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Voting by Conformity

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Abstract: A group of agents has to decide whether to accept or reject a proposal. Agents vote in favor or against the proposal and, if the number of agents in favor is greater to certain quota, the proposal is accepted. The *socially optimal decision* is the one adopted when all agents vote truthfully. Conformist agents vote based not only on their opinion but also on the vote of other agents. Independent agents only care about their opinion. If all agents are conformists and vote simultaneously, for any quota there are undominated Nash equilibria where the socially optimal decision is not obtained. Next, we provide the number of independents needed for the socially optimal decision to be obtained in any equilibria. It depends on the total number of agents, the quota and the conformity measure. If agents vote sequentially, the socially optimal decision is obtained in any subgame perfect Nash equilibrium.

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"Sooner or later if a guy sits in the General Assembly for ten years, I think it is crazy not to think that he is gonna make at least one judgement on, maybe, his principle...[But] what good is it for me to sit there and vote what I feel would be my principle - in terms of the philosophy that I would have on how government ought to be run relating to an issue - and I voted against my constituency and voted my political philosophy, and then still when they took the tally, I was still on the losing side?...When you are really in a position where you can make it happen, then it would be rewarding enough to say, "I'll see you guys later; beat me in an election!" I don't care whatever it is, that is where it makes it worthwhile. Otherwise you are crazy, in my estimation."
A pro-Equal Rights Amendment legislator in *Why we lost the ERA,* by Jane J. Mansbridge (p. 162).

1 Introduction

A group of agents has to decide whether to accept or to reject a proposal. Each agent is either in favor or against the proposal (we suppose no agent is indifferent). Agents vote in favor or against the proposal and if the number of agents in favor is greater to certain quota, the proposal is accepted. In this simple context, the decision adopted when all agents vote according to their true opinion is called the *socially optimal decision.* However, it is not unusual to find situations where agents are not truthful when the decision does not depend on their votes. In fact, in these situations, the vote of the agents can be guided by their desire to coincide with the vote of certain number of agents instead of by their true opinion (as our pro-Equal Rights Amendment legislator from the initial quote, these agents will make the same reasoning when voting: why confront others, voting following my philosophy if I cannot obtain any gain doing so?). The tendency of agents to adapt their vote to the vote of other agents is known as *conformity.* We call *minimal conformity* to the minimal number of agents that voters want to coincide with in situations where their vote does not determine the decision. This minimal conformity number can take any value from one to n, the number of agents.

In this paper, we study the consequences that agents vote considering not only their true opinion about the proposal but also the vote of the rest of the voters. We refer to these agents as *conformists.* In contrast, when voters only consider their opinion when voting, we call them *independents.*

First, we consider a situation in which agents vote simultaneously. When all agents are independents, the unique equilibrium is announcing their true opinion and the socially optimal decision is obtained. The question that arises is: when all agents are conformists, announcing the true opinion is also the unique equilibrium strategy[1]? Do new equilibrium strategies emerge? In such a case, we check whether the decision associated to any equilibrium when all agents are conformists coincide with the socially optimal decision.

When all agents are conformists, we show that for any required quota to accept the proposal, there are undominated Nash equilibria where the decision does not coincide with

[1] In the simultaneous version of the game the equilibrium concept that we use is weakly undominated Nash equilbirum while in the sequential version it is the subgame perfect Nash equilibrium.

the socially optimal decision. This result holds for any minimal conformity. Surprisingly, for certain quotas, announcing the true opinion is not even a strategy equilibrium. Our next step is to ask ourselves whether there is a number of independents which makes that the decision adopted in every equilibrium coincides with the socially optimal decision. The answer is yes. We show that the minimum number of independents depends on the total number of agents, the required quota to accept the proposal and the minimal conformity. This allow us to make a double exercise of comparative static analysis: compare the different quotas in terms of the minimal number of independents required to obtain the socially optimal decision or given certain quota, study how the minimal number of independents evolve as the minimal conformity changes. When we compare the required quotas to accept the proposal, we fix the minimal conformity and the total number of agents and find that the less demanding quota in terms of independents is unanimity to accept the proposal. The next less demanding quota is the majority. After majority, the greater is the quota, the greater the required minimal number of independents. To study the performance of each quota when the minimal conformity changes, given the number of agents, n, we find that the minimum number of independents decreases as the minimal conformity moves away from $\frac{n}{2}$.

In the corporate world, we find some examples in which the regulator makes the presence of independent agents explicit and essential when taking decisions. One example is the Dodd-Frank law[2], passed by the US Congress in 2010. We mention some institutions affected by this law. For the Remuneration Committees, the Dodd-Frank establishes more requirements when hiring external consultants in retributive matters. In the case of credit rating agencies, the Dodd-Frank requires that at least half of the members of the Nationally Recognize Statistical Ratings Organizations boards (NRSROB) be independent, with no financial stake in credit ratings. For compensation committees, the Dodd-Frank requires including only independent directors who have authority to hire compensation consultants in order to strengthen their independence from the executives they are rewarding or punishing.

Finally, we consider a situation in which agents vote sequentially. We show that for any required quota to accept the proposal, in any subgame perfect Nash equilibrium, the decision is socially optimal regardless of the number of conformist and independent agents, the minimal conformity and the sequence in which agents vote.

Next section contains a revision of the literature regarding the conformity phenomenon. In Section 3, we present the model and a basic result using undominated Nash equilibrium. In Section 4, we determine the number of independent agents that guarantees the socially optimal decision. In Section 5, we study the sequential version of the problem. Some concluding remarks are given in Section 6. Appendix A and B present the proofs of the results in Section 4 and Section 5, respectively.

2 Previous Research

Social psychologists start to study conformity in the 1930s. Asch (1951), conducts a conformity experiment in the laboratory to show the degree to which the own individual's vote

[2] A brief summary of the Dodd-Frank law can be found in http://www.banking.senate.gov/public/_files/070110_Dodd_Frank_Wall_Street_Reform_comprehensive_summary_Final.pdf

could be persuaded by the vote of a majority within a group. During the experiment, a group of agents is asked, one at a time, to match a line of a specific length to one of three lines (the answer being obvious). Among the agents, only one participant is not an accomplice of the researcher. The aim is to see whether this agent would vote in the same way as the rest of the members of the group, despite it being the wrong answer. Results show that, on average, about a third of the tested subjects conform to the wrong judgement. Deutsch and Gerard (1955) shows that the conformity effect is weaker when agents' reports are secret, although they observe that even in anonymous choices several individuals conform to the wrong choice of the others (see also Hogg and Vaughan, 2009). Besides, Deutsch and Gerard (1955) identifies two types of social influence: informational and normative. The former refers to updating an opinion taking into account others' opinions, whereas the latter describes the behavior of stating an opinion that fits with the group choice.

The agents of our model do have a clear and defined opinion, so we limit our study to the normative version of conformity. In fact, we introduce conformity in the preferences of the agents by considering that agents want their message to coincide with the messages of other agents. Dutta and Sen (2012) also defines agents' preferences based on the decision and the messages of the rest of the agents. In their model, agents strictly prefer to report the "true" state rather than a "false" state when reporting the former leads to an outcome which is at least as preferred to the outcome obtained when reporting the latter. Then, their agents are what they call "partially honest" agents. On the contrary, our agents are not necessarily partially honest. A conformist agent strictly prefers to conform to the opinion of some reference groups when her message does not have any influence on the decision, regardless of whether her message corresponds with her true opinion or not.[3]

In the literature, there are many papers analyzing conformity both from a normative and an informational perspective, and assuming that conformity is either exogenous or endogenous. Among the normative studies, Bernheim (1994) analyzes a model of social interaction where agents' preferences are based on two characteristics: an "intrinsic" utility (derived from consumption), and status (popularity, esteem or respect within a group). Status is then taken as an endogenous conformity measure. He shows that when status is sufficiently important relative to intrinsic utility, many individuals conform to a single, homogeneous standard of behavior, despite heterogeneous preferences. Bernheim (1994) also finds that small departures from the norm can seriously impair agents' popularity. Besides, the author identifies the role of independent agents, as he finds a group of agents whose preferences are extreme enough to refuse to conform.

Luzzati (1999) deals with the problem of how to model social influence. It models conformity exogenously within the standard theoretical framework for the issue of voluntary contributions, even though he acknowledges that conformism can be partially endogenised within economic models (e.g. conformism as a lack of information). Luzzati (1999) introduces a penalty term to the traditional utility function that depends on the distance between the individual choice and the "prevailing social choice". According to Luzzati (1999), what social psychology rather seems to suggest is that agents, although they sometimes consciously

[3] We acknowledge that it could be very natural to think that when agents have different groups to conform to, they prefer to conform to those whose message coincide with their opinion. Our results hold under this assumption.

use conformism as a strategy, are more truly (unconsciously) conformist.

Herrera and Martinelli (2006) departs from the idea that participation in elections is a group activity. Herrera and Martinelli (2006) borrows from Glaeser et al. (1996) the fact that most citizens imitate the behavior of a group to which they belong, while some act independently. Herrera and Martinelli (2006) also considers two type of agents in their model: conformist and independent agents. Herrera and Martinelli (2006) presents a model in which leaders are interested in influencing citizens and, therefore, the election outcome. It uses a type of conformity that is called "identification" to study the voter turnout and the winning margin depending on the strength or weakness of social interactions.

Among the informational studies, Rivas and Rodríguez-Alvarez (2014) also considers different types of agents, being the objective voters what we have categorized as independents and, as Herrera and Martinelli (2006), distinguishes between leaders and followers (conformists). Rivas and Rodríguez-Alvarez (2014) shows that the introduction of a leader affects information revealed by followers, who misreport the information to conform to the leader. Buechel et al. (2015) studies a model of opinion formation in a social network framework including leaders, conformist and honest agents. As in our model, agents express their decision according to their true opinions and their preferences for conformity. Buechel et al. (2015) obtains that reducing prominence of individuals increases the accuracy of information aggregation.

In the literature, there are other papers studying voting procedures involving different quotas. Maggi and Morelli (2006) presents a model with a self-enforcing voting system. It concludes that unanimity is the optimal system if there is no external enforcement and majority rule is the ex-ante efficient rule. Buchanan and Tullock (1962) also supports the use of unanimity. The idea of considering other quotas apart from simple majority and unanimity rule appears also in Feddersen and Pesendorfer (1998), which suggest to combine a super-majority rule with a larger jury for cases where they want to reduce the probability of convicting the innocent.

3 The model and the basic result

Let $N = \{1, ..., n\}$ be any finite set of agents. Let capital letters $C, S \subset N$ denote subsets of agents while lower case letters c, s denote their cardinality. This group of agents have to decide whether to accept or to reject a proposal. We refer to the **true opinion** of agent $i \in N$ as $t_i \in \{0, 1\}$, where $t_i = 0$ stands for agent i rejecting the proposal and $t_i = 1$ for accepting the proposal. Let $t = (t_1, ..., t_n) \in \{0, 1\}^n$ be a list of true opinions. True opinions will be denoted in two possible ways. When they are part of a given list of opinions, we will use the notation $t_i \in \{0, 1\}$ to denote that this is the true opinion of agent i in the profile. In other cases, we want to have a name for a given true opinion, and we will use superscripts, i.e. t_i^0 stands for agent $i \in N$ against the proposal. We also write t_C^1, and t_C^0, when $t_i = 1$ and $t_i = 0$, respectively, for any $i \in C$ and $C \subseteq N$, $t^1 = (t_1^1, ..., t_n^1)$ when all agents prefer 1 to 0 and $t^0 = (t_1^0, ..., t_n^0)$ when all agents prefer 0 to 1.

In order to accept or reject the proposal, we ask agents to vote in favor or against such a proposal. Therefore, agents will be asked to announce a message in this sense.

A profile of messages is denoted by $m \in M$ where M is the **set of messages**. For any agent $i \in N$ and any profile of messages $m \in M$, let m_i denote the message of agent i and $m_{-i} \in M_{-i} = \times_{j \in N \setminus \{i\}} M_j$ the messages of all agents except i.

In most of the paper, we assume that agents vote simultaneously[4]. Then, $M_i = \{0, 1\}$ is the set of messages for agent $i \in N$, where $m_i = 0$ means that agent i votes against the proposal and $m_i = 1$ that agent i votes in favor of the proposal. As for the list of opinions, profiles of messages will be denoted in two possible ways. When they are part of a given profile of messages, we use the notation $m_i \in M_i$ to denote that this is the message of agent i in the profile. In other cases, we want to have a name for a given message, and we will use superscripts, i.e. m_i^0 stands for agent $i \in N$ voting against the proposal. We will also write m_S^1 when $m_i = 1$ for all $i \in S$ and $S \subseteq N$, $m^1 = (m_1^1, ..., m_n^1)$ and $m^0 = (m_1^0, ..., m_n^0)$.

The proposal is accepted if a given number of agents are in favor of the proposal and is rejected otherwise. Let $q \in \{1, 2, .., n\}$ be the required number of agents for the proposal to be accepted. Therefore, q is a mapping from M to $\{0, 1\}$. Given any quota q, the **socially optimal decision** would be the decision obtained as a result of applying q to the profile of messages m where $m_i = t_i$ for any $i \in N$, (i.e., the decision obtained when all agents are truthful and the number of agents needed to accept the proposal is q).

The description of the preferences is more complicated in our case than in the standard case. The complication arises since the preferences of our agents depend not only on the decision taken but also on the profile of messages announced by the agents. Let $\{0, 1\} \times M$ be the set of alternatives, and $(x; m) \in \{0, 1\} \times M$ be an alternative where x stands for the decision taken and m for the profile of messages.

Example 1 *Let $N = \{1, 2, 3\}$. Then $\{0, 1\} \times M = \{(0; 0, 0, 0),\ (0; 0, 0, 1),\ (0; 1, 1, 0),$ $(0; 1, 1, 1), (0; 0, 1, 0), (0; 1, 0, 0), (0; 1, 0, 1), (0; 0, 1, 1), (1; 0, 0, 0), (1; 0, 0, 1), (1; 1, 1, 0), (1; 1, 1, 1),$ $(1; 0, 1, 0),\ (1; 1, 0, 0),\ (1; 1, 0, 1),\ (1; 0, 1, 1)\}$.*

Let $\mathcal{R}_i$ be the set of all possible preference relations for agent $i \in N$ defined on $\{0, 1\} \times M$ satisfying reflexivity, transitivity and completeness. Let $R_i \in \mathcal{R}_i$ be a preference relation for agent $i \in N$, and $R = (R_1, ..., R_n) \in \times_{i \in N} \mathcal{R}_i$ be an admissible preference profile.

We introduce two properties regarding agents' preference relations. We call the first property, *selfishness*. We say that an agent's preference relation satisfies selfishness if, when comparing two different pairs of alternatives, she prefers that alternative where the decision matches her opinion, whatever the decision is.

Definition 1 *Agent i's preference relation $R_i \in \mathcal{R}_i$ is **selfish** if for any true opinion, $t_i \in \{0, 1\}$ and for any $(x; m), (x'; m') \in \{0, 1\} \times M$ such that $x = t_i$ and $x' \neq t_i$, we have that $(x; m) P_i (x'; m')$.*

Example 2 *Let $N = \{1, 2, 3\}$. Let $(x; m)$ and $(x'; m') \in \{0, 1\} \times M$ be such that $x = 0$ and $x' = 1$. Let $t_i = 1$ for some $i \in N$. If $R_i \in \mathcal{R}_i$ satisfies selfishness we have that, $(x'; m') P_i (x; m)$.*

[4] Section 5 analyzes a sequential version of the game.

In order to introduce the second property, we need some additional notation. We first introduce the idea of a committee structure. A committee structure for a voter consists of all subsets of voters that she takes into account. We assume that if a subset of agents belongs to an agent's committee structure, then any set with equal or higher cardinality also belongs to her committee structure. Admittedly, alternative committee structures can be contemplated but we think that the one we propose is quite canonical[5].

Definition 2 *A committee structure for agent i, W_i^h is a set of subsets of agents satisfying:*
- For any $C \in W_i^h$, $i \in C$ and
- If $C \in W_i^h$ and C' is such that $|C'| \geq |C|$ then $C' \in W_i^h$ and
- h is the cardinality of H, where $|C| \geq |H|$ for any $C \in W_i^h$.

Example 3 *Let $N = \{1, 2, 3, 4\}$. $W_1^3 = \{\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\}$ and $W_1^4 = \{\{1, 2, 3, 4\}\}$ are two admissible committee structures for agent* 1*.*

We call the second property, *conformity relative to a committee structure*. Given W_i^h, we say that an agent's preference relation satisfies *conformity relative to* W_i^h if, when comparing two different pairs of alternatives with identical decision, she prefers the alternative where the number of agents with the same message as her is greater or equal to h. We refer to h as the **minimal conformity**.

Before presenting this concept, we define when two agents conform.

Definition 3 *For any $m \in \times_{i \in N} M_i$, any $i, j \in N$, we say that agent i* ***conforms*** *to agent j if and only if $m_i = m_j$.*

Definition 4 *Given a committee structure W_i^h, agent i's preference relation satisfies* ***conformity relative to*** *W_i^h if, for any true opinion $t_i \in \{0, 1\}$, any $(x; m), (x; m') \in \{0, 1\} \times M$, we have $(x; m) P_i (x; m')$ if and only if $\{j \in N :$ such that $m_i = m_j\} \in W_i^h$, but $\{j \in N :$ such that $m'_i = m'_j\} \notin W_i^h$.*

The following example clarifies these concepts for the case of three agents. We present two committee structures for agent 1, W_1^2 with minimal conformity $h = 2$ and W_1^3 with minimal conformity $h = 3$. When the preference relation of agent 1 satisfies conformity relative to W_1^2, agent 1 has to conform at least to agent 2 or agent 3. If the preference relation of agent 1 satisfies conformity relative to W_1^3, agent 1 has to conform to agent 2 and agent 3.

[5] We thank Kenneth Shepsle for proposing us an interesting alternative committee structure. An agent i may want to conform to agent j and, at the same time, he does not want to conform to agent k. Note that in this case the identity of the agents play an important role and there can be many ways to define a committee structure.

Example 4 *Let $N = \{1, 2, 3\}$ and $(x; m)$ and $(x; m') \in \{0, 1\} \times M$ be such that $m = (0, 1, 0)$ and $m' = (1, 0, 0)$.*
For committee structure $W_1^2 = \{\{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$, we have that $\{j \in N : \text{such that } m_j = m_1\} = \{1, 3\}$, $\{j \in N : \text{such that } m'_j = m'_1\} = \{1\}$, $\{1\} \notin W_1^2$ and $\{1, 3\} \in W_1^2$. If R_1 satisfies conformity relative to W_1^2 we have that, $(x; m)P_1(x; m')$ for any $t_1 \in \{0, 1\}$.
For committee structure $W_1^3 = \{\{1, 2, 3\}\}$, we have that $\{j \in N : \text{such that } m_j = m_1\} = \{1, 3\}$ and $\{j \in N : \text{such that } m'_j = m'_1\} = \{1\}$, $\{1, 3\} \notin W_1^3$ and $\{1\} \notin W_1^3$. If R_1 satisfies conformity relative to W_1^3 we have that, $(x; m')I_1(x; m)$ for any $t_1 \in \{0, 1\}$.

Once defined conformity relative to a committee structure, we are able to distinguish among a conformist and an independent agent. A conformist agent takes into account at least the message of one more agent before voting. However, an independent agent dedicates no attention to what the rest of the agents are voting for and, therefore, she is only considering her own opinion when voting.

Definition 5 *Given W_i^h, agent $i \in N$ is **conformist** if the minimal conformity is greater to one, that is $h > 1$.*

Definition 6 *Given W_i^h, agent $i \in N$ is **independent** if the minimal conformity is equal to one, that is $h = 1$.*

In our model, there can be two set of agents, the set of independent agents and the set of conformist agents. We assume through the paper that the minimal conformity is the same for all the agents belonging to the same group. Let $D = \{i \in N : h = 1\}$ be the set of independent agents where d refers to its cardinality. Henceforth, when we refer to a list of committee structures we will write $W = (W_D^1, W_{N \setminus D}^h)$ to emphasize the partition between the set of independent and conformist agents. Thus, W_D^1 stands for the committee structure of the independent agents and $W_{N \setminus D}^h$ refers to the committee structure of the conformist agents.

Given $W = (W_D^1, W_{N \setminus D}^h)$, a list of true opinions, $t = (t_1, ..., t_n) \in \{0, 1\}^n$ observed by all agents, $\mathcal{R}_i^h$ denote the class of preference relations satisfying selfishness and conformity relative to W_i^h. When a preference relation is part of a given profile of preferences, we use the notation $R_i \in \mathcal{R}_i^h$ to denote that this is the preference relation of agent i in the profile. In other cases, since an agent has only two admissible preference relations, as we have seen in the previous example, we want to have a name for a given preference relation, and we use superscripts, i.e. R_i^0 and R_i^1 corresponding to the case in which $t_i = 0$ and $t_i = 1$, respectively. We will also write R_C^1 when $R_i = R_i^1$ for all $i \in C$ and $C \subseteq N$, $R^1 = (R_1^1, ..., R_n^1)$ and $R^0 = (R_1^0, ..., R_n^0)$.

We now illustrate that the properties of selfishness and conformity completely determine the set of admissible preferences of the agents. We also want to stress, as evidenced in the following example that, when comparing alternatives, agents first look at the decision and, only after, look at the vote of the agents they consider relevant. Agents' preferences are lexicographic.

Example 5 *Let $N = \{1, 2, 3\}$. Given $W_1^2 = \{\{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$, the set of admissible preference relations for agent 1 satisfying selfishness and conformity relative to the committee structure W_1^2 are given in the following table:*

R_1^0
$\{(0; 0, 0, 0), (0; 0, 0, 1), (0; 1, 1, 0), (0; 1, 1, 1), (0; 0, 1, 0), (0; 1, 0, 1)\}$
$\{(0; 1, 0, 0), (0; 0, 1, 1)\}$
$\{(1; 0, 0, 0), (1; 0, 0, 1), (1; 1, 1, 0), (1; 1, 1, 1), (1; 0, 1, 0), (1; 1, 0, 1)\}$
$\{(1; 1, 0, 0), (1; 0, 1, 1)\}$

R_1^1
$\{(1; 0, 0, 0), (1; 0, 0, 1), (1; 1, 1, 0), (1; 1, 1, 1), (1; 0, 1, 0), (1; 1, 0, 1)\}$
$\{(1; 1, 0, 0), (1; 0, 1, 1)\}$
$\{(0; 0, 0, 0), (0; 0, 0, 1), (0; 1, 1, 0), (0; 1, 1, 1), (0; 0, 1, 0), (0; 1, 0, 1)\}$
$\{(0; 1, 0, 0), (0; 0, 1, 1)\}$

The first table refers to the case in which agent 1 is against the proposal (R_1^0) and the second table refers to the case in which agent 1 is in favor of the proposal (R_1^1). Therefore, the committee structure and the true opinion of an agent completely determine her preference relation. Note that given the committee structure, there are only two admissible preference relations, each of them corresponding to the two possible opinions of the agent.

Definition 7 *For any h and any $q \in \{1, ..., n\}$, $m \in M$ is a* ***weakly undominated Nash equilibrium*** *of (M, q) at $R \in \times_{i \in N} \mathcal{R}_i^h$ if for all $i \in N$, (1) m_i is not weakly dominated and (2) for all $m_i' \in M_i$, $g(m_i, m_{-i}) R_i g(m_i', m_{-i})$.*

Given $W = (W_D^1, W_{N \setminus D}^h)$ and $R \in \times_{i \in N} \mathcal{R}_i^h$, $UN((M, q), R, W)$ is the set of weakly undominated Nash equilibria of (M, q) at R.

If all agents are conformists, we show that for any required quota to accept the proposal, there are undominated Nash equilibria where the decision does not coincide with the socially optimal decision.

Theorem 1. *Let $W = (W_D^1, W_{N \setminus D}^h)$ be such that $D = \emptyset$. Then, for any $q \in \{1, ..., n\}$,* there exists undominated Nash equilibria not yielding to the socially optimal decision.

Proof. Let $W = \left(W_D^1, W_{N \setminus D}^h\right)$ be such that $D = \emptyset$. In order to prove the result, we present the following three cases that apply to $q = 1$, $q = n$ and $q \in \{2, ..., n - 1\}$, respectively.

Case 1. $\mathbf{q = 1}$. If all agents prefer to reject the proposal, all agents voting in favor of the proposal is a weakly undominated Nash equilibrium.

Let $t = t^0$ and $q = 1$. Note that $q(m^1) = 1 = q(m_i^0, m_{-i}^1)$ for any $i \in N$. Since all agents are conformists, $q(m^1) P_i^0 q(m_i^0, m_{-i}^1)$ for any $i \in N$. Since $q(m^1) \neq 0 = q(t^0)$ and $m^1 \in UN((M, q), R, W)$, the socially optimal decision is not always obtained[6].

Case 2. $\mathbf{q = n}$. If all agents prefer to accept the proposal, all agents voting against the

[6] Note that for any profile of preferences where not all the agents reject the proposal, the weakly undominated Nash equilibrium yield to the socially optimal decision.

proposal is a weakly undominated Nash equilibrium.
Let $t = t^1$ and $q = n$. Note that $q(m^0) = 0 = q(m_i^1, m_{-i}^0)$ for any $i \in N$. Since all agents are conformists, $q(m^0)P_i^1 q(m_i^1, m_{-i}^0)$ for any $i \in N$. Since $q(m^0) \neq 1 = q(t^1)$ and $m^0 \in UN((M,q),R,W)$, the socially optimal decision is not always obtained[7].
Case 3. $\mathbf{q \in \{2,...,n-1\}}$**.** For any profile of preferences, all agents voting in favor or against the proposal is a weakly undominated Nash equilibrium.
Let any $t \in \{0,1\}^n$, and any $q \in \{2,...,n-1\}$. Note that $q(m^0) = 0 = q(m_i^1, m_{-i}^0)$ for any $i \in N$. Since agents are conformists, $q(m^0)P_i^t q(m_i^1, m_{-i}^0)$ for any $i \in N$. Therefore, $m^0 \in UN((M,q),R,W)$ for any $t \in \{0,1\}^n$. Note also that $q(m^1) = 1 = q(m_i^0, m_{-i}^1)$ for any $i \in N$. Since agents are conformists, $q(m^1)P_i^t q(m_i^0, m_{-i}^1)$ for any $i \in N$. Therefore, $m^1 \in UN((M,q),R,W)$ for any $t \in \{0,1\}^n$. Since $q(m^1) \neq 0 = q(t^0)$, $q(m^0) \neq 1 = q(t^1)$, $m^1, m^0 \in UN((M,q),R,W)$ for any $q \in \{2,...,n-1\}$ and the socially optimal decision is not always obtained. ■

Theorem 1 offers a negative result[8]. It shows that asking the voters about their opinions may not lead us to the socially optimal decision when all agents are conformists. Besides, there are weakly undominated Nash equilibrium driving to alternatives that are Pareto dominated. In the case of any quota different to n, when all agents are against the proposal, all agents voting in favor of the proposal is a weakly undominated Nash equilibrium. However, having all agents voting against the proposal produces an alternative where all of them are strictly better off. In the case of quota n, when all agents prefer the proposal, all agents voting against the proposal is a weakly undominated Nash equilibrium. However, all agents are strictly better off voting in favor of the proposal.

4 Having independent and conformist agents

In the previous section, we have shown that if all agents are conformists and for any required quota to accept the proposal, there are undominated Nash equilibria where the decision is not the socially optimal. By contrast, it is well known that if all agents are independents and for any quota, the unique weakly undominated Nash equilibrium is all agents telling their true opinion and the socially optimal decision obtains. Then, it is almost imperative to question whether there is a number of independents which makes that the decision adopted in every equilibrium coincide with the socially optimal one. There are some examples in real life, in which the regulator makes the presence of independent agents explicit and essential when taking decisions.[9]

We now present a collection of theorems where we show what is the number of independent agents, d, guaranteeing that the socially optimal decision is obtained in any weakly undominated Nash equilibrium for any number of agents, any minimal conformity, and any

[7]Note that for any profile of preferences where not all the agents are in favor of the proposal, the weakly undominated Nash equilibrium yield to the socially optimal decision.

[8]This negative result also holds if we consider different committee structures for each of the agents and/or more general committee structures in which the identity of the other agents play a role.

[9]In the Introduction we mention the Dodd-Frank law and how the presence of independent agents is made explicit in some institutions.

quota. We will assume that independent agents vote according to their true opinion and that conformist agents know the number of independents and the orientation of their vote.

In Theorem 2, we provide the number of independent agents for $q = 1$, $q = n$, and any minimal conformity, $h \in \{2, .., n\}$. The strategy of proof is the following. For $q = 1$, the only situation where the socially optimal decision may be not obtained in equilibrium is when all agents are against the proposal. If the number of independents agents is $h - 1$, by telling the truth any agent conforms to h agents and the truth is a weakly dominant strategy. If $n - (h - 1)$ is lower than $h - 1$ and the number of independents is $n - (h - 1)$, by telling the truth any agent conforms to h agents for some voting configurations and not telling the truth would never allow to conform to h agents. Similarly reasoning applies for $q = n$.

Theorem 2 ($\mathbf{q} \in \{\mathbf{1}, \mathbf{n}\}$). *Given any $W = (W_D^1, W_{N\setminus D}^h)$, for $q \in \{1, n\}$, any undominated Nash equilibrium yields to the socially optimal decision if*

$$d = \min\{h - 1, n + 1 - h\}.$$

Proof. See Appendix A. ■

In Theorem 3, we provide the number of independent agents for $q \in \{2, .., n - 1\}$, and any minimal conformity, $h \in \{2, .., n\}$. The strategy of proof is similar to that of the proof of Theorem 2. The idea is to obtain the number of independents for the truth to be a weakly dominant strategy. But, in this case, that number depends also on the quota.

Theorem 3 ($\mathbf{q} \in \{\mathbf{2}, .., \mathbf{n - 1}\}$). *Given any $W = (W_D^1, W_{N\setminus D}^h)$, for $q \in \{2, ..., n - 1\}$, any undominated Nash equilibrium yields to the socially optimal decision if*

$$d = \min\{n - 1, \max\{q - 1, n - q\} + \min\{h - 1, n + 1 - h\}\}.$$

Proof. See Appendix A. ■

As a Corollary of the above Theorems, we obtain that for any number of voters and any minimal conformity, the number of independent agents is the same for any two quotas symmetric with respect to $\frac{n+1}{2}$. Before state the corollary, we present an example.

Example 6 *In the following table, we provide the number of independent agents guaranteeing the socially optimal decision when $n = 12$ and the minimal conformity is either $h = 9$, or $h = 11$ or $h = 12$. Take, for instance, $h = 11$. Then for $q = \{1, 12\}$, we have that $\min\{h-1, n+1-h\} = \min\{10, 2\}$ and $d = 2$. For $q = \{5, 8\}$, we have that $\left|q - \left(\frac{n+1}{2}\right)\right| = 1.5$, $\min\{n - 1, \max\{q - 1, n - q\}+ \min\{h - 1, n + 1 - h\}\} = \min\{11, \max\{4, 7\} + \min\{10, 2\} = \min\{11, 9\}$ and $d = 9$.*

q/h	$h=9$	$h=11$	$h=12$
$q=\{1,12\}$	4	2	1
$q=\{6,7\}$	10	8	7
$q=\{5,8\}$	11	9	8
$q=\{4,9\}$	11	10	9
$q=\{3,10\}$	11	11	10
$q=\{2,11\}$	11	11	11

Corollary 1. *For any n, any $h \in [2,n]$, and any $q < q'$ such that $q' = n - q + 1$, we have that the number of independent agents guaranteeing the socially optimal decision under* q and q' *is the same.*
Proof. See Appendix A. ■

From the above Theorems, Corollary 2 states that for any number of voters and any quota, the number of independent agents is the same for any minimal conformity symmetric with respect to $\frac{n}{2}+1$. We also present an example of this.

Example 7 *In the following table, we provide the number of independent agents guaranteeing the socially optimal decision when $n = 12$ and any quota from 7 to 12. Take, for instance, $q = 7$. Then for $h = \{2, 12\}$, we have that $\left|h - \left(\frac{n}{2}+1\right)\right| = 5$, $\min\{n-1, \max\{q-1, n-q\}+ \min\{h-1, n+1-h\}\} = \min\{11, \max\{6,5\} + \min\{1,11\} = \min\{11,7\}$ and $d = 7$.*

$q\backslash h$	*Minimal Conformity,h*										
Quota q	2	3	4	5	6	7	8	9	10	11	12
$q=7$	7	8	9	10	11	11	11	10	9	8	7
$q=8$	8	9	10	11	11	11	11	11	10	9	8
$q=9$	9	10	11	11	11	11	11	11	11	10	9
$q=10$	10	11	11	11	11	11	11	11	11	11	10
$q=11$	11	11	11	11	11	11	11	11	11	11	11
$q=12$	1	2	3	4	5	6	5	4	3	2	1

Corollary 2. *For any n, any $q \in [1,n]$, and any $h < h'$ such that $h' = n + 2 - h$, we have that the number of independent agents guaranteeing the socially optimal decision under any q is the same for* h and h'.
Proof. See Appendix A. ■

The requirement of independent agents for each quota yielding to the socially optimal decision, allows us to provide a complete order of the quotas. Since finding independent agents may not be an easy task, we establish that a quota is better than other if it is less demanding in terms of independent agents. It turns out that the unanimous quota (either to accept or to reject the proposal) is the less demanding one. Interestingly, after unanimity, the less demanding quota is the majority.

From the above two Corollaries, we know that for any h, quotas are symmetric around $\frac{n+1}{2}$. In the following example, we make explicit these ideas for the case in which there are 12 agents.

Example 8 *Suppose that there are 12 agents. From the above two corollaries, w.l.o.g., we restrict to* $q = \{1, ..., 6\}$ *and* $h = \{2, ..., 7\}$. *We provide a picture in which we represent for each quota the number of independent agents guaranteeing the socially optimal decision in terms of the minimal conformity. Each quota appears in the picture with a different dotted line. For any* h, *the best quota is* $q = 1$. *If* $h = 2$, *the order is strict. If* $h = 6$, *any* $q = \{2, 3, 4, 5, 6\}$ *requires the same number of independent agents. And so on and so forth.*

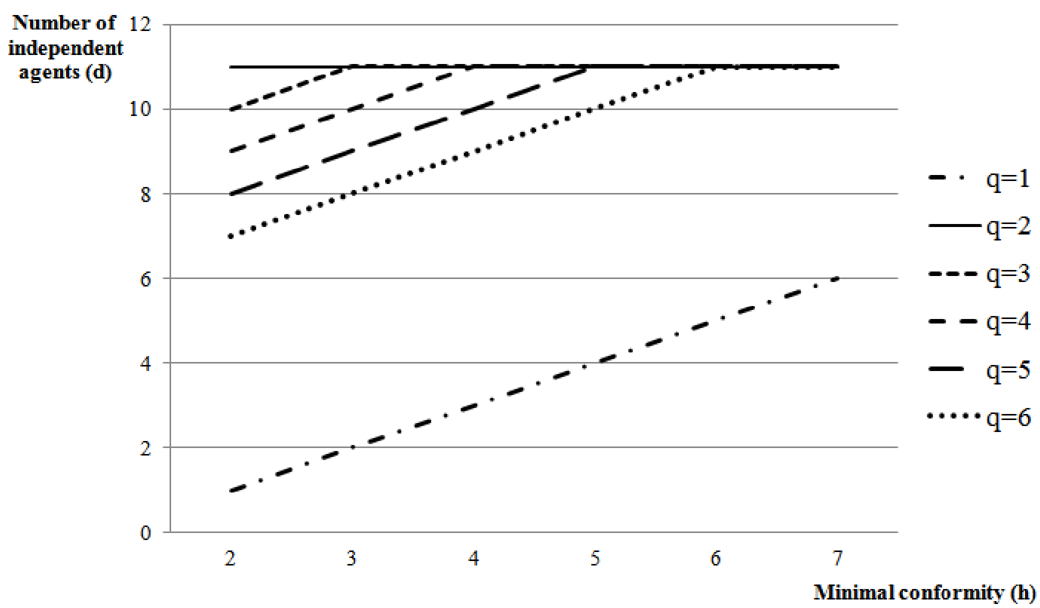

Figure 1: Representation of the number of independent agents for each $q = \{1, .., 6\}$ relative to the minimal conformity $h = \{2, ..., 7\}$.

Remark 1 *For any number of agents,* n, *and any* $h = \{2, .., n\}$, *there is a complete ordering of the quotas in terms of the number of independent agents guaranteeing the socially optimal decision. Moreover, unanimous quotas are the less demanding. For any* $q, q' \in \{2, .., n-1\}$, q *is weakly less demanding than* q' *if and only if* $\left|q - \frac{n+1}{2}\right| \leq \left|q' - \frac{n+1}{2}\right|$.

5 Sequential Voting

In this section, we study situations in which agents take turns when voting. Suppose that there is a fixed order of the agents indicating the sequence in which the agents vote and, when voting, each agent knows what preceding agents have voted for.

Let O be the set of all linear orderings of the agents and $o \in O$ be a particular ordering. In what follows, without loss of generality, we suppose that $o = \{1, 2, ..., n\}$, that is, agent 1 plays in the first stage, agent 2 plays in the second stage, and so on and so forth. Note that for any other ordering of the agents, say $o' \in O$, we can rename the agents and call agent 1 to the first agent in o', agent 2 to the second agent in o', etc.

Since agents vote following an ordering, each agent's message space consists of an action at each node in the stage that she plays. For instance, for any $i \in N$, $M_i = \{0,1\}^{2^{i-1}}$. In order to introduce conformity in this set-up, we define the following function $\alpha : \times_{i \in N} M_i \to \{0,1\}^n$, mapping each profile of messages, $m \in \times_{i \in N} M_i$, to the profile of actions, $\alpha(m) = (\alpha_1(m), ..., \alpha_n(m))$, where for each agent i, α_i is the action in m_i corresponding to the node in which agent i plays given m. We first define when two agents conform in this context.

Definition 8 *For any $m \in \times_{i \in N} M_j$, and any $i, j \in N$, we say that agent i* ***conforms*** *to agent j if and only if $\alpha_i(m) = \alpha_j(m)$.*

For any $R \in \mathcal{R}^h$ and any sequential game (M, q), let $G^{\{1,..,n\}} = ((M, q), R)$ be the game played by agents 1 to n. We now introduce the equilibrium concept that we use throughout this section.

Definition 9 *For any h, and any $q \in \{1, ..., n\}$, $m \in M$ is a* ***subgame perfect Nash equilibrium*** *of $G^{\{1,..,n\}}$ at $R \in \mathcal{R}^h$ if for all $i \in N$, m is a Nash equilibrium of every subgame of $G^{\{1,..,n\}}$.*

We refer to $G^{\{1,..,n\},q} = ((M, q), R)$ as a game for agents 1 to n, quota q, and $R \in \mathcal{R}^h$. Given game $G^{\{1,..,n\},q} = ((M, q), R)$, $SPN(G^{\{1,..,n\},q})$ is the set of *subgame perfect Nash equilibria* of (M, q) at R.

We now define when an agent is pivotal or not pivotal. These remain valid for any type of the agent $t \in \{0,1\}^n$ at any stage of the game $G^{\{1,..,n\},q}$.

Definition 10 *Agent $i \in N$ is* ***pivotal*** *relative to $m_{-i} \in M_{-i}$ and q if there exist $m_i, m'_i \in M_i$ such that $q(m_i, m_{-i}) \neq q(m'_i, m_{-i})$. Agent $i \in N$ is* ***not pivotal*** *relative to $m_{-i} \in M_{-i}$ and q if for any $m_i, m'_i \in M_i$ we have that $q(m_i, m_{-i}) = q(m'_i, m_{-i})$.*

In the following three theorems, we show that for any q, the socially optimal decision obtains in subgame perfect Nash Equilibrium at any (M, q). In Theorems 4 and 5, we offer a direct proof for the cases of $q = 1$ and $q = n$, respectively.

In each proof, we first analyze all preference profiles for which the socially optimal decision is to reject the proposal and then those for which the proposal is accepted. In both cases, we solve the game from stage n, where agent n votes, to stage 1. In each stage, we analyze the equilibrium strategies for each of the voters, who vote taking into account the actions of the voters who have voted before them. In equilibrium, each agent votes according to her true opinion when she is pivotal relative to what the rest of agents have previously voted for, and according to her minimal conformity when she is not pivotal. This completes the proof.

Theorem 4. Let $q = 1$. *For any number of agents n,* the socially optimal decision obtains *in any subgame perfect Nash Equilibrium on $\mathcal{R}^h$.*
Proof. See Appendix B. ■

Theorem 5. Let $q = n$. *For any number of agents n, the socially optimal decision obtains in any subgame perfect Nash Equilibrium on $\mathcal{R}^h$.*
Proof. See Appendix B. ■

Finally, Theorem 6 applies for quotas where $q \in \{2, .., n-1\}$. We introduce some additional notation. We denote as $G^{\{1,...,k\},q} = \left(\times_{i=1}^{k} M_i, q, \{R_i\}_{i=1}^{k}\right)$ a game consisting of agents 1 to k and quota q, where $R_i \in \mathcal{R}_i^h$ for any $i = \{1, ..., k\}$. The strategy of proof is to show that given the equilibrium strategy of the last agent in the sequence, the reduced game is either $G^{\{1,...,n-1\},q}$ if $t_n = 0$ or $G^{\{1,...,n-1\},q-1}$ if $t_n = 1$. First, we describe the equilibrium strategies of this last agent in two claims: a and b. Claim a applies when $t_n = 0$ and Claim b when $t_n = 1$. Note that the new last agent in the reduced game is agent $n-1$. Then, we show that applying iteratively this line of reasoning, we eventually end up in a game such that either $G^{\{1,...,k\},q}$ where $q = 1$ or $G^{\{1,...,k\},q}$ where $q = k$. Once we obtain one of these two reduced games, the proof finish applying either Theorem 4 when $G^{\{1,...,k\},q}$ and $q = 1$ or Theorem 5 when $G^{\{1,...,k\},q}$ and $q = k$.

Theorem 6 Let $q \in \{2, .., n-1\}$. *For any number of agents n, the socially optimal decision obtains in any subgame perfect Nash Equilibrium on $\mathcal{R}^h$.*
Proof. See Appendix B. ■

Note that the socially optimal decision obtains regardless of the minimal conformity, h. This implies that, under sequential voting, it does not depend on the number of independent agents as it is the case under simultaneous voting. In both models, conformity is present. Whereas under simultaneous voting, those equilibria yielding a not socially optimal decision are generated by the presence of conformity, under sequential voting, conformity does not affect the decision since agents vote selfishly in any situation they have the opportunity to do so. That is, when agents play strategies that are SPNE, they behave as if they were pivotal at nodes that are out of the equilibrium path.

Finally, solving the model sequentially, we find that the conformity phenomenon does not affect the results since we only obtain equilibria where the decision is the socially optimal one. This is due to agents, under sequential voting, reach some nodes in which they can be pivotal and that are unreachable when solving the game simultaneously. It could be said then that agents are pivotal "more times" when voting sequentially rather than simultaneously. This does not imply that conformity is not present, instead it only applies when the decision is already set. Therefore, a sequential voting model as, for instance, the one used in athletics competitions, can be an option to consider even with the presence of the conformity phenomenon and under an open ballot system, as it does derive the socially optimal decision.

6 Concluding Remarks

In this section, we compare secret versus open ballot in terms of conformity. In any democratic system, after elections are held, the support obtained by each option is made public. Therefore, the difference between open and secret ballot is that in the former, what each agent has voted for is made public whereas in the latter it is not. If agents only pay attention to the number of people they want to conform to, assuming open or secret ballot does not provide different results. Admittedly, when agents pay attention also to the identity of the people they want to conform to, assuming open or secret ballot makes a difference.

Finally, a word about the presence of independent agents when taking decisions. We have shown that when voters only take into account the number of agents to conform to, in order to mitigate the negative effect of conformity, introducing independent agents help. It is the case that in the corporate world, a great majority of companies normally include in their committees independent agents that contribute with an external and unbiased opinion to the decision making process. We ignore what motivates the companies to do so, however we do know that this helps in obtaining the true choice.

Appendix A

This appendix includes the proofs developed in Section 4. It includes the proof of Theorem 2 and Theorem 3. Additionally, proofs of Corollary 1 and Corollary 2 are also included.

Theorem 2. *Given any $W = (W^1_D, W^h_{N\backslash D})$, for $q \in \{1, n\}$, any undominated Nash equilibrium yields to the socially optimal decision if*

$$d = \min\{h - 1, n + 1 - h\}.$$

Proof. We distinguish two cases.
Case 1. Let $W = (W^1_D, W^h_{N\backslash D})$ and $q = 1$. From Case 1 in Theorem 1, if $t \neq t^0$, the socially optimal decision is obtained in equilibrium. Therefore, the only case that is left is when $t = t^0$. We now show that if $d = \min\{h - 1, n + 1 - h\}$, the socially optimal decision is also obtained in equilibrium. To do that, we show that m^1_i is weakly dominated by m^0_i for any $i \in N\backslash D$. By assumption, any conformist agent knows that there are d independent agents that vote for 0. We distinguish two cases:
Case 1a. $n + 1 - h > h - 1$. Let $d = h - 1$. Take any agent $i \in N\backslash D$. For any $m = (m^1_S, m^0_{N\backslash S}) \in UN((M, q), R^0, W)$ we have that $D \subseteq N\backslash S$. Since $n - s \geq h - 1$ and by conformity of agent $i \in S$ relative to W^h_i, we have that $q(m^0_i, m^1_{S\backslash\{i\}}, m^0_{N\backslash S}) R^0_i q(m^1_S, m^0_{N\backslash S})$. By selfishness, $q(m^0_i, m^0_{-i}) P^0_i q(m^1_i, m^0_{-i})$ and m^1_i is weakly dominated by m^0_i for any $i \in N\backslash D$. Therefore, the result follows.
Finally, if $d < h - 1$ we show that there exists $m \in UN((M, q), R^0, W)$ such that $q(m) = 1$. Let $m = (m^1_S, m^0_{N\backslash S})$ where $S = N\backslash D$ and $N\backslash S = D$. Take $i \in S$, a conformist agent. By conformity relative to W^h_i and since $N\backslash S \cup \{i\} \notin W^h_i$, $q(m^1_S, m^0_{N\backslash S}) P^0_i q(m^0_i, m^1_{S\backslash\{i\}}, m^0_{N\backslash S})$.
Case 1b. $n + 1 - h \leq h - 1$. Let $d = n + 1 - h$. Take any agent $i \in N\backslash D$. For any

$m = (m_S^1, m_{N\backslash S}^0) \in UN((M,q), R^0, W)$ we have that $D \subseteq N\backslash S$. Since $n - s \geq n + 1 - h$ or equivalently, $s \leq h - 1$ and by conformity of agent $i \in S$ relative to W_i^h, we have that $q(m_i^0, m_{S\backslash\{i\}}^1, m_{N\backslash S}^0) R_i^0 q(m_S^1, m_{N\backslash S}^0)$. By selfishness, $q(m_i^0, m_{-i}^0) P_i^0 q(m_i^1, m_{-i}^0)$ and m_i^1 is weakly dominated by m_i^0 for any $i \in N\backslash D$. Therefore, the result follows.

Finally, if $d < n + 1 - h \leq h - 1$, we show that there exists $m \in UN((M,q), R^0, W)$ such that $q(m) = 1$. Let $m = (m_S^1, m_{N\backslash S}^0)$ where $S = N\backslash D$ and $N\backslash S = D$. Take $i \in S$, a conformist agent. By conformity relative to W_i^h and since $N\backslash S \cup \{i\} \notin W_i^h$, $q(m_S^1, m_{N\backslash S}^0) P_i^0 q(m_i^0, m_{S\backslash\{i\}}^1, m_{N\backslash S}^0)$.

Case 2. Let $W = (W_D^1, W_{N\backslash D}^h)$ and $q = n$. From Case 2 in Theorem 1, if $t \neq t^1$, the socially optimal decision is obtained in equilibrium. Therefore, the only case that is left is when $t = t^1$. To do that, we show that m_i^0 is weakly dominated by m_i^1 for any $i \in N\backslash D$. By assumption, any conformist agent knows that there are d independent agents that vote for 1. We now show that if $d = \min\{h-1, n+1-h\}$, the socially optimal decision is also obtained in equilibrium. We distinguish two cases:

Case 2a. $n + 1 - h > h - 1$. Let $d = h - 1$. Take any agent $i \in N\backslash D$. For any $m = (m_S^1, m_{N\backslash S}^0) \in UN((M,q), R^1, W)$ we have that $D \subseteq S$. Since $s \geq h - 1$ and by conformity of agent $i \in N\backslash S$ relative to W_i^h, we have that $q(m_i^1, m_S^1, m_{N\backslash S\cup\{i\}}^0) R_i^1 q(m_S^1, m_{-S}^0)$. By selfishness, $q(m_i^1, m_{-i}^1) P_i^1 q(m_i^0, m_{-i}^1)$ and m_i^0 is weakly dominated by m_i^1 for any $i \in N\backslash D$. Therefore, the result follows.

Finally, if $d < h - 1$ we show that there exists $m \in UN((M,q), R^1, W)$ such that $q(m) = 0$. Let $m = (m_S^1, m_{N\backslash S}^0)$ where $S = N\backslash D$ and $N\backslash S = D$. Take $i \in S$, a conformist agent. By conformity relative to W_i^h and since $N\backslash S \cup \{i\} \notin W_i^h$, $q(m_S^1, m_{N\backslash S}^0) P_i^1 q(m_i^0, m_{S\backslash\{i\}}^1, m_{N\backslash S}^0)$.

Case 2b. $n + 1 - h \leq h - 1$. Let then $d = n + 1 - h$. Take any agent $i \in N\backslash D$. For any $m = (m_S^1, m_{N\backslash S}^0) \in UN((M,q), R^1, W)$ we have that $D \subseteq S$. Since $s \geq n + 1 - h$ or equivalently, $n - s \leq h - 1$ and by conformity of agent $i \in N\backslash S$ relative to W_i^h, we have that $q(m_i^1, m_S^1, m_{N\backslash S\cup\{i\}}^0) R_i^1 q(m_S^1, m_{N\backslash S}^0)$. By selfishness, $q(m_i^1, m_{-i}^1) P_i^1 q(m_i^0, m_{-i}^1)$ and m_i^0 is weakly dominated by m_i^1 for any $i \in N\backslash D$. Therefore, the result follows.

Finally, if $d < n + 1 - h \leq h - 1$, we show that there exists $m \in UN((M,q), R^1, W)$ such that $q(m) = 0$. Let $m = (m_S^1, m_{N\backslash S}^0)$ where $S = D$ and $N\backslash S = N\backslash D$. Take $i \in N\backslash S$, a conformist agent. By conformity relative to W_i^h and since $S \cup \{i\} \notin W_i^h$, $q(m_S^1, m_{N\backslash S}^0) P_i^1 q(m_i^1, m_S^1, m_{N\backslash S\cup\{i\}}^0)$. ■

In Theorem 3, we show the number of independent agents d guaranteeing the socially optimal decision when the quota is $q \in \{2, .., n-1\}$.

Theorem 3 ($\mathbf{q} \in \{2, .., n-1\}$). *Given any $W = (W_D^1, W_{N\backslash D}^h)$, for $q \in \{2, ..., n-1\}$, any undominated Nash equilibrium yields to the socially optimal decision if*

$$d = \min\{n-1, \max\{q-1, n-q\} + \min\{h-1, n+1-h\}\}.$$

Proof. Let $W = (W_D^1, W_{N\backslash D}^h)$. We now show that if $d = \min\{n-1, \max\{q-1, n-q\} + \min\{h-1, n+1-h\}\}$, the socially optimal decision is obtained in equilibrium for any $q \in \{2, .., n-1\}$ and any $t \in \{0,1\}^n$. To do that, we show that for any t_i, $m_i \neq t_i$ is weakly dominated by $m_i^{t_i}$ for any $i \in N\backslash D$. By assumption, any conformist agent knows that there are d independent agents, knows their opinion and also that they are telling the truth. For clarifying purposes, we divide the proof in four cases depending on the quota.

Case 1. $q \in \{2, ..., \lfloor \frac{n+1}{2} \rfloor - 1\}$. Then, $q - 1 < n - q$. We distinguish two cases:

Case 1.1. $q - 1 \leq \min\{h-1, n+1-h\}$**.** Then, $q \leq h$. Let $d = n-1$, and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. Let agent i be the only conformist agent.

If $c < q-1$, we have that $q(m_i^1, m_{-i}) = q(m_i^0, m_{-i}) = 0$. Since $c < q-1 \leq h-1$ and by conformity relative to W_i^h, $q(m_i^0, m_{-i}) R_i^{t_i} q(m_i^1, m_{-i})$.

If $c = q-1$ and $t_i = 1$ or $c = q$ and $t_i = 0$, we have that $q(m_i^1, m_{-i}) = q(m_i^0, m_{-i}) = 0$. Since $c = q-1 \leq h-1$ and by conformity relative to W_i^h, $q(m_i^0, m_{-i}) R_i^{t_i} q(m_i^1, m_{-i})$.

If $c = q-1$ and $t_i = 0$, by selfishness $0 = q(m_i^0, m_C^1, m_{N\backslash C\cup\{i\}}^0) P_i^0 q(m_i^1, m_C^1, m_{N\backslash C\cup\{i\}}^0) = 1$.

If $c = q$ and $t_i = 1$, by selfishness $q(m_i^1, m_{C\backslash\{i\}}^1, m_{N\backslash C}^0) P_i^1 q(m_i^0, m_{C\backslash\{i\}}^1, m_{N\backslash C}^0)$.

If $c > q$, we have that $q(m_i^1, m_{-i}) = q(m_i^0, m_{-i}) = 1$. If $c < h$, by conformity relative to W_i^h, $q(m_i^0, m_{-i}) R_i^{t_i} q(m_i^1, m_{-i})$. If $c \geq h$, by conformity relative to W_i^h, $q(m_i^1, m_{-i}) I_i^{t_i} q(m_i^0, m_{-i})$.

Finally, suppose that $d < n-1$. Let $d = n-2$ and suppose w.l.o.g. that $1, 2 \in N\backslash D$. Let also $t = (t_C^1, t_{N\backslash C}^0)$ be such that $c = q$, and $t_1 = t_2 = 1$. For $m = (m_1^0, m_2^0, m_{-\{1,2\}})$ such that for any $k \in D$, $m_k = t_k$, and $i, j \in \{1, 2\}$, we have that $q(m_i^0, m_j^0, m_{-\{i,j\}}) R_i^1 q(m_i^1, m_j^0, m_{-\{i,j\}})$.

Case 1.2. $q - 1 > \min\{h-1, n+1-h\}$**.** We distinguish two subcases:

Subcase 1.2.a. $h - 1 < n + 1 - h$**.** Then, $h \leq \frac{n}{2}$ and $q > h$. Let $d = n-1-(q-h)$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. Let $n - d = 1 + q - h$ be the group of conformist agents.

If $c < q$, there are at most $q-1$ independent agents voting for 1, and then there are at least $h-1$ independent agents voting for 0. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 0 for any conformist agent such that $t_i = 0$.

If $c \geq q$, there are at least $h-1$ independent agents voting for 1. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 1 for any conformist agent such that $t_i = 1$.

Finally, suppose a case where the number of independent agents is lower than d. Let $d = n-2-(q-h)$ and suppose $t = (t_C^1, t_{N\backslash C}^0)$ is such that for $h-2$ independent agents $t_i = 1$, for $n-q$ independent agents $t_j = 0$ and all conformist agents are such that $t_k = 1$. Note that the socially optimal decision is 1. Let m be such that all independent agents tell the truth, and the conformist agents all report $m_k = 0$. Since $h \leq \frac{n}{2}$, by conformity, $0 = q(m_k^0, m_{-k}) P_k^1 q(m_k^1, m_{-k}) = 0$.

Subcase 1.2.b. $h - 1 \geq n + 1 - h$**.** Then, $h > \frac{n}{2}$ and $q < h$. Let $d = n-q+n-h+1$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. Let $n - d = h - n + q - 1$ be the group of conformist agents.

If $c < q$, there are at most $q - 1 < h - 1$ independent agents voting for 1. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 0 for any conformist agent such that $t_i = 0$.

If $c \geq q$, there are at least $c - (n-d) \geq n+1-h$ independent agents voting for 1, and then there are strictly less than $h-1$ independent agents voting for 0. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 1 for any conformist agent such that $t_i = 1$.

Finally, suppose a case where the number of independent agents is lower than d. Let $d = n-q+n-h$ and suppose $t = (t_C^1, t_{N\backslash C}^0)$ is such that for $n-h$ independent agents $t_i = 1$, for $n-q$ independent agents $t_j = 0$ and all conformist agents are such that $t_k = 1$. Note that the socially optimal decision is 1. Let m be such that all independent agents tell the truth, and the conformist agents all report $m_k = 0$. Then, since $h > \frac{n}{2}$ and by conformity, $0 = q(m_k^0, m_{-k})P_k^1 q(m_k^1, m_{-k}) = 0$.

Case 2. $q \in \{\lceil \frac{n+1}{2} \rceil + 1, ..., n-1\}$. Then, $q-1 > n-q$. We distinguish two cases:
Case 2.1. $n - q \leq \min\{h-1, n+1-h\}$. Then, $n-q \leq h-1 \leq q$.Let $d = n-1$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. Let agent i be the only conformist agent.
If $c < q-1$, we have that $q(m_i^1, m_{-i}) = q(m_i^0, m_{-i}) = 0$.
If $c = q-1$ and $t_i = 1$ or $c = q$ and $t_i = 0$, since $q-1 > n-q$ and by conformity relative to W_i^h, $q(m_i^1, m_{-i})R_i^{t_i} q(m_i^0, m_{-i})$.
If $c = q-1$ and $t_i = 0$, by selfishness $0 = q(m_i^0, m_C^1, m_{N\backslash C\cup\{i\}}^0)P_i^0 q(m_i^1, m_C^1, m_{N\backslash C\cup\{i\}}^0) = 1$.
If $c = q$ and $t_i = 1$, by selfishness $q(m_i^1, m_{C\backslash\{i\}}^1, m_{N\backslash C}^0)P_i^1 q(m_i^0, m_{C\backslash\{i\}}^1, m_{N\backslash C}^0)$.
If $c > q$, we have that $q(m_i^1, m_{-i}) = q(m_i^0, m_{-i}) = 1$. If $c < h$, at least q independent agents vote for 1 and at most $h-1$ agents vote for 1. Since $h \geq n+1-q$ and $h > q > n-q$, then $q(m_i^0, m_{-i})I_i^{t_i} q(m_i^1, m_{-i})$ as either voting for 0 or for 1, any conformist agents does not conform to h agents.
If $q < c < h$, at least q independent agents are voting for 1 and at most $h-1$ agents are voting for 1. As $h \geq n+1-q$ and $h > q > n-q$ then $q(m_i^0, m_{-i})I_i^{t_i} q(m_i^1, m_{-i})$ as either voting for 0 or for 1, any conformist agents does not conform to h agents.
If $q \leq h \leq c$, it could be either $q \leq h < c$ or $q < h \leq c$. If $q \leq h < c$, at least h independent agents are voting for 1. If $q < h \leq c$, at least $h-1$ independent agents are voting for 1. Then, by conformity $q(m_i^1, m_{-i})R_i^{t_i} q(m_i^0, m_{-i})$.
Finally, suppose that $d < n-1$. Let $d = n-2$ and suppose w.l.o.g. that $1, 2 \in N\backslash D$. Let also $t = (t_C^1, t_{N\backslash C}^0)$ be such that $c = q-1$, and $t_1 = t_2 = 0$. For $m = (m_1^1, m_2^1, m_{-\{1,2\}})$ such that for any $k \in D$, $m_k = t_k$, and $i, j \in \{1, 2\}$, since $n-2-(q-1) = n-q-1$ and $q-1 > n-q$, we have that $q(m_i^1, m_j^1, m_{-\{i,j\}})R_i^0 q(m_i^0, m_j^1, m_{-\{i,j\}})$.
Case 2.2. $n-q > \min\{h-1, n+1-h\}$**.** We distinguish two subcases:
Subcase 2.2.a. $h-1 < n+1-h$**.** Then, $h \leq \frac{n}{2}$ and $q > h$. Let $d = q-1+h-1$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. Let $n-d = n-q+1-h+1$ be the group of conformist agents.
If $c < q$, there are at most $q-1$ independent agents voting for 1 and there are at least $h-1$ independent agents voting for 0. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 0 for a conformist agent such that $t_i = 0$. For a conformist agent such that $t_i = 1$, they vote for 1 when the independent agents voting for 1 are equal or greater to $h-1$. Otherwise, by conformity, they vote for 0. For the particular case where $q-1$ independent agents vote for 1 and $h-1$ independent agents vote for 0, since $q > h$ all conformist agents such that $t_i = 0$ will vote for 0 and all conformist agents such that $t_i = 1$ will vote for 1 as $q > h$ either.
If $c \geq q$, there are at least $h-1$ independent agents voting for 1. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 1 for any conformist agent such that $t_i = 1$.
Finally, suppose a case where the number of independent agents is lower than d. Let $d = q-1+h-2$ and suppose $t = (t_C^1, t_{N\backslash C}^0)$ is such that for $h-1$ independent agents

$t_i = 0$, for $q-2$ independent agents $t_j = 1$ and all conformist agents are such that $t_k = 0$. Note that the socially optimal decision is 0. Let m be such that all independent agents tell the truth, and the conformist agents all report $m_k = 1$. Then, since $h \leq \frac{n}{2}$ and at least $q+1$ are voting for 1, by conformity, $q(m_k^1, m_{-k})P_k^1 q(m_k^0, m_{-k})$.
Subcase 2.2.b. $h-1 \geq n+1-h$**.** Note that $h > \frac{n}{2}$ and $h-1 > q$. Let $d = n+q-h$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. Let $n-d = h-q$ be the group of conformist agents.
If $c < q$, there are at most $q-1$ independent agents voting for 1. If all conformist agents vote for 1, then at most $h-1$ agents vote for 1. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 0 for any conformist agent such that $t_i = 0$.
If $c \geq q$, there are strictly more than $n+1-h$ independent agents voting for 1 and strictly less than $h-1$ independent agents voting for 0. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 1 for any conformist agent such that $t_i = 1$.
Finally, suppose a case where the number of independent agents is lower than d. Let $d = n+q-h-1$ and suppose $t = (t_C^1, t_{N\backslash C}^0)$ is such that for $q-1$ independent agents $t_i = 1$, for $n-h$ independent agents $t_j = 0$ and all conformist agents are such that $t_k = 0$. Note that the socially optimal decision is 0. Let m be such that all independent agents tell the truth, and the conformist agents all report $m_k = 1$. Then, since $q-1+n-[n+q-h-1] = h$ and by conformity, $q(m_k^1, m_{-k})P_k^0 q(m_k^0, m_{-k})$.

Case 3. n even and $q \in \{\frac{n}{2}, \frac{n}{2}+1\}$. Then, $n-q > q-1$. We distinguish two cases:
Case 3.1. $q-1 \leq \min\{h-1, n+1-h\}$**.** Then, $q \leq h$. Let $d = n-1$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. Let agent i be the only conformist agent.
If $c < q-1$, $q(m_i^1, m_{-i}) = q(m_i^0, m_{-i}) = 0$.
If $c = q-1$ and $t_i = 1$, by conformity relative to W_i^h and $q \leq h \leq q+1$, we have that $q(m_i^0, m_{-i})R_i^{t_i} q(m_i^1, m_{-i})$.
If $c = q-1$ and $t_i = 0$, by selfishness $q(m_i^0, m_C^1, m_{N\backslash C\cup\{i\}}^0)P_i^0 q(m_i^1, m_C^1, m_{N\backslash C\cup\{i\}}^0)$.
If $c = q$ and $t_i = 0$, by conformity relative to W_i^h, and $q \leq h \leq q+1$, we have that $q(m_i^0, m_{-i})R_i^{t_i} q(m_i^1, m_{-i})$.
If $c = q$ and $t_i = 1$, by selfishness $q(m_i^1, m_{C\backslash\{i\}}^1, m_{N\backslash C}^0)P_i^1 q(m_i^0, m_{C\backslash\{i\}}^1, m_{N\backslash C}^0)$.
If $c > q$, $q(m_i^1, m_{-i}) = q(m_i^0, m_{-i}) = 1$.
If $q < c < h$, at least q independent agents are saying 1 and at most $h-1$ agents are saying 1. Since $q \leq h \leq q+1$,by conformity we have that $q(m_i^0, m_{-i})R_i^{t_i} q(m_i^1, m_{-i})$.
If $q \leq h \leq c$, it could be either $q \leq h < c$ or $q < h \leq c$. If $q \leq h < c$, at least h independent agents are voting for 1. If $q < h \leq c$, at least $h-1$ independent agents are voting for 1. Then, by conformity $q(m_i^1, m_{-i})R_i^{t_i} q(m_i^0, m_{-i})$.
Finally, suppose that $d < n-1$. Let $d = n-2$ and suppose w.l.o.g. that $1, 2 \in N\backslash D$. Let also $t = (t_C^1, t_{N\backslash C}^0)$ be such that $c = q-1$, and $t_1 = t_2 = 1$. For $m = (m_1^0, m_2^0, m_{-\{1,2\}})$ such that for any $k \in D$, $m_k = t_k$, and $i, j \in \{1, 2\}$, since $q-1 < n-q$ we have that $q(m_i^0, m_j^0, m_{-\{i,j\}})R_i^1 q(m_i^1, m_j^0, m_{-\{i,j\}})$.
Case 3.2. $q-1 > \min\{h-1, n+1-h\}$**.** We distinguish two subcases:
Subcase 3.2.a. $h-1 < n+1-h$**.** Then, $h \leq \frac{n}{2}$ and $q > h$. Let $d = n-1-(q-h)$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. Let $n-d = q-h+1$ be the group of conformist agents.
If $c < q$, there are at most $q-1$ independent agents voting for 1, and there are at least $h-1$

independent agents voting for 0. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 0 for any conformist agent such that $t_i = 0$. For a conformist agent such that $t_i = 1$, they vote for 1 when the independent agents voting for 1 are enough to conform to at least $h-1$. Otherwise, by conformity, they vote for 0. For the particular case where $q-1$ independent agents vote for 1 and $h-1$ independent agents vote for 0, since $q > h$ all conformist agents such that $t_i = 0$ will vote for 0 and all conformist agents such that $t_i = 1$ will vote for 1 as $q > h$.

If $c \geq q$, there are at least $h-1$ independent agents voting for 1. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 1 for any conformist agent such that $t_i = 1$.

Finally, suppose a case where the number of independent agents is lower than d. Let $d = n-2-(q-h)$ and suppose $t = (t^1_C, t^0_{N\setminus C})$ is such that for $h-2$ independent agents $t_i = 1$, for $n-q$ independent agents $t_j = 0$ and all conformist agents are such that $t_k = 1$. Note that the socially optimal decision is 1. Let m be such that all independent agents tell the truth, and the conformist agents all report $m_k = 0$. Then, since $h \leq \frac{n}{2}$ and by conformity, $0 = q(m^0_k, m_{-k}) P^1_k q(m^1_k, m_{-k}) = 0$.

Subcase 3.2.b. $h-1 \geq n+1-h$**.** Then, $h > \frac{n}{2}$. Let $d = n-q+n-h+1$, and $t = (t^1_C, t^0_{N\setminus C}) \in \{0,1\}^n$. Let $n-d = q-n+h-1$ be the group of conformist agents.

If $c < q$, there are at most $q-1$ independent agents voting for 1. Then, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 0 for any conformist agent such that $t_i = 0$.

If $c \geq q$, there are at least $n+1-h$ independent agents voting for 1, and then there are strictly less than $n-q$ independent agents voting for 0. Given that $n-q > q-1$ and $q < h-1$, either by selfishness or by conformity, it is a weakly dominant strategy to vote for 0 for any conformist agent such that $t_i = 0$.

Finally, suppose a case where the number of independent agents is lower than d. Let $d = n-q+n-h$ and suppose $t = (t^1_C, t^0_{N\setminus C})$ is such that for $n-h$ independent agents $t_i = 1$, for $n-q$ independent agents $t_j = 0$ and all conformist agents are such that $t_k = 1$. Note that the socially optimal decision is 1. Let m be such that all independent agents tell the truth, and the conformist agents all report $m_k = 0$. Then, since $h > \frac{n}{2}$ and $q < h-1$, by conformity, $0 = q(m^0_k, m_{-k}) P^1_k q(m^1_k, m_{-k}) = 0$.

Case 4. n odd and $q = \frac{n+1}{2}$. Then, $n-q = q-1 = \frac{n-1}{2}$. We distinguish two cases:

Case 4.1. $\frac{n-1}{2} \leq \min\{h-1, n+1-h\}$**.** Then, $q \leq h$. Let $d = n-1$ and $t = (t^1_C, t^0_{N\setminus C}) \in \{0,1\}^n$. Let agent i be the only conformist agent.

If $c < q-1$, $q(m^1_i, m_{-i}) = q(m^0_i, m_{-i}) = 0$.

If $c = q-1$ and $t_i = 1$, by conformity relative to W^h_i and $q \leq h \leq q+1$, we have that $q(m^0_i, m_{-i}) R^{t_i}_i q(m^1_i, m_{-i})$.

If $c = q-1$ and $t_i = 0$, by selfishness $q(m^0_i, m^1_C, m^0_{N\setminus C\cup\{i\}}) P^0_i q(m^1_i, m^1_C, m^0_{N\setminus C\cup\{i\}})$.

If $c = q$ and $t_i = 0$, by conformity relative to W^h_i, and $q \leq h \leq q+1$, we have that $q(m^0_i, m_{-i}) R^{t_i}_i q(m^1_i, m_{-i})$.

If $c = q$ and $t_i = 1$, by selfishness $q(m^1_i, m^1_{C\setminus\{i\}}, m^0_{N\setminus C}) P^1_i q(m^0_i, m^1_{C\setminus\{i\}}, m^0_{N\setminus C})$.

If $c > q$, $q(m^1_i, m_{-i}) = q(m^0_i, m_{-i}) = 1$.

If $q \leq h \leq c$, it could be either $q \leq h < c$ or $q < h \leq c$. If $q \leq h < c$, at least h independent agents are voting for 1. If $q < h \leq c$, at least $h-1$ independent agents are voting for 1.

Then, by conformity $q(m_i^1, m_{-i})R_i^{t_i}q(m_i^0, m_{-i})$.
Finally, suppose that $d < n-1$. Let $d = n-2$ and suppose w.l.o.g. that $1, 2 \in N\backslash D$. Let also $t = (t_C^1, t_{N\backslash C}^0)$ be such that $c = q-1$, and $t_1 = t_2 = 1$. For $m = (m_1^0, m_2^0, m_{-\{1,2\}})$ such that for any $k \in D$, $m_k = t_k$, and $i, j \in \{1, 2\}$, since $q-1 < n-q$ we have that $q(m_i^0, m_j^0, m_{-\{i,j\}})R_i^1 q(m_i^1, m_j^0, m_{-\{i,j\}})$.
Case 4.2. $\frac{n-1}{2} > \min\{h-1, n+1-h\}$**.** We distinguish two subcases:
Subcase 4.2.a. $h-1 < n+1-h$**.** Then, $h \leq \frac{n}{2}$ and $q > h$. Let $d = n-1-(q-h)$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. This case is analogous to Subcase 3.2.a.
Subcase 4.2.b. $h-1 \geq n+1-h$**.** Then $h \geq \frac{n}{2}+1$. Let $d = n-q+n-h+1$ and $t = (t_C^1, t_{N\backslash C}^0) \in \{0,1\}^n$. This case is analogous to Subcase 3.2.b. ■

Corollary 1. *For any n, any $h \in [2, n]$, and any $q < q'$ such that $q' = n-q+1$, we have that the number of independent agents guaranteeing the socially optimal decision of q* and *q' is the same.*
Proof. Let *any n, any $h \in [2, n]$, and q, q' such that $q' = n-q+1$.* If $q = 1$ and $q' = n$, from Theorem 2 $d = \min\{h-1, n+1-h\}$, that does not depend on q. For any $q, q' \neq \{1, n\}$, from Theorem 3, $d = \min\{n-1, \max\{q-1, n-q\} + \min\{h-1, n+1-h\}\}$.
Let h be such that $2 \leq h < \frac{n}{2}+1$. Then, the number of independent agents is as follows:

$$d = \begin{cases} n-1 & \text{when } 2 \leq q \leq h \\ n-q+h-1 & \text{when } h \leq q \leq \frac{n}{2} \\ q-1+h-1 & \text{when } \frac{n}{2} \leq q \leq n-h+1 \\ n-1 & \text{when } n-h+1 \leq q \leq n-1 \end{cases}$$

If q is such that $2 \leq q \leq h$, we have that $n-h+1 \leq q' \leq n-1$ and the number of independent agents is $n-1$ for q, q'. If q is such that $h \leq q \leq \frac{n}{2}$, we have that $\frac{n}{2} < q' < n-h+1$ and the number of independent agents is $n-q+h-1 = q'-1+h-1$ for q, q'.

Let h be such that $\frac{n}{2}+1 \leq h \leq n$. Then, the number of independent agents is as follows:

$$d = \begin{cases} n-1 & \text{when } 2 \leq q \leq n+2-h \\ n-q+n+1-h & \text{when } n+2-h \leq q \leq \frac{n}{2} \\ q-1+n+1-h & \text{when } \frac{n}{2} < q < h-1 \\ n-1 & \text{when } h-1 \leq q \leq n-1 \end{cases}$$

If q is such that $2 \leq q \leq n+2-h$, we have that $h-1 \leq q' \leq n-2$ and the number of independent agents is $n-1$ for q, q'. If q is such that $n+2-h \leq q \leq \frac{n}{2}$, we have that $\frac{n}{2} < q' < h-1$ and the number of independent agents is $n-q+n+1-h = q'-1+n+1-h$ for q, q'. ■

Corollary 2. *For any n, any $q \in [1, n]$, and any $h < h'$ such that $h' = n+2-h$, we have that the number of independent agents guaranteeing the socially optimal decision under any quota q is the same for h* and *h'.*
Proof. Let *any n, any $q \in [1, n]$, and h, h' such that $h' = n+2-h$.* If $q = 1$ or $q = n$, from Theorem 2 $d = \min\{h-1, n+1-h\}$. In particular:

$$d = \begin{cases} h-1 & \text{when } 2 \leq h \leq \frac{n}{2}+1 \\ n+1-h & \text{when } \frac{n}{2}+1 \leq h \leq n \end{cases}$$

If h is such that $2 \leq h \leq \frac{n}{2}+1$ we have that $\frac{n}{2}+1 \leq h' \leq n$ and the number of independent agents is $h-1=n+1-h'$ for h, h'.
Let q be such that $q \in \{2, n-1\}$. According to Theorem 3, $d=\min\{n-1, \max\{q-1, n-q\}+\min\{h-1, n+1-h\}\}$.
Let q be such that $2 \leq q < \frac{n}{2}+1$. Then, the number of independent agents is as follows:

$$d = \begin{cases} n-q+h-1 & \text{when } 2 \leq h \leq q \\ n-1 & \text{when } q \leq h \leq n+2-q \\ n-q+n+1-h & \text{when } n+2-q \leq h \leq n \end{cases}$$

If h is such that $2 \leq h \leq q$ we have that $n+2-q \leq h' \leq n$ and the number of independent agents is $n-q+h-1=n-q+n+1-h'$ for h, h'.
Let q be such that $\frac{n}{2}+1 \leq q < n-1$. Then, the number of independent agents is as follows:

$$d = \begin{cases} q-1+h-1 & \text{when } 2 \leq h \leq n-q+1 \\ n-1 & \text{when } n-q+1 \leq h \leq q+1 \\ q-1+n+1-h & \text{when } q+1 \leq h \leq n \end{cases}$$

If h is such that $2 \leq h \leq n-q+1$ we have that $q+1 \leq h' \leq n$ and the number of independent agents is $q-1+h-1=q-1+n+1-h'$ for h, h'. $\blacksquare$

Appendix B

This appendix includes the proofs of Theorem 4, Theorem 5 and Theorem 6 developed in Section 4.
Theorem 4. Let $q=1$. *For any number of agents n,* the socially optimal decision obtains *in any subgame perfect Nash Equilibrium on $\mathcal{R}^h$.*

Proof. Let any n be any finite number of agents. Note that under quota $q=1$, if an agent is not pivotal relative to some $m_{-i} \in M_{-i}$, it means that for any $m_i, m_i' \in M_i$ we have that $q(m_i, m_{-i}) = q(m_i', m_{-i}) = 1$.
Let $W=(W_D^1, W_{N\backslash D}^h)$ and $R \in \mathcal{R}^h$ be such that $t_i=0$ for any $i \in N$. Then, the socially optimal decision is 0. We have to show that $SPN(G^{\{1,..,n\},1}) \neq \emptyset$ and that for any $m \in SPN(G^{\{1,..,n\},1})$, $q(m)=0$. We solve the game starting at stage n where agent n votes (remember that $o=(1,..,n)$).
Since R_n is such that $t_n=0$, in equilibrium m_n is such that for any $\widetilde{m}_i \in M_i$ where $i<n$, by selfishness $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n)=0$ if agent n is pivotal relative to $(\widetilde{m}_1,..,\widetilde{m}_{n-1})$, and by conformity relative to W_n^h, $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n)$ would be 0 or 1 depending on her committee structure if agent n is not pivotal. We proceed to stage k, that is, to agent $n-1$.
Applying iteratively the same reasoning we reach stage 1.
Again since R_1 is such that $t_1=0$, in equilibrium m_1 is such that $(m_2,..,m_n)$ as described above, by selfishness $\alpha_1(m_1,..,m_n)=0$ since agent 1 is pivotal relative to $(m_2,..,m_n)$. Note that this is the unique situation agent 1 can find, as it is the first stage of the game in which there is only one node.

Note that $q(m_1,..,m_n) = 0$. Then m as described above belongs to $SPN(G^{\{1,..,n\},1})$ and $q(m) = 0$.
Let $W = (W_D^1, W_{N\backslash D}^h)$ and $R \in \mathcal{R}^h$ be such that for some R_i, $t_i = 1$. Then, the socially optimal decision is 1. We have to show that $SPN(G^{\{1,..,n\},1}) \neq \emptyset$ and that for any $m \in SPN(G^{\{1,..,n\},1})$, $q(m) = 1$. As before, we solve the game starting at stage n where agent n votes.
Let, without loss of generality, agent k be the highest agent in order o for whom R_k is such that $t_k = 1$.
Since R_n is such that $t_n = 0$, in equilibrium m_n is such that for any $\widetilde{m}_i \in M_i$ where $i < n$, by selfishness $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n) = 0$ if agent n is pivotal relative to $(\widetilde{m}_1,..,\widetilde{m}_{n-1})$, and by conformity relative to W_n^h, $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n)$ would be 0 or 1 depending on her committee structure if agent n is not pivotal. We proceed to stage $n-1$, that is, to agent $n-1$.
Applying iteratively the same reasoning we reach stage k.
Since R_k is such that $t_k = 1$, in equilibrium m_k is such that for any $\widetilde{m}_i \in M_i$ where $i < k$ and $(m_{k+1},..,m_n)$, by selfishness $\alpha_k(\widetilde{m}_1,..,\widetilde{m}_{k-1}, m_k, m_{k+1},..,m_n) = 1$ if agent k is pivotal relative to $(\widetilde{m}_1,..,\widetilde{m}_{k-1}, m_{k+1},..,m_n)$ and by conformity relative to W_k^h, $\alpha_k(\widetilde{m}_1,..,\widetilde{m}_{k-1}, m_k, m_{k+1},..,m_n)$ would be 0 or 1 depending on her committee structure if agent k is not pivotal. We proceed to stage $k-1$, that is, to agent $k-1$.
Note that for any $\widetilde{m}_i \in M_i$ where $i < k$, $q(\widetilde{m}_1,..,\widetilde{m}_{k-1}, m_k,..,m_n) = 1$. Therefore, any $i < k$ is not pivotal relative to any $(\widetilde{m}_1,..,\widetilde{m}_{i-1}, \widetilde{m}_{i+1},..,\widetilde{m}_{k-1}, m_k,..,m_n)$ for any $\widetilde{m}_j \in M_j$ for $j \neq i$, $j < k$. In equilibrium, for any $i < k$, m_i would be such that by conformity relative to W_i^h, agent i votes according to her committee structure at any node in which she plays. Then m as described above belongs to $SPN(G^{\{1,..,n\},1})$ and $q(m) = 1$. ■

Theorem 5. Let $q = n$. *For any number of agents n, the socially optimal decision obtains in any subgame perfect Nash Equilibrium on $\mathcal{R}^h$.*
Proof. Let any n be any finite number of agents. Note that under quota $q = n$, if an agent is not pivotal relative to some $m_{-i} \in M_{-i}$, it means that for any $m_i, m_i' \in M_i$ we have that $q(m_i, m_{-i}) = q(m_i', m_{-i}) = 0$.
Let $W = (W_D^1, W_{N\backslash D}^h)$ and $R \in \mathcal{R}^h$ be such that $t_i = 0$ for any $i \in N$. Then, the socially optimal decision is 0. We have to show that $SPN(G^{\{1,..,n\},n}) \neq \emptyset$ and that for any $m \in SPN(G^{\{1,..,n\},n})$, $q(m) = 0$. We solve the game starting at stage n where agent n votes (remember that $o = (1,..,n)$).
Let, without loss of generality, agent k be the highest agent in order o for whom R_k is such that $t_k = 0$.
Since R_n is such that $t_n = 1$, in equilibrium m_n is such that for any $\widetilde{m}_i \in M_i$ where $i < n$, by selfishness $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n) = 1$ if agent n is pivotal relative to $(\widetilde{m}_1,..,\widetilde{m}_{n-1})$, and by conformity relative to W_n^h, $\alpha_n(\widetilde{m}_1,..,\widetilde{m}_{n-1}, m_n)$ would be 0 or 1 depending on her committee structure if agent n is not pivotal. We proceed to stage $n-1$, that is, to agent $n-1$.
Applying iteratively the same reasoning we reach stage k.
Since R_k is such that $t_k = 0$, in equilibrium m_k is such that for any $\widetilde{m}_i \in M_i$ where $i < k$ and $(m_{k+1},..,m_n)$, by selfishness $\alpha_k(\widetilde{m}_1,..,\widetilde{m}_{k-1}, m_k, m_{k+1},..,m_n) = 0$ if agent k is pivotal relative to $(\widetilde{m}_1,..,\widetilde{m}_{k-1}, m_{k+1},..,m_n)$ and by conformity relative to W_k^h, $(\alpha_k\widetilde{m}_1,..,\widetilde{m}_{k-1}, m_k, m_{k+1},..,m_n)$ would be 0 or 1 depending on her committee structure if agent k is not

pivotal. We proceed to stage $k-1$, that is, to agent $k-1$.
Note that for any $\widetilde{m}_i \in M_i$ where $i < k$, $q(\widetilde{m}_1, .., \widetilde{m}_{k-1}, m_k, .., m_n) = 0$. Therefore, any $i < k$ is not pivotal relative to any $(\widetilde{m}_1, .., \widetilde{m}_{i-1}, \widetilde{m}_{i+1}, .., \widetilde{m}_{k-1}, m_k, .., m_n)$ for any $\widetilde{m}_j \in M_j$ for $j \neq i$, $j < k$. In equilibrium, for any $i < k$, m_i would be such that by conformity relative to W_i^h, agent i votes according to her committee structure at any node in which she plays. Then m as described above belongs to $SPN(G^{\{1,..,n\},n})$ and $q(m) = 0$.
Let $W = (W_D^1, W_{N \setminus D}^h)$ and $R \in \mathcal{R}^h$ be such that $t_i = 1$ for any $i \in N$. Then, the socially optimal decision is 1. We have to show that $SPN(G^{\{1,..,n\},n}) \neq \emptyset$ and that for any $m \in SPN(G^{\{1,..,n\},n})$, $q(m) = 1$. As before, we solve the game starting at stage n where agent n votes.
Since R_n is such that $t_n = 1$, in equilibrium m_n is such that for any $\widetilde{m}_i \in M_i$ where $i < n$, by selfishness $\alpha_n(\widetilde{m}_1, .., \widetilde{m}_{n-1}, m_n) = 1$ if agent n is pivotal relative to $(\widetilde{m}_1, .., \widetilde{m}_{n-1})$, and by conformity relative to W_n^h, $\alpha_n(\widetilde{m}_1, .., \widetilde{m}_{n-1}, m_n)$ would be 0 or 1 depending on her committee structure if agent n is not pivotal. We proceed to stage $n-1$, that is, to agent $n-1$.
Applying iteratively the same reasoning we reach stage 1.
Again since R_1 is such that $t_1 = 1$, in equilibrium m_1 is such that $(m_2, .., m_n)$ as described above, by selfishness $\alpha_1(m_1, .., m_n) = 1$ since agent 1 is pivotal relative to $(m_2, .., m_n)$. Note that this is the unique situation agent 1 can find, as it is the first stage of the game in which there is only one node.
Note that $q(m_1, .., m_n) = 1$. Then m as described above belongs to $SPN(G^{\{1,..,n\},n})$ and $q(m) = 1$. ■

Theorem 6 Let $q \in \{2, .., n-1\}$. *For any number of agents n, the socially optimal decision obtains in any subgame perfect Nash Equilibrium on $\mathcal{R}^h$.*

Proof. Let n be any finite number of agents and $q \in \{2, ..., n-1\}$. We first show that for any game with a given number of agents n and quota $q \in \{2, ..., n-1\}$, if agents play their equilibrium strategies, the reduced game obtained can be either $G^{\{1,...,k\},q}$ where $q = k$ or to $G^{\{1,...,k\},q}$ where $q = 1$, which will depend on the preferences of the agents.
Let $p \in \{1, .., n-1\}$ be any finite number of agents, $q \in \{2, ..., p-1\}$, and $R_i \in \mathcal{R}_i^h$ for any $i \in \{1, .., p\}$. We distinguish two claims depending on the preferences of agent p:
Claim a. R_p such that $t_p = 0$. In equilibrium m_p is such that for any $\widetilde{m}_i \in M_i$ where $i < p$, by selfishness $\alpha_p(\widetilde{m}_1, ..., \widetilde{m}_{p-1}, m_p) = 0$ if agent p is pivotal relative to $(\widetilde{m}_1, ..., \widetilde{m}_{p-1})$, and by conformity relative to W_p^h, $\alpha_p(\widetilde{m}_1, ..., \widetilde{m}_{p-1}, m_p)$ would be 0 or 1 depending on her committee structure if agent p is not pivotal. Then, the reduced game obtained after agent p plays her equilibrium strategy coincides with a game with agents 1 to $p-1$ and quota q, i.e. $G^{\{1,...,p-1\},q}$.
Claim b. R_p such that $t_p = 1$. In equilibrium m_p is such that for any $\widetilde{m}_i \in M_i$ where $i < p$, by selfishness $\alpha_p(\widetilde{m}_1, ..., \widetilde{m}_{p-1}, m_p) = 1$ if agent p is pivotal relative to $(\widetilde{m}_1, .., \widetilde{m}_{p-1})$, and by conformity relative to W_p^h, $\alpha_p(\widetilde{m}_1, ..., \widetilde{m}_{p-1}, m_p)$ would be 0 or 1 depending on her committee structure if agent p is not pivotal. Then, the reduced game obtained after agent p plays her equilibrium strategy coincides with a game with agents 1 to $p-1$ and quota $q-1$, i.e. $G^{\{1,...,p-1\},q-1}$.
We now solve the game starting at stage n where agent n votes (remember that $o = (1, ..., n)$).

Let $R \in \mathcal{R}^h$. We distinguish several cases depending on the number of agents supporting the proposal, that is $t_i = 1$.
Case 1. R such that $\#\{i \in N : t_i = 1\} < q-1$. We distinguish several situations depending on the preferences of the agents 2 to n. If R_k is such that $t_k = 0$ for $k = \{q+1, ..., n\}$, Claim a applies at stages n to $q+1$. Then, the reduced game obtained after agents n to $q+1$ play their equilibrium strategies coincides with a game with agents 1 to q and quota q, i.e. $G^{\{1,...,q\},q}$.
If R_q is such that $t_q = 0$, and R_j is such that $t_j = 1$ for exactly one agent $j \in \{q+1, ..., n\}$. Claim b applies when we reach stage j, otherwise Claim a applies. Then, the reduced game obtained after agents n to $q+1$ play their equilibrium strategies coincides with a game with agents 1 to q and quota $q-1$, i.e. $G^{\{1,...,q\},q-1}$. To solve stage q, we apply Claim a and the reduced game obtained consists of a game with agents 1 to $q-1$ and quota $q-1$, i.e. $G^{\{1,...,q-1\},q-1}$.
If R_{q-1} is such that $t_{q-1} = 0$, and R_j, R_k are such that $t_j = 1 = t_k$ for exactly two agents, $j, k \in \{q, ..., n\}$. Claim b applies when we reach stages j and k, otherwise Claim a applies. Then, the reduced game obtained after agents n to q play their equilibrium strategies coincides with a game with agents 1 to $q-1$ and quota $q-2$, i.e. $G^{\{1,...,q-1\},q-2}$. To solve stage $q-1$, we apply Claim a and the reduced game obtained consists of a game with agents 1 to $q-2$ and quota $q-2$, i.e. $G^{\{1,...,q-2\},q-2}$.
By a similar reasoning, we end up in a case in which R_3 is such that $t_3 = 0$, and R_j is such that $t_j = 1$ for exactly $q-2$ agents $j \in \{4, ..., n\}$. Claim b applies in the stages in which each of the above $q-2$ agents play, otherwise Claim a applies. Then, the reduced game obtained after agents n to 4 play their equilibrium strategies coincides with a game with agents 1 to 3 and quota 2, i.e. $G^{\{1,2,3\},2}$. To solve stage 3, we apply Claim a and the reduced game obtained consists of a game with agents 1 and 2 and quota 2, i.e. $G^{\{1,2\},2}$.
In all of the above cases, for the obtained reduced games the quota coincides with the number of agents remaining in the game. Applying Theorem 5, the result follows.
Case 2. R such that $q < \#\{i \in N : t_i = 1\}$. We distinguish several situations depending on the preferences of the agents 2 to n. If R_k is such that $t_k = 1$ for $k = \{n+2-q, ..., n\}$, Claim b applies at stages n to $n+2-q$. Then, the reduced game obtained after agents n to $n+2-q$ play their equilibrium strategies coincides with a game with agents 1 to $n+1-q$ and quota 1, i.e. $G^{\{1,...,n+1-q\},1}$.
If R_{n+1-q} is such that $t_{n+1-q} = 1$, and R_j is such that $t_j = 0$ for exactly one agent $j \in \{n+2-q, ..., n\}$. Claim a applies when we reach stage j, otherwise Claim b applies. Then, the reduced game obtained after agents n to $n+2-q$ play their equilibrium strategies coincides with a game with agents 1 to $n+1-q$ and quota 2, i.e. $G^{\{1,...,n+1-q\},2}$. To solve stage $n+1-q$, we apply Claim b and the reduced game obtained consists of a game with agents 1 to $n-q$ and quota 1, i.e. $G^{\{1,...,n-q\},1}$.
If R_{n-q} is such that $t_{n-q} = 1$, and R_j, R_k are such that $t_j = 0 = t_k$ for exactly two agents, $j, k \in \{n+1-q, ..., n\}$. Claim a applies when we reach stages j and k, otherwise Claim b applies. Then, the reduced game obtained after agents n to $n+1-q$ play their equilibrium strategies coincides with a game with agents 1 to q and quota 2, i.e. $G^{\{1,...,n+1-q\},2}$. To solve stage $n-q$, we apply Claim b and the reduced game obtained consists of a game with agents 1 to $n-q-1$ and quota 1, i.e. $G^{\{1,...,n-q-1\},1}$.

By a similar reasoning, we end up in a case in which R_3 is such that $t_3 = 1$, and R_j is such that $t_j = 0$ for exactly $n-q-1$ agents $j \in \{4, ..., n\}$. Claim a applies in the stages in which each of the above $n-q-1$ agents play, otherwise Claim b applies. Then, the reduced game obtained after agents n to 4 play their equilibrium strategies coincides with a game with agents 1 to 3 and quota 2, i.e. $G^{\{1,2,3\},2}$. To solve stage 3, we apply Claim b and the reduced game obtained consists of a game with agents 1 and 2 and quota 1, i.e. $G^{\{1,2\},1}$.
In all of the above cases, for the obtained reduced games the quota is 1. Applying Theorem 4, the result follows.
Case 3. R such that $q-1 \leq \#\{i \in N : t_i = 1\} \leq q$. We distinguish two subcases.
Subcase 3.1. $n-q < q-1$. We distinguish several situations depending on the preferences of the agents 2 to n. If R_k is such that $t_k = 0$ for $k = \{q+1, ..., n\}$, Claim a applies at stages n to $q+1$. Then, the reduced game obtained after agents n to $q+1$ play their equilibrium strategies coincides with a game with agents 1 to q and quota q, i.e. $G^{\{1,...,q\},q}$.
If R_{r+1} is such that $t_{r+1} = 0$, and R_j is such that $t_j = 1$ for exactly $q-r$ agents, $q > r > 1$, $j \in \{r+2, ..., n\}$. Claim b applies in the stages in which each of the above $q-r$ agents play, otherwise Claim a applies. Then, the reduced game obtained after agents n to $r+2$ play their equilibrium strategies coincides with a game with agents 1 to $r+1$ and quota r, i.e. $G^{\{1,...,r+1\},r}$. To solve stage $r+1$, we apply Claim a and the reduced game obtained consists of a game with agents 1 to r and quota r, i.e. $G^{\{1,...,r\},r}$. Theorem 5 applies.
If R_2 is such that $t_2 = 0$, and R_j is such that $t_j = 1$ for exactly $q-1$ agents, $j \in \{3, ..., n\}$. Claim b applies in the stages in which each of the above $q-1$ agents play, otherwise Claim a applies and the reduced game obtained consists of a game with agents 1 and 2 and quota 1, i.e. $G^{\{1,2\},1}$.
Subcase 3.2. $n-q > q-1$. We distinguish several situations depending on the preferences of the agents 2 to n. If R_k is such that $t_k = 1$ for $k = \{n+2-q, ..., n\}$, Claim a applies at stages n to $n+2-q$. Then, the reduced game obtained after agents n to $n+2-q$ play their equilibrium strategies coincides with a game with agents 1 to $n+1-q$ and quota 1, i.e. $G^{\{1,...,n+1-q\},1}$.
If R_{r+1} is such that $t_{r+1} = 1$, and R_j is such that $t_j = 0$ for exactly $n-q-r$ agents, $n-q > r > 0$, $j \in \{r+2, ..., n\}$. Claim a applies in the stages in which each of the above $n-q-r$ agents play, otherwise Claim b applies. Then, the reduced game obtained after agents n to $r+2$ play their equilibrium strategies coincides with a game with agents 1 to $r+1$ and quota r, i.e. $G^{\{1,...,r+1\},2}$. To solve stage $r+1$, we apply Claim b and the reduced game obtained consists of a game with agents 1 to r and quota 1, i.e. $G^{\{1,...,r\},1}$. Theorem 4 applies.
If R_3 is such that $t_3 = 1$, and R_j is such that $t_j = 0$ for exactly $n-q-r$ agents, $j \in \{3, ..., n\}$. Claim a applies in the stages in which each of the above $n-q-r$ agents play, otherwise Claim b applies and the reduced game obtained consists of a game with agents 1 and 2 and quota 1, i.e. $G^{\{1,2\},1}$.
Subcase 3.3. $n-q = q-1$. We distinguish several situations depending on the preferences of the agents 2 to n.
If R_{r+1} is such that $t_{r+1} = 0$, and R_j is such that $t_j = 1$ for exactly $q-r$ agents, $q > r > 1$, $j \in \{r+2, ..., n\}$. Claim b applies in the stages in which each of the above $q-r$ agents play, otherwise Claim a applies. Then, the reduced game obtained after agents n to $r+2$ play

their equilibrium strategies coincides with a game with agents 1 to $r+1$ and quota r, i.e. $G^{\{1,...,r+1\},r}$. To solve stage $r+1$, we apply Claim a and the reduced game obtained consists of a game with agents 1 to r and quota r, i.e. $G^{\{1,...,r\},r}$. Theorem 5 applies.
If R_{r+1} is such that $t_{r+1} = 1$, and R_j is such that $t_j = 0$ for exactly $n-q-r$ agents, $n-q > r > 0$, $j \in \{r+2,...,n\}$. Claim a applies in the stages in which each of the above $n-q-r$ agents play, otherwise Claim b applies. Then, the reduced game obtained after agents n to $r+2$ play their equilibrium strategies coincides with a game with agents 1 to $r+1$ and quota r, i.e. $G^{\{1,...,r+1\},2}$. To solve stage $r+1$, we apply Claim b and the reduced game obtained consists of a game with agents 1 to r and quota 1, i.e. $G^{\{1,...,r\},1}$. Theorem 4 applies.
Case 1, 2 and 3 covers all the cases and then the result follows. ■

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