N,Y,Y,Y) \succ_{D^-} (N,N,Y,Y)$, $(Y,Y,N,N) \succ_{R^-} (Y,Y,Y,N)$, $(N,N,Y,N) \succ_{D^+} (Y,N,Y,N)$, $(N,Y,N,N) \succ_{R^-} (N,Y,Y,N)$, $(N,Y,Y,N) \succ_{D^-} (N,N,Y,N)$, $(Y,Y,N,N) \succ_{R^+} (Y,Y,N,Y)$, $(Y,N,N,N) \succ_{R^+} (Y,N,N,Y)$, $(N,Y,N,N) \succ_{R^+} (N,Y,N,Y)$, $(N,Y,N,Y) \succ_{D^-} (N,N,N,Y)$, $(N,Y,N,N) \succ_{D^+} (Y,Y,N,N)$, $(Y,Y,N,N) \succ_{D^-} (Y,N,N,N)$, and $(N,Y,N,N) \succ_{D^-} (N,N,N,N)$. Moreover, if the equilibrium strategies in the second and third stages are such that D^+ or D^- (respectively R^-) wins the general election if all candidates are running, then $(Y,Y,N,Y) \succ_{R^-} (Y,Y,Y,Y)$ (respectively $(Y,N,Y,Y) \succ_{D^-} (Y,Y,Y,Y)$). Therefore, any profile of equilibrium strategies is such that $s^1 = (N,Y,N,N)$ and D^- wins the general election.[30] The case where $\succ^m = \succ^2_{R^-}$ is analogous. ■

PROOF OF PROPOSITION 5:

i) According to Lemma 1, when $\succ^m = \succ^1_{D^-}$, the democratic nominee wins the general election and therefore, for each democratic voter this is a weakly dominant strategy to vote for his/her most preferred candidate in the primary stage. Consider that there are at least three (or more) moderated democratic candidates $\left\{D_1^-, D_2^-, D_3^-\right\}$. Consider that weak democratic partisan voters split their vote among these three candidates whereas strong democratic partisans vote for the unique D^+. Thus, even if $\succ^m_D = \succ^1_{D^-}$, candidate D^+ may eventually be nominated and can defeat the republican nominee at the general election. We show that this electoral outcome can be immune to the strategic exit stage. If one of the moderated democratic candidates withdraws from the contest, still the remaining moderated democratic candidates may split their vote among the two remaining democratic candidates and candidate D^+ is not defeated in the democratic primary. No other candidate has incentives to withdraw.

Suppose that $\succ^m = \succ^2_{D^-}$ then, the preferences of the median voter are such that $D^- \succ^m R^- \succ^m D^+ \succ^m R^+$ and thus, only the moderate democratic candidate can defeat the moderate republican candidate at the general election. For every republican voter, voting for R^+ is weakly dominated by voting for R^- at the primary

[29] Observe that, in this case, Y is not a weakly dominant strategy for D^+, R^- and R^+ given the continuation equilibrium strategies. Moreover, if the equilibrium strategies in the second and third stages are such that D^+ wins the general election if all candidates are running (or R^- wins the general election and the preferences of candidate D^- are type $\succ^2_{D^-}$), then Y is not a weakly dominant strategy for D^+ either.

[30] Observe that, in this case, Y is not a weakly dominant strategy for D^+, R^- and R^+ given the continuation equilibrium strategies. Moreover, if the equilibrium strategies in the second and third stages are such that R^- wins the general election if all candidates are running (or D^+ wins the general election and the preferences of candidate D^- are type $\succ^2_{D^-}$), then Y is not a weakly dominant strategy for D^+ either.

stage. Even if there are several moderate republican candidates, one of them is nominated by the republican party. Therefore, if D^+ is nominated, R^- defeats D^+ at the general election.

ii) According to Lemma 1, when $\succ^m = \succ^2_{D^-}$, for every republican voter in the primary stage, voting for R^- is a weakly dominant strategy; for every weak or lean democratic voter, voting for D^- is a weakly dominant strategy; and for every strong democratic voter, voting for D^+ is not weakly dominated. Then, if there are at least three (or more) moderate democratic candidates $\left\{D_1^-, D_2^-, D_3^-\right\}$, the weak and lean democratic voters may split their vote among the several moderate democrats. Consider that there are at least two extreme democrats $\left\{D_1^+, D_2^+\right\}$. Then, even when the strong democratic voters can also split their vote among the two extreme democratic candidates, one of the extreme democratic candidates can be elected in the democratic primary. Thus, if the two nominees are $\left\{D^+, R^-\right\}$, then candidate R^- defeats D^+. We show that this electoral outcome is immune to the strategic exit of candidates. If one of the extreme democratic partisans opts out, e.g. D_1^+, still the other extreme democratic candidate D_2^+ may win the partisan primary. If one of the moderate democratic candidates withdraws from the contest, there are still other candidates with the same ideology remaining in the race and the primary result may not be modified. Finally, neither R^- nor R^+ have incentives to opt out.[31]

Suppose that $\succ^m = \succ^1_{D^-}$ then, the preferences of the median voter are such that $D^+ \succ^m D^- \succ^m R^- \succ^m R^+$ and thus, the democratic nominee will always win the general election. ■

PROOF OF PROPOSITION 6:

i) Suppose that $\succ^m = \succ^1_{D^-}$ and that $\mathcal{C} = \left\{D^+, D^-, R_1^-, R_2^-, R^+\right\}$. By Claim 1 in Lemma 3, when there are four candidates, the only weakly dominated strategies for weak democratic voters are voting for R^-, R^+, and there are no other weakly dominated strategies for any other voter in the primary stage. For the proposed set of candidates, voting for R_1^-, R_2^-, R^+ is also weakly dominated for weak democratic voters. Then, one possibility is that candidate D^+ together with R^+ or R_1^- or R_2^-, pass to the next round, in which case, D^+ is elected. We show that this electoral outcome can be immune to the strategic entry of candidates. If one candidate R_1^- or R_2^- opts out, still there is another candidate with ideology R^- in the race, and D^+ can be elected. Similarly, if candidate R^+ opts out, the votes of R^+ split between R_1^- or R_2^- and D^+ can be still elected. Candidates D^+ and D^- have no incentives to withdraw.[32]

Suppose that $\succ^m = \succ^2_{D^-}$ and that $\mathcal{C} = \left\{D^+, D^-, R^-, R_1^+, R_2^+\right\}$. By Claim 2 in Lemma 3, when there are four candidates, the only weakly dominated strategies for lean democratic voters are voting for D^+ or R^+. Voting for D^+ or R^+ is not weakly dominated for any other voter in the primary stage. For the proposed set of candidates, voting for D^+, R_1^+, R_2^+, is also weakly dominated for lean democratic voters. Consider

[31] Note that once we have eliminated the strategy of voting for R^+ in the republican primaries, the strategy of voting for D^+ is weakly dominated for every democratic voter (given the equilibrium strategies in the continuation game). However, we are not considering this additional equilibrium refinement.

[32] Even when $\mathcal{C} = \left\{D^+, R_1^-, R_2^-, R^+\right\}$ we can derive a similar result.

that there is a small fraction of lean democratic candidates. Then, one possibility is that candidate D^+ together with R_1^+ or R_2^+ pass to the next round, in which case, D^+ is elected. We show that this electoral outcome can be immune to the strategic exit of candidates. If candidate R^- opts out, votes for R^- can split so that D^+ can still be elected. Similarly, if candidate R_1^+ or R_2^+ opts out, the remaining candidate R^+ gets more votes, moves to the next round, and D^+ can be elected. Finally, candidate D^-, if his/her ideology is $\succ^1_{D^-}$, may not have incentives to opt out.[33]

ii) Suppose that $\succ^m = \succ^1_{D^-}$ and that $\mathcal{C} = \left\{D^+, D_1^-, D_2^-, D_3^-, R^-, R^+\right\}$. By Claim 1 in Lemma 3, when there are four candidates, the only weakly dominated strategies for weak democratic voters are voting for R^-, R^+, and there are no other weakly dominated strategies for any other voter in the primary stage. For the proposed set of candidates, voting for R^-, R^+ is also weakly dominated for weak democratic voters. Then, if weak democratic voters split their vote among the four democratic candidates, candidates R^- and R^+ can eventually be nominated for the general election. In this case, candidate R^- is elected. We show that this election outcome can be immune to the strategic exit of candidates. If D^+ withdraws, for all the democratic voters (which are a majority of voters) voting for R^-, R^+ is a weakly dominated strategy. Then, if democratic voters split their vote among the candidates D_1^-, D_2^-, D_3^- and the republican partisan voters vote for candidates R^- and R^+, still R^- and R^+ can pass to the general election and R^- is elected. If any of the moderated democratic voters D^- withdraws, weak democratic voters can vote for one of the three democratic candidates D^+, D_1^-, D_2^- and all the other voters (the strong or lean democrats and republican partisans), can vote for the republican candidates R^- and R^+. As a result, R^- and R^+ can be nominated and R^- becomes elected. Finally, nor R^- or R^+ have no incentives to withdraw from the contest.

Suppose that $\succ^m = \succ^2_{D^-}$ and that $\mathcal{C} = \left\{D^+, D_1^-, D_2^-, D_3^-, R^-, R^+\right\}$. By Claim 2 in Lemma 3, when there are four candidates, the only weakly dominated strategies for lean democratic voters are voting for D^+ or R^+ but, voting for these two candidates, D^+ and R^+, is not weakly dominated for any other voter in the primary stage. For the proposed set of candidates, voting for D^+, R^+ is also weakly dominated for lean democratic voters. If lean democrats split their vote among the moderated candidates, the nominees can be D^+ and R^-. Hence, candidate R^- can be elected. We show that this electoral outcome can be immune to the strategic exit of candidates. If D^+ withdraws, for all the democratic voters (which are a majority of voters) voting for R^-, R^+ is a weakly dominated strategy. Then, if democratic voters split their vote among the candidates D_1^-, D_2^-, D_3^-, and all the republican candidates vote for either R^- or R^+, then R^- and R^+ can pass to the general election and still, R^- can be elected. If any of the moderated democratic candidates D^- withdraws, lean democratic voters can split their vote among the candidates D_1^-, D_2^- or R^-, the strong or weak democrats can vote for D^+ and republican partisans, can vote for the republican candidate R^-. As a result, R^- and D^+ can be nominated and R^- becomes elected. Finally, neither R^- nor R^+ have no incentives to withdraw from the contest. ■

[33] Even when $\mathcal{C} = \left\{D^+, R^-, R_1^+, R_2^+\right\}$, we can derive a similar result.

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